University of Toronto

FACULTY OF APPLIED SCIENCE AND ENGINEERING

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CONTENTS

THE THEORETICAL FOUNDATION OF THE QUATERNARY LINEAR REAL TRANSFORMATION CONNECTING INPUT AND OUTPUT QUANTITIES IN THE GENERAL ELECTRIC CIRCUIT TOGETHER WITH ITS APPLICATION BY ALGEBRAIC AND GEOMETRICAL METHODS

By T. R. ROSEBRUGH[1]

SYNOPSIS.—This paper deals mainly with a more general and systematic method o handling certain classes of problems involving steady state transmission calculations in terms of real quantities than has hitherto been used.

Part I is introductory, however, and shows that the binary linear transformation which the writer showed in 1919 to exist for all ladder circuits in terms of complex quantities, and with determinant unity, occurs also in operational form for the general input-output network and likewise with determinant unity, thus including sine-wave as a particular case.

Part II views the general case of input-output circuits under sine wave conditions as a quaternary linear transformation in real quantities, likewise with determinant unity, but subject to a certain condition ("absolute covariant").

The subject matter involves on equal terms voltage-squared (z), current-squared (w), power (x), reactive voltamperes (y) for both input and output ends in the general circuit, and in the special case of a transmission line per se it also introduces mean values of these quantities throughout the length of the line, suggesting the engineering importance of some of them.

For this latter purpose the sixteen real coefficients of the transformation are given as functions of s the length of the line leading to their mean values as tabulated.

Twenty identities involving values of the "a-set" of coefficients are given which are useful for checking numerical values and simplifying unnecessarily cumbrous results.

The relations between the "a-set" and the "b-set" coefficients are given. The diagram of the admittance-plane is used and it is pointed out that it is also a three-origin vector diagram, the exact interpretation being given. A table of statistical data about the points of a "kite framework" shows the proportionate values of each of the eight fundamental quantities at each of these points in extremely simple form. There are eight fundamental circles in the plane and 28 families of simple ratio-loci (circles) each family having its own radical axis. About half of these have obviously useful conceptions either in power or in communication engineering.

A wider point of view is now introduced in which linear functions of the eight fundamental quantities are dealt with. A classification is made of problems of complete and incomplete data. The former may or may not involve maximum conditions. Problems of maxima may be one-conditioned or two-conditioned.

The type-cases of each are studied and the other cases reduced to the type.

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One-conditioned maxima are associated graphically with (Poncelet's) limit-points and analytically with a matrix which defines a line and three circles intersecting in these limit-points.

Two-conditioned maxima are represented graphically by the points of intersection of a circle cutting three others orthogonally, with one of the circles. Analytically the determinantal equation, rules for whose formation are given in each case, gives the radical centre and the radius of the circle intersecting the other orthogonally.

Two simple examples are worked out algebraically and arithmetically. *Acknowledgment is made of the assistance of Mr. V. G. Smith and Mr. G. DeB. Robinson.*

PART I

The present paper deals with the general case of an electric system with input and output terminals, having constant coefficients, when in the steady state. It has reference mainly to a system of calculation in terms of real qualities by means of a quaternary linear transformation conditioned by a quadratic covariant.

While several papers,[1,2,3,4,5] dealing with certain problems of this kind in terms of real quantities, have already appeared, this paper is intended, within limits which will appear later, as a systematic study of the whole subject with a view to the greatest possible generality of treatment.

2. The binary substitution or transformation in complex quantities

$$\left.\begin{aligned} E_1 &= aE_0 + bI_0 \\ I_1 &= cE_0 + dI_0 \end{aligned}\right\} \qquad (1)$$

whereby two input quantities E_1 and I_1 are obtainable from the two output quantities E_0 and I_0 is now fairly well known.

The author has shown[6] that this transformation together with its inverse namely

$$\left.\begin{aligned} E_0 &= dE_1 - bI_1 \\ I_0 &= -cE_1 + aI_1 \end{aligned}\right\} \qquad (2)$$

would lead to many practical conclusions with regard to the behaviour of series and parallel combinations of transmission lines, transformers, tie

[1]H. B. Dwight, *Elec. Jour.*, Aug. 1914, Vol. XI, p. 487.

[2]R. D. Evans and H. K. Sels, *Power Limitations of Transmission Systems*, A. I. E. E. Trans., 1924, Vol. 43, p. 26.

[3]C. L. Fortescue and C. F. Wagner, *Some Theoretical Considerations of Power Transmission*, A. I. E. E. Trans., 1924, Vol. 43, p. 16.

[4]E. B. Shand, *The Limitations of Output of a Power System Involving Long Transmission Lines*, A. I. E. E. Trans., 1924, Vol. 43, p. 59.

[5]R. D. Evans and R. C. Bergvall, *Experimental Analysis of Stability and Power Limitations*, A. I. E. E. Trans., 1924, Vol. 43, p. 39.

[6]"*The Calculation of Transmission Line Networks*" Bulletin No. 1, 1919, School of Engineering Research, University of Toronto.

lines, etc., as well as theoretically at least to solve some problems on the general network of which the units consist of such items. Some of these facts about circuits have now become widely known through the publications[7] of the Westinghouse Company.

3. It is fundamental in the establishment of these formulas that the determinant $ad-bc$ is unity. In the paper of 1919 referred to this is proved for quite general circuits but still not so general as the case of the general circuit of constant coefficients, and of any number of meshes and any couplings between meshes. It will therefore be shown in the next section how the proof may be extended to include such cases.

4. For a circuit of any number of meshes and having in general localized resistance inductance, and capacity, and coupled in any manner by such means and by mutual inductance and having only two meshes in which external e.m.f. is impressed called respectively the input and the output, the n differential equations as is well known may be written

$$\sum_{s=1}^{n} f_{rs} q_s = e_r, \ (r=1, 2 \ldots n) \tag{3}$$

where
$$f_{rs} = f_{sr} = a_{rs} p^2 + b_{rs} p + c_{rs} \tag{4}$$

These are in fact direct consequences of "Lagrange's Equations of Motion in Generalized Coordinates" and here two only of the set e_r differ from zero, say e_2 and e_1 (input and output).

Now treating these equations as though they were algebraical instead of differential equations, and solving for q_1 and q_2

$$\begin{aligned} q_1 D &= e_1 F_{11} + e_2 F_{21} \\ q_2 D &= e_1 F_{12} + e_2 F_{22} \end{aligned} \tag{5}$$

in which the "F" quantities are the corresponding co-factors to the "f's" and D is the determinant of the "f's." Put $-e_1 = e_1'$.

Then

$$\left.\begin{aligned} e_2 &= \frac{F_{11}}{F_{21}} e_1' + \frac{D}{F_{21}} q_1 &&= a' e_1' + b' q_1 \\ q_2 &= \left(\frac{F_{11} F_{22}}{F_{21} D} - \frac{F_{12}}{D} \right) e_1' + \frac{F_{22}}{F_{21}} q_1 &&= c' e_1' + d' q_1 \end{aligned}\right\} \tag{6}$$

Then
$$\begin{vmatrix} a', & b' \\ c', & d' \end{vmatrix} = \frac{F_{12}}{F_{21}} = 1 \tag{7}$$

[7] "*Electrical Characteristics of Transmission Circuits*," by Westinghouse Engineers —William Nesbit, 1926.

Now put $$i_1 = pq_1,\ i_2 = pq_2$$

then
$$\left.\begin{aligned} e_2 &= a'e_1' + \frac{b'}{p}\, i_1 = ae_1' + bi_1 \\ i_2 &= pc'e_1' + d'i_1 = ce_1' + di_1 \end{aligned}\right\} \qquad (8)$$

Then
$$\begin{vmatrix} a, & b \\ c, & d \end{vmatrix} = 1 \qquad (9)$$

That is $ad - bc = 1$

Here a, b, c, and d are rational functions of p the time-differentiator. This relation true for all operational values of p will then hold for the special equivalent ωj. Thus the truth of this equation is established for the case of sine-wave.

PART II

5. (a) There are great advantages to be gained by proceeding in a uniform and symmetrical manner to develop the consequences of Equation (1) fully with regard to the values of four homogeneous real quantities at both ends of the circuit, namely voltage-squared, current-squared, power and reactive volt-amperes. Individual formulas have been used by previous writers but a consideration of the set of relations as a whole is the guiding idea in what follows.

Another advantage of the more general point of view here presented is that by the use of twenty algebraic identities thereby developed many expressions may be greatly simplified.

From the two Equations (1) expressing the binary linear transformation, may be deduced the quaternary linear transformation in real quantities which in combination with the equation of its covariant is to be the subject of Part II. The procedure is as follows:

From Equations (1) deduce Equations (10) by taking the conjugate of each quantity on the left hand side. Conjugates are here denoted by a circumflex.

Thus
$$\left.\begin{aligned} E_1 &= aE_0 + bI_0 \\ I_1 &= cE_0 + dI_0 \end{aligned}\right\} \text{(1) bis.} \quad \left.\begin{aligned} \hat{E}_1 &= \hat{a}\hat{E}_0 + \hat{b}\hat{I}_0 \\ \hat{I}_1 &= \hat{c}\hat{E}_0 + \hat{d}\hat{I}_0 \end{aligned}\right\} \qquad (10)$$

Hence, expressing the four products obtained by taking each of the quantities on the left of one set multiplied into each of the like quantities of the other, four equations are obtained giving values in terms of similar quantities from the right hand side. The coefficients will be sixteen quantities similar to $a\hat{a}$.

On making the substitutions
$$\begin{aligned} &E_0\hat{E}_0 = z,\ I_0\hat{I}_0 = w,\ I_0\hat{E}_0 = x + yj,\ \hat{I}_0E_0 = x - yj \qquad (11) \\ &E_1\hat{E}_1 = z',\ I_1\hat{I}_1 = w',\ I_1\hat{E}_1 = x' + y'j,\ \hat{I}_1E_1 = x' - y'j \end{aligned}$$

the quaternary linear real transformation is obtained in the form

$$
\begin{aligned}
z' &= a_{11}z + a_{12}w + a_{13}x + a_{14}y \\
w' &= a_{21}z + a_{22}w + a_{23}x + a_{24}y \\
x' &= a_{31}z + a_{32}w + a_{33}x + a_{34}y \\
y' &= a_{41}z + a_{42}w + a_{43}x + a_{44}y
\end{aligned}
\tag{12}
$$

The sixteen coefficients may be expressed as follows (if $a = a_0 + a_1 j$, $b = b_0 + b_1 j$, $c = c_0 + c_1 j$, $d = d_0 + d_1 j$)

$$
\begin{array}{ll}
a_{11} = a_0^2 + a_1^2 & a_{12} = b_0^2 + b_1^2 \\
a_{13} = 2(a_0 b_0 + a_1 b_1) & a_{14} = 2(b_0 a_1 - a_0 b_1) \\
a_{21} = c_0^2 + c_1^2 & a_{22} = d_0^2 + d_1^2 \\
a_{23} = 2(c_0 d_0 + c_1 d_1) & a_{24} = 2(d_0 c_1 - c_0 d_1) \\
a_{31} = c_0 a_0 + c_1 a_1 & a_{32} = d_0 b_0 + d_1 b_1 \\
a_{33} = 1 + 2(a_1 d_1 + b_0 c_0) & a_{34} = 2(d_0 a_1 - b_1 c_0) \\
a_{41} = a_0 c_1 - c_0 a_1 & a_{42} = b_0 d_1 - d_0 b_1 \\
a_{43} = 2(a_0 d_1 - b_1 c_0) & a_{44} = 1 + 2(a_1 d_1 - b_1 c_1)
\end{array}
\tag{13}
$$

Again starting from the inverse binary transformation (2) the inverse quaternary transformation is found which may be denoted by

$$
\begin{aligned}
z &= b_{11}z' + b_{12}w' + b_{13}x' + b_{14}y' \\
w &= b_{21}z' + b_{22}w' + b_{23}x' + b_{24}y' \\
x &= b_{31}z' + b_{32}w' + b_{33}x' + b_{34}y' \\
y &= b_{41}z' + b_{42}w' + b_{43}x' + b_{44}y'
\end{aligned}
\tag{14}
$$

The values of the b-set when found are seen to be connected with the a-set by the following very simple relations:

$$
\begin{array}{l|l|l|l}
b_{11} = a_{22}, & b_{12} = a_{12}, & b_{13} = -2a_{32}, & b_{14} = -2a_{42} \\
b_{21} = a_{21}, & b_{22} = a_{11}, & b_{23} = -2a_{31}, & b_{24} = -2a_{41} \\
b_{31} = -\tfrac{1}{2}a_{23}, & b_{32} = -\tfrac{1}{2}a_{13}, & b_{33} = a_{33}, & b_{34} = a_{43} \\
b_{41} = -\tfrac{1}{2}a_{24}, & b_{42} = -\tfrac{1}{2}a_{14}, & b_{43} = a_{34}, & b_{44} = a_{44}
\end{array}
\tag{15}
$$

These relations are for the most general input-output circuit and naturally lead to simpler ones where the circuit is symmetrical as in the case of the transmission line per se or the transmission line with similar transformers at both ends.

This may easily be shown to be identical with the cases where $a = d$.

In this case of input-output symmetry then the following additional relations hold:

$$
\begin{aligned}
&a_{11} = a_{22},\ a_{13} = 2a_{32},\ a_{14} = 2a_{42} \\
&a_{23} = 2a_{31},\ a_{24} = 2a_{41},\ a_{34} = a_{43}
\end{aligned}
\tag{16}
$$

So that in this case the upper left and lower right quarters of the tables of the a-set and the b-set contain identical coefficients. The other eight pairs of a and b coefficients differ only in sign. This difference

in sign is due to the convention with regard to current which is reckoned by an input rule of sense, at one end, and output at the other. The input rule of sense at both ends in the symmetrical circuit would identify completely the two sets of coefficients.

5. (*b*) As an example the following are the values of the sixteen a-coefficients for the Paugan Falls-Leaside (Toronto) 25-cycle transmission line of the Hydro-Electric Power Commission of Ontario, the length of which is 229 miles, where the units are:—100,000 volts, 100 amperes, 10,000 kv-a.

a_{11}	a_{12}	a_{13}	a_{14}
0.961,71	0.006,438,0	0.053,583	−0.147,97
a_{21}	a_{22}	a_{23}	a_{24}
0.257,22	0.961,71	0.004,677,8	0.994,70
a_{31}	a_{32}	a_{33}	a_{34}
0.002,338,9	0.026,791	1.000,031,82	0.013,673
a_{41}	a_{42}	a_{43}	a_{44}
0.497,35	−0.073,984	0.013,673	0.923,38

These have been checked by a set of 10 identities.

6. Determinants. According to a theorem in determinants imputed to Kronecker by E. Pascal[8] the determinant of sixteen coefficients of type $a\ \hat{a}$ must be unity since it would be the product of the squares of the determinants of (1) and (10).

The determinant of the sixteen real coefficients a_{11} to a_{44} may then be shown to be unity also.

7. A covariant under the transformation.

There are fundamental facts with regard to the translation from complex form to real quantities expressible as follows:

$$\begin{aligned} wz &= I_0\hat{I}_0 . E_0\hat{E}_0 = I_0\hat{E}_0 . \hat{I}_0E_0 \\ &= (x+yj)\ (x-yj) = x^2+y^2 \qquad (17) \\ w'z' &= I_1\hat{I}_1 . E_1\hat{E}_1 = I_1\hat{E}_1 . \hat{I}_1E_1 \\ &= (x'+y'j)\ (x'-y'j) = x'^2+y'^2 \end{aligned}$$

that is

$$wz\ -x^2-y^2=0 \qquad (18)$$

$$w'z'-x'^2-y'^2=0 \qquad (19)$$

---

[8]E. Pascal, "*I Determinanti*," Milan 1897. But it is not to be found in Kronecker's "Werke."

This suggests and it is in fact true that $wz - x^2 - y^2$ is an absolute co-variant under the transformation (12). In ordinary language the situation is that the four Equations (12) when applied to any set of $zwxy$ will produce another possible (consistent) set of values for the b-end.

8. Consequences in twenty identities. Useful for checks of values of co-efficients and for simplifying formulas.

Since a, b, c, and d are complex quantities they are defined by eight real numbers, which, however, since $ad - bc = 1$ must be equivalent to six independent real parameters on which the sixteen coefficients of the a-set depend.

There must therefore be ten independent relations between these sixteen coefficients. These ten identities are then to be found by noting the conditions which are consequences of the covariance of $wz - x^2 - y^2$.

By substituting for $w'z' - x^{2\prime} - y^{2\prime}$ in terms of w, z, x and y by (12) and identifying the result with $wz - x^2 - y^2$ the following ten identities are obtained:

$$a_{31}^2 + a_{41}^2 - a_{11}a_{21} = 0 \tag{20}$$
$$a_{32}^2 + a_{42}^2 - a_{12}a_{22} = 0 \tag{21}$$
$$a_{33}^2 + a_{43}^2 - a_{13}a_{23} = 1 \tag{22}$$
$$a_{34}^2 + a_{44}^2 - a_{14}a_{24} = 1 \tag{23}$$
$$a_{11}a_{22} + a_{12}a_{21} - 2a_{31}a_{32} - 2a_{41}a_{42} = 1 \tag{24}$$
$$a_{11}a_{23} + a_{13}a_{21} - 2a_{31}a_{33} - 2a_{41}a_{43} = 0 \tag{25}$$
$$a_{11}a_{24} + a_{14}a_{21} - 2a_{31}a_{34} - 2a_{41}a_{44} = 0 \tag{26}$$
$$a_{12}a_{23} + a_{13}a_{22} - 2a_{32}a_{33} - 2a_{42}a_{43} = 0 \tag{27}$$
$$a_{12}a_{24} + a_{14}a_{22} - 2a_{32}a_{34} - 2a_{42}a_{44} = 0 \tag{28}$$
$$a_{13}a_{24} + a_{14}a_{23} - 2a_{33}a_{34} - 2a_{43}a_{44} = 0 \tag{29}$$

Again by making similar use of (14) another set of ten identities as follows are obtained:

$$a_{23}^2 + a_{24}^2 - 4a_{21}a_{22} = 0 \tag{30}$$
$$a_{13}^2 + a_{14}^2 - 4a_{11}a_{12} = 0 \tag{31}$$
$$a_{33}^2 + a_{34}^2 - 4a_{31}a_{32} = 1 \tag{32}$$
$$a_{43}^2 + a_{44}^2 - 4a_{41}a_{42} = 1 \tag{33}$$
$$2a_{22}a_{11} + 2a_{12}a_{21} - a_{23}a_{13} - a_{24}a_{14} = 2 \tag{34}$$
$$2a_{22}a_{31} + 2a_{32}a_{21} - a_{23}a_{33} - a_{24}a_{34} = 0 \tag{35}$$
$$2a_{22}a_{41} + 2a_{42}a_{21} - a_{23}a_{43} - a_{24}a_{44} = 0 \tag{36}$$
$$2a_{12}a_{31} + 2a_{32}a_{11} - a_{13}a_{33} - a_{14}a_{34} = 0 \tag{37}$$
$$2a_{12}a_{41} + 2a_{42}a_{11} - a_{13}a_{43} - a_{14}a_{44} = 0 \tag{38}$$
$$2a_{32}a_{41} + 2a_{42}a_{31} - a_{33}a_{43} - a_{34}a_{44} = 0 \tag{39}$$

These evidently must be equivalent to the first set but are nevertheless as useful practically.

In obtaining formulas for particular conditions it often occurs that very great simplification in the result may be attained by their use. For example:

Suppose one wished to know the value of $y' \div z$ under conditions of maximum power x' received for fixed transmitting z. Its value is

$$a_{41}+a_{42}\div(4a_{32}^2)+(2a_{42}a_{31}-a_{43}a_{33}-a_{44}a_{34})\div(2a_{32})$$

But by using identity (39) it is found that all but the second term $a_{42}\div(4a_{32}^2)$ is identically zero.

These identities are also of value in checking the accuracy of the sixteen coefficients.

9. There may be instances where mean values of current-squared, voltage-squared, etc., along a transmission line are desired. The former is a measure of the conductor line loss as well as the line reactive volt-amperes due to inductance. The latter measures the reactive volt-amperes due to capacity as well as the insulation loss, if any, and may then perhaps serve also as a suitable comparative measure of the load to which the line insulators are subject if a single number is desired for this purpose. For this purpose the sixteen coefficients must be regarded not as the constants which they are in the case of the length of the whole line, $(s=1)$, but as functions of the variable s for the point in question. Hence the set of four fundamental quantities, z', w', x', y', are likewise functions of s. Then if bars above the symbols denote the corresponding mean quantities

$$\bar{z}=\int_0^1 z'ds=z\int_0^1 a_{11}ds+w\int_0^1 a_{12}ds$$
$$+x\int_0^1 a_{13}ds+y\int_0^1 a_{14}ds \tag{40}$$

similarly for the others, and if we write

$$\int_0^1 a_{11}ds=\bar{a}_{11} \text{ etc.}$$

then

$$\begin{aligned}\bar{z}&=\bar{a}_{11}z+\bar{a}_{12}w+\bar{a}_{13}x+\bar{a}_{14}y\\ \bar{w}&=\bar{a}_{21}z+\bar{a}_{22}w+\bar{a}_{23}x+\bar{a}_{24}y\\ \bar{x}&=\bar{a}_{31}z+\bar{a}_{32}w+\bar{a}_{33}x+\bar{a}_{34}y\\ \bar{y}&=\bar{a}_{41}z+\bar{a}_{42}w+\bar{a}_{43}x+\bar{a}_{44}y\end{aligned} \tag{41}$$

In order to find these sixteen new coefficients conveniently introduce the following abbreviations:

$$A_0+B_0j=\sqrt{(R+Xj)\ (G+Bj)} \tag{42}$$
$$m\ +nj\ =\sqrt{Z}\div\sqrt{Y} \tag{43}$$
$$p\ +qj\ =\sqrt{Y}\div\sqrt{Z} \tag{44}$$

$$\lambda=\tfrac{1}{2}(\cosh\ 2A_0s-\cos\ 2B_0s) \tag{45}$$

Then at any point s units from A where unity is the length of the line we have

$$a_{11}=\tfrac{1}{2}\cosh\ 2A_0s+\tfrac{1}{2}\cos\ 2B_0s \tag{46}$$
$$a_{12}=(m^2+n^2)\lambda \tag{47}$$
$$a_{13}=m\sinh\ 2A_0s-n\sin\ 2B_0s \tag{48}$$
$$a_{14}=-n\sinh\ 2A_0s-m\sin\ 2B_0s \tag{49}$$
$$a_{21}=(p^2+q^2)\lambda \tag{50}$$
$$a_{22}=a_{11} \tag{51}$$
$$a_{23}=p\sinh\ 2A_0s-q\sin\ 2B_0s \tag{52}$$
$$a_{24}=q\sinh\ 2A_0s+p\sin\ 2B_0s \tag{53}$$
$$a_{31}=\tfrac{1}{2}a_{23} \tag{54}$$
$$a_{32}=\tfrac{1}{2}a_{13} \tag{55}$$
$$a_{33}=a_{11}+(pm+qn)\lambda \tag{56}$$
$$a_{34}=(qm-pn)\lambda \tag{57}$$
$$a_{41}=\tfrac{1}{2}a_{24} \tag{58}$$
$$a_{42}=\tfrac{1}{2}a_{14} \tag{59}$$
$$a_{43}=a_{34} \tag{60}$$
$$a_{44}=a_{11}-(pm+qn)\lambda \tag{61}$$

10. If now $\bar{a}_{ij}=\int_0^1 a_{ij}ds$ and we put

$$\bar{\lambda}=\frac{1}{4A_0}\sinh\ 2A_0-\frac{1}{4B_0}\sin\ 2B_0 \tag{62}$$

we have mean values of the sixteen coefficients for the line:

$$\bar{a}_{11}=\frac{1}{4A_0}\sinh\ 2A_0+\frac{1}{4B_0}\sin\ 2B_0 \tag{63}$$

$$\bar{a}_{12}=(m^2+n^2)\bar{\lambda} \tag{64}$$

$$\bar{a}_{13}=\frac{m}{2A_0}(\cosh\ 2A_0-1)+\frac{n}{2B_0}(\cos\ 2B_0-1) \tag{65}$$

$$\bar{a}_{14}=-\frac{n}{2A_0}(\cosh\ 2A_0-1)+\frac{m}{2B_0}(\cos\ 2B_0-1) \tag{66}$$

$$\bar{a}_{21}=(p^2+q^2)\bar{\lambda} \tag{67}$$

$$\bar{a}_{22}=\bar{a}_{11} \tag{68}$$

$$\bar{a}_{23} = \frac{p}{2A_0}(\cosh 2A_0 - 1) + \frac{q}{2B_0}(\cos 2B_0 - 1) \tag{69}$$

$$\bar{a}_{24} = \frac{q}{2A_0}(\cosh 2A_0 - 1) - \frac{p}{2B_0}(\cos 2B_0 - 1) \tag{70}$$

$$\bar{a}_{31} = \tfrac{1}{2}\bar{a}_{23} \tag{71}$$

$$\bar{a}_{32} = \tfrac{1}{2}\bar{a}_{13} \tag{72}$$

$$\bar{a}_{33} = \bar{a}_{11} + (pm + qn)\bar{\lambda} \tag{73}$$

$$\bar{a}_{34} = (qm - pn)\bar{\lambda} \tag{74}$$

$$\bar{a}_{41} = \tfrac{1}{2}\bar{a}_{24} \tag{75}$$

$$\bar{a}_{42} = \tfrac{1}{2}\bar{a}_{14} \tag{76}$$

$$\bar{a}_{43} = \bar{a}_{34} \tag{77}$$

$$\bar{a}_{44} = \bar{a}_{11} - (pm + qn)\bar{\lambda} \tag{78}$$

The loss of power in the line and the change in value of y is evidently given as follows:

$$\begin{aligned} x' &= x + R\bar{w} + G\bar{z} \\ y' &= y + B\bar{z} - X\bar{w} \end{aligned} \tag{79}$$

By solving the system of four pairs of equations which follow from these two equations, the following values of eight of the mean values may be found. These appear likely to be the most useful, and the check with the previous formulas may be applied. Where $K = 2A_0B_0$

$$\begin{aligned}
&\bar{a}_{11} = (a_{31}X + a_{41}R) \div K, \quad \bar{a}_{21} = (a_{31}B - a_{41}G) \div K \\
&\bar{a}_{12} = (a_{32}X + a_{42}R) \div K, \quad \bar{a}_{22} = (a_{32}B - a_{42}G) \div K \\
&\bar{a}_{13} = \{(a_{33} - 1)X + a_{43}R\} \div K. \\
&\bar{a}_{23} = \{(a_{33} - 1)B - a_{43}G\} \div K \\
&\bar{a}_{14} = \{a_{34}X + (a_{44} - 1)R\} \div K, \\
&\bar{a}_{24} = \{a_{34}B - (a_{44} - 1)G\} \div K
\end{aligned} \tag{80}$$

and therefore

$$\begin{aligned} K\bar{z} &= X(x' - x) + R(y' - y), \\ K\bar{w} &= B(x' - x) - G(y' - y) \end{aligned} \tag{81}$$

11. Just as in ordinary vector notation three numbers, namely the value of the current, the voltage, and the angle between their directions are necessary for specifying conditions at an end of a circuit, so here also three numbers are necessary and sufficient. Any three out of the four z, w, x, y (barring the usual ambiguities arising from quadratics), will specify exactly the conditions. It may thence be seen that apart from possible ambiguity any three independent data about z, w, x, y, z', w', x', y' (or for a simple transmission line, also $\bar{z}$, $\bar{w}$, $\bar{x}$, $\bar{y}$,) will fix all conditions. Thus if L_1, L_2, and L_3 be any three linear functions of these

eight (or twelve) variables, the fixing of these three L's will in general serve. For the points of a model to represent definite states of the circuit three dimensions would then be needed and the values of L_1, L_2 and L_3 would serve to define a point, the "state-point" of the system such that all other numerical data would be implied by its position.

Using only the twelve quantities above, simplex, there would be 220 choices of such models. However, the combinations x, y, z, or $x'y'z'$ have evident advantages, though something might be said for certain other combinations.

Now in view of the fact that all the equations of the a-set, the b-set and those of the mean value quantities, and the covariant conditions, are homogeneous in the variables, it follows that the equations deal essentially with ratios, and that these all hold when every quantity involved, namely z, w, x, y, or z', w', x', y', $\bar{z}$, $\bar{w}$, $\bar{x}$, $\bar{y}$ and consequently also L_i (any homogeneous linear function of them) is multiplied by any common factor.

12. Any three-dimensional model of this type therefore will have the property that as the state-point moves on a straight line from the origin all the variables and hence all L's will vary in direct proportion to its distance from the origin. The essence of any such model will therefore consist in the direction of the line from the origin to the state-point and this is indicated by the intersection of this line with any fixed plane not through the origin.

The simplest plane of representation in the xyz model would be a plane for constant z. This amounts to what has been called the "Evans and Sels modified diagram." However, the three-dimensional model which has been suggested uses $|E|$, that is, in the present notation the square root of z for third dimension. This naturally does not possess the simple linear projective property described above.

Instead of straight lines in the model joining points, in the plane above mentioned, to the origin there would be parabolas and for circles in the plane instead of cones there would be complicated surfaces. Hence at least for visualization the model here proposed is preferable.

13. Now since all variables are functions of any three, and other consistent values of all may be found by using any constant factor, it follows that all ratios of variables are determined when any two ratios are given. It will also follow that if three ratios be given there will generally be an inconsistency in the data.

Taking $x \div z = u$ and $y \div z = v$, u and v are the representatives of x and y obtained by using the points where the line from the origin to the state-point (xyz) intersects the plane $z = 1$.

Hence all the 28 ratios of the eight (or 66 of the twelve) variables have defined values for assigned u and v.

So far as ratios are concerned the state-point may now be thought of as in this plane. At any rate the perspective image of the state-point lies there.

14. Although all the quantities thus dealt with in the x, y or in the uv plane are real, there are nevertheless vector relations at their basis which have not disappeared beyond recovery.

For
$$E_1 = b.AS.E_0 \tag{82}$$
and
$$I_1 = d.BS.E_0 \tag{83}$$

These relations which are to be interpreted vectorially may be obtained as follows:

For
$$E_1 \div E_0 = b.(a/b + u + vj) \tag{84}$$
and (as vector)
$$AO = (a_{13} + a_{14}j) \div 2a_{12} = a/b \tag{85}$$
and
$$u + vj = OS \tag{86}$$
hence (82) is true.

Also
$$I_1 \div E_0 = d.(c/d + u + vj) \tag{87}$$
and (as vector)
$$BO = (a_{23} + a_{24}j) \div a_{22} = c/d \tag{88}$$
hence (83) follows.

That is the vectors AS and BS defined by the state-point S fix E_1 and I_1 in proper vector relation to E_0 when respective base lines are used which take account of b and d respectively.

More precisely stated the vector ratio of E_1 to E_0 is that of AS to $1/b$ and the vector ratio of I_1 to E_0 is that of BS to $1/d$.

Thus a second interpretation which may be put on the u, v plane is that it is a three-origin vector diagram. That is to say O, A, and B are three origins from which vectors drawn to a point S represent correctly in magnitude and phase (when properly referenced) I_0, E_1, and I_1 respectively.

15. The framework. Eight circles. Continuing with the ideas of Sec. 13 with a view to the general treatment of Sec. 16, it is noted that the fundamental quantities individually present the simplest instances of linear functions of their own values.

Thus z, w, x, y, z', w', x', y' are each an L where L denotes a linear homogeneous function of such quantities.

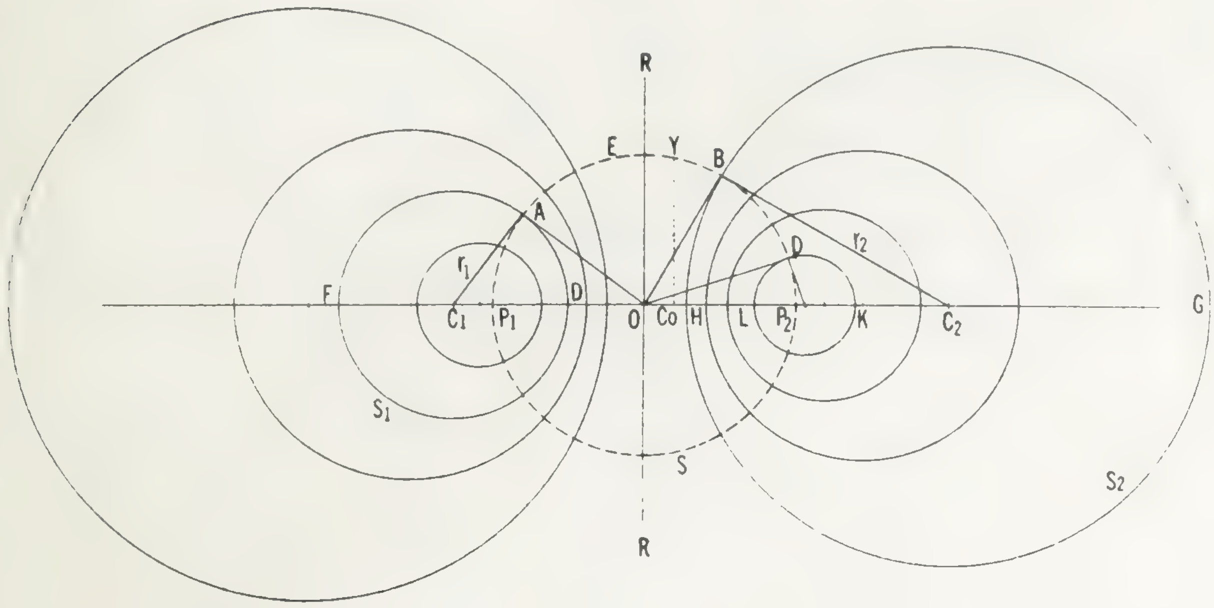

Fig. 1.—Systems of Circular Loci with Common Radical Axis and r^2 Positive. Also Type Case QM_r

Each of these is comprised in the equation

$$L \equiv az + bw + cx + dy$$

where the coefficients are general.
Then putting as before $u = x \div z$, $v = y \div z$

$$S \equiv L \div z = a + b(u^2 + v^2) + cu + dv$$

Then $S = 0$ would, when $b \neq 0$, denote a circle in the u, v plane, and this will be the case for $z'w'x'y'$ and w, at least in a formal sense. Also S denotes for any given point (u, v) the square of the tangent to the

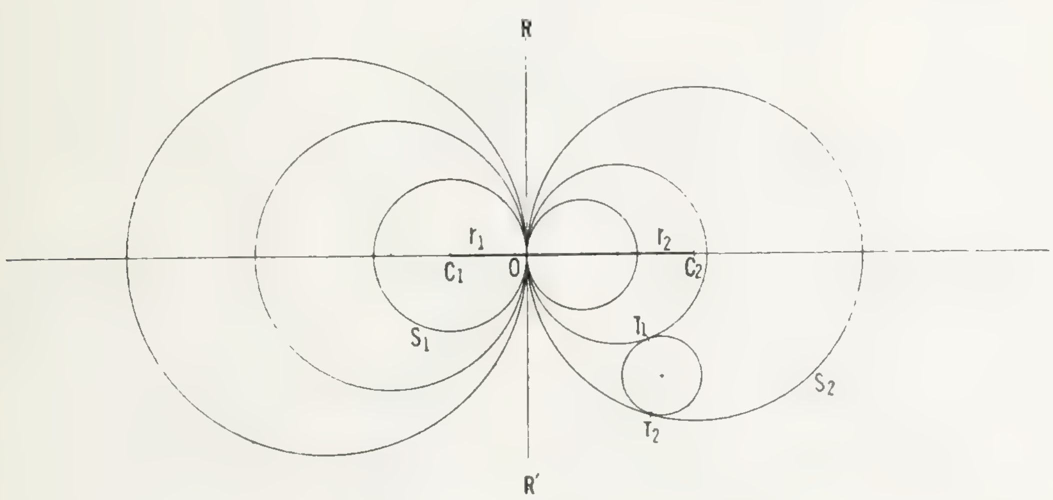

Fig. 2.—Systems of Circular Loci with Common Radical Axis and r^2 Zero

given circle multiplied by b when the point is external, or the negative of the square of half the chord which it bisects multiplied by b when it is internal to a real circle.

For $L = x$ and y respectively $S = u$ and v respectively. Thus $L = 0$, and hence $S = 0$ denotes, here, one of five circles or two straight lines, the axes.

The case $L=z$ gives $S=L\div z=1\neq 0$. This case is called in geometry the "line at infinity." Here a better name would be "circle at infinity."

The centre of $S=0$ is at $u=-c\div(2b)$, $v=-d\div(2b)$ and its radius squared is $(c^2+d^2-4ab)\div(4b^2)$.

The quantity c^2+d^2-4ab on reference to identities (31) (30) (32) and (33) will be seen to have values, 0, 0, 1 and 1 for z', w', x' and y' respectively. The radii for these circles will then be 0, 0, $1\div(2a_{32})$ and $1\div(2a_{42})$.

If now the centres of these circles, $S=0$ for z', w', x' and y' be located in the plane with the coordinates,

for $z'=0$, $-a_{13}\div(2a_{12})$ and $-a_{14}\div(2a_{12})$
" $w'=0$, $-a_{23}\div(2a_{22})$ and $-a_{24}\div(2a_{22})$
" $x'=0$, $-a_{33}\div(2a_{32})$ and $-a_{34}\div(2a_{32})$
" $y'=0$, $-a_{43}\div(2a_{42})$ and $-a_{44}\div(2a_{42})$

these centres will appear as in the "kite" framework shown in Fig. 4 at A, B, C, and D respectively. A and B will then be point circles while the circles $x'=0$, $y'=0$ must each pass through the points $z'=0$, $w'=0$, that is A and B.

The other circles will be $w=0$, a point circle at O, the two axes as infinite circles, and the "circle at infinity."

By use of (12) and the identities given, the relative values of the fundamental quantities at the centres $ABCD$ and O are as given in Table I where being relative only they have been arranged for convenience to avoid fractions.

TABLE I

RELATIVE VALUES OF

At	z	w	x	y	z'	w'	x'	y'
A	$2a_{12}$,	$2a_{11}$	$-a_{13}$	$-a_{14}$	0	2	0	0
B	$2a_{22}$,	$2a_{21}$	$-a_{23}$	$-a_{24}$	2	0	0	0
C	$4a_{32}^2$,	$1+4a_{31}a_{32}$,	$-2a_{33}a_{32}$,	$-2a_{34}a_{32}$,	a_{12}	a_{22}	$-a_{32}$	a_{42}
D	$4a_{42}^2$,	$1+4a_{41}a_{42}$,	$-2a_{43}a_{42}$,	$-2a_{44}a_{42}$,	a_{12}	a_{22}	a_{32}	$-a_{42}$
O	1	0	0	0	a_{11}	a_{21}	a_{31}	a_{41}

The distances in Fig. 4 are given as follows:

$$AB^2=1\div(a_{12}a_{22}),\ AC^2=BC^2=1\div(4a_{32}^2)$$
$$AD^2=BD^2=1\div(4a_{42}^2)$$
$$CD^2=a_{12}a_{22}\div(4a_{32}^2a_{42}^2),\ AO^2=a_{11}\div a_{12}$$
$$BO^2=a_{21}\div a_{22}$$
$$CO^2=AC^2+a_{31}\div a_{32},\ DO^2=AD^2+a_{41}\div a_{42}$$

Since $CD^2=CA^2+AD^2$, by using (21), the angle CAD is a right angle and the circles $x'=0$, $y'=0$ intersect orthogonally.

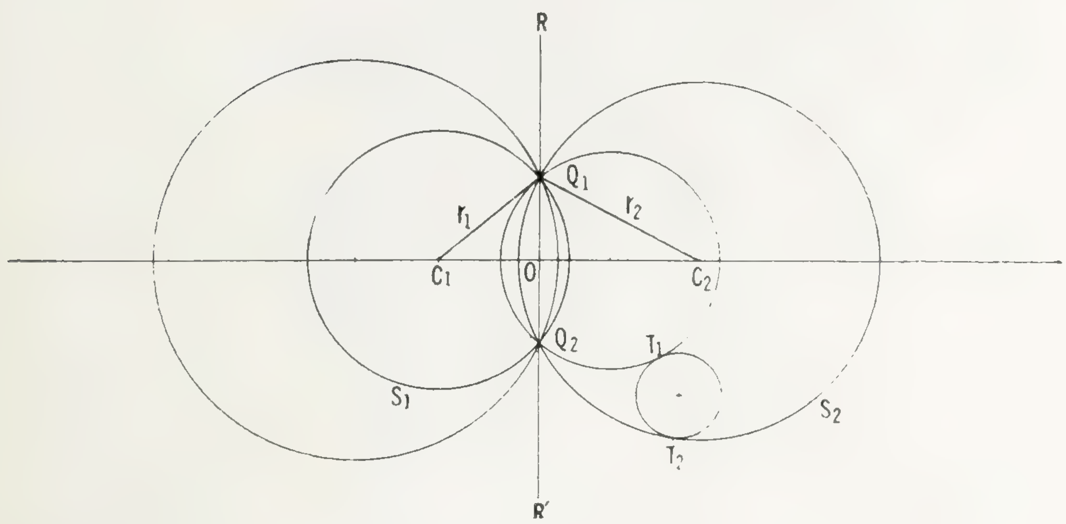

Fig. 3.—Systems of Circular Loci with Common Radical Axis and r^2 Negative

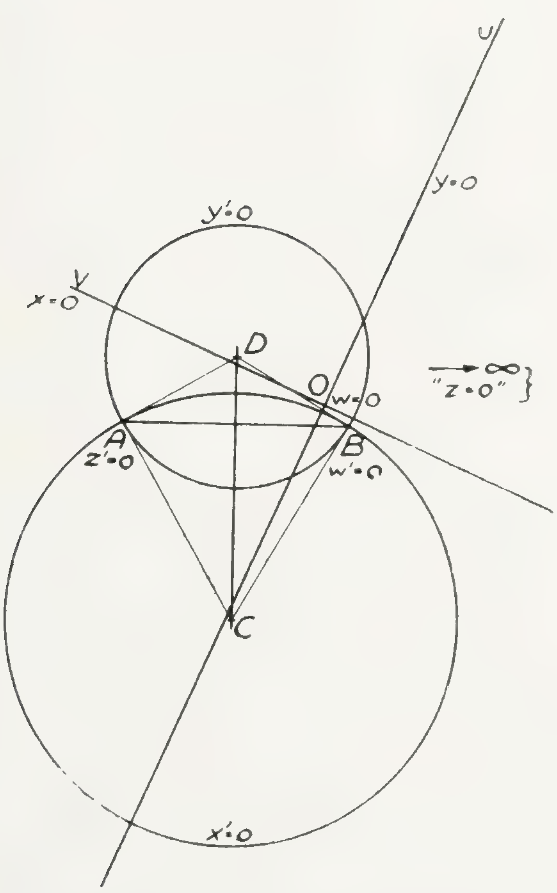

Fig. 4.—The Framework. Eight Circles the Basis for 28 Families of Circular Ratio-Loci

16. I. Problems involving linear functions, L of the fundamental quantities z, w, x, and y in such a way as to lead to fixed values of them may be classified into three divisions, in each of which definite solutions

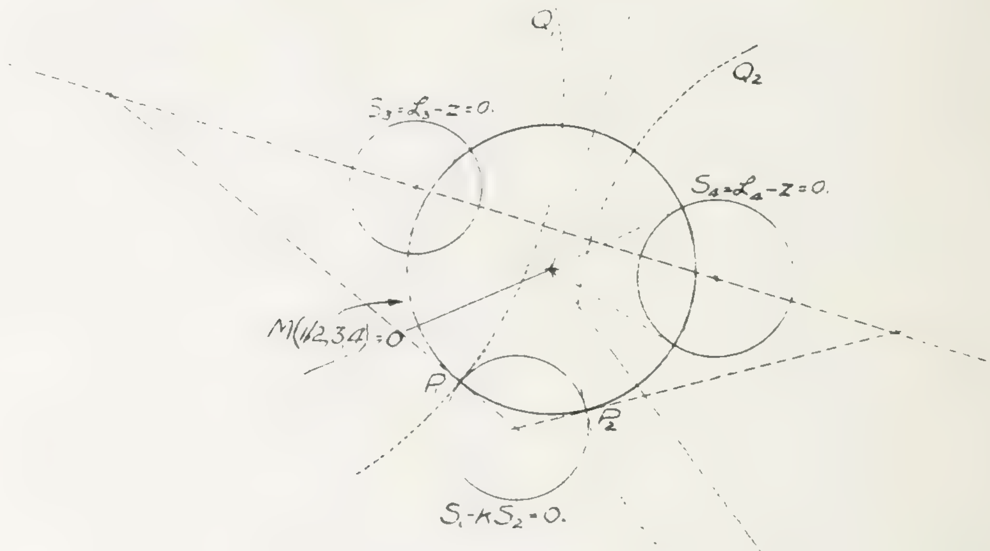

Fig. 5.—Type Case Q^2M_r. Two Quantities Given. Maximum of Ratio at P_1P_2

are possible both analytically and graphically. For simplicity the term maximum will here be used in place of extremum to include minimum.

(A) Three linear functions given numerically of which one at least is not zero.

(B) One-conditioned maximum. The type case is one linear function not zero given, and a ratio of two linear functions to have a maximum value.

(C) Two-conditioned maximum. The type case is two linear functions given of which one at least is not zero, and a ratio of two linear functions to have a maximum value.

II. Problems involving linear functions may also be presented with less data than as described above thus:

(D) Two linear functions given of which one at least is not zero.

(E) Two linear functions given equal to zero.

(F) One linear function given equal to zero.

While these do not lead to definite solutions they do lead to loci whose examination may be very instructive and yield much information.

17. Algebraic Solutions.

For convenience in the further examination of these six divisions let the linear functions be referred to as L_1L_2 etc. with numerical values denoted by q_1, q_2 etc.

Let Q denote a case of $L = q \neq 0$

O denote a case of $L = 0$

R denote a case of $L_1 \div L_2 = k$

M_q denote a maximum of a quantity.

M_r denote a maximum of a ratio.

An exponent is used to indicate the number of each kind of datum. Thus Q^2O denotes a case where two linear functions not equal to zero are given and one that is zero.

I. (A) This case of three data may then occur in the forms Q^3, Q^2O, QO^2 and to these may be added Q^2R and QR^2 because R means a relation of the type: $L_1 \div L_2 = k$, or $L_1 - kL_2 = 0$ that is $L = 0$ which is denoted by O.

In all these five cases then there are three equations such as

$$\begin{aligned} a_1z+b_1w+c_1x+d_1y&=q_1\\ a_2z+b_2w+c_2x+d_2y&=q_2\\ a_3z+b_3w+c_3x+d_3y&=q_3 \end{aligned} \tag{89}$$

of which not more than two values on the right may be zero. These are to be combined with

$$wz - x^2 - y^2 = 0 \tag{90}$$

This is readily done by obtaining linear expressions for w, x, and y from (89) in terms of z and these on substitution in (90) give a quadratic equation determining z and thence w, x, and y.

I. (B) One-conditioned maximum. Two subdivisions are (1), L_1, $L_2 \div L_3$ maximum, regarded as the type case QM_r and (2) QM_q which may also be presented in the form QM_r.

For QM_q means $L_1 = q_1$, and L_2 to be maximum. Now this is equivalent to $L_2 \div L_1$ to be maximum. Hence it may be thus put into the form QM_r.

A solution of the type case QM_r gives the following matrix equal to zero:

$$\begin{bmatrix} a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ w & z & -2x & -2y \end{bmatrix} \tag{91}$$

in the sense that each of the three-rowed square determinants which can be made from it is zero. Omitting the first column the line of centres for the two circles S_2 and S_3 associated with L_2 and L_3 respectively is found and omitting any other column a circle is found which intersects this line of centres in the limit-points of the system.

For brevity the matrix conditions above, (91), may be denoted by $M(2, 3) = 0$.

Then since the second case QM_q corresponds as mentioned above to a denominator L_1 instead of L_3 the condition for it must be $M(2, 1) = 0$.

In both these cases the matrix gives a linear and also a quadratic equation in u and v which are thus sufficient to determine them, and then the covariantive equation and the equation $L_1 = q_1$ together fix z, w, x and y.

2

I. (C) Two-conditioned maximum. The type case is Q^2M_r, the other cases being Q^2M_q, QOM_r, QRM_r, QRM_q and QOM_q. For as in the previous divisions, an R condition may be put in the O form and an M_q in the M_r form.

Type case Q^2M_r: This denotes a problem of the form

$$L_1 = a_1z + b_1w + c_1x + d_1y = q_1$$
$$L_2 = a_2z + b_2w + c_2x + d_2y = q_2$$

$$L_3 \div L_4 = \frac{a_3z + b_3w + c_3x + d_3y}{a_4z + b_4w + c_4x + d_4y} \text{ to be maximum.}$$

Let $q_2 \div q_1 = k$, then $L_2 - kL_1 = 0$ is a homogeneous equation conditioning the maximum and it may be shown that $D(2-k\,1,\ 3,\ 4) = 0$ is the solution where this for brevity denotes that the following determinant is zero:

$$\begin{vmatrix} a_2-ka_1, & b_2-kb_1, & c_2-kc_1, & d_2-kd_1 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \\ w & z & -2x & -2y \end{vmatrix} = 0 \qquad (92)$$

This may be expressed also as

$$D(2,\ 3,\ 4) = kD(1,\ 3,\ 4) \qquad (93)$$

The other cases may then be dealt with as follows:

Case Q^2M_q:

Say that $L_1 = q_1$, $L_2 = q_2$ and that L_3 is to be maximum and therefore also $L_3 \div L_1$. L_1 now may replace L_4 and the equation becomes $D(2,\ 3,\ 1) = kD(1,\ 3,\ 1)$. But $D(1,\ 3,\ 1) = 0$ from the properties of determinants.

Therefore $$D(2,\ 3,\ 1) = 0 \qquad (94)$$

Case QOM_r: This is $L_1 = q_1$, $L_2 = 0$, $L_3 \div L_4$ to be maximum.

That is to say a particular case of Q^2M_r with $k = 0$.

Hence the equation is $$D(2,\ 3,\ 4) = 0 \qquad (95)$$

Case QRM_r: This is $L_1 = q_1$, $L_2 - kL_3 = 0$, $L_4 \div L_5$ to be maximum.

That is, it is a variant of QOM_r and the equation will be

$$D(2-k3,\ 4,\ 5) = 0 \text{ or } D(2,\ 4,\ 5) = kD(3,\ 4,\ 5) \qquad (96)$$

Case QRM_q: This is $L_1 = q_1$, $L_2 - kL_3 = 0$, L_4 to be maximum.

Substituting for the latter $L_4 \div L_1$ to be maximum, the equation becomes

$$D(2-k3,\ 4,\ 1) = 0, \text{ or } D(2,\ 4,\ 1) = kD(3,\ 4,\ 1) \qquad (97)$$

Case QOM_q: This is $L_1 = q_1$, $L_2 = 0$, L_3 to be maximum.

Since the conditions are as in Q^2M_q, with $k=0$ (which however does not alter the result) the equation is

$$D(2, 3, 1) = 0 \tag{98}$$

18. Geometrical Solutions. The ratio relation $L_1 = kL_2$, leads to

$$S_1' - kS_2' = 0$$

and thus to the corresponding loci. As sufficient detail is not readily found, if at all, in standard works on analytical geometry, a classification of the different forms, real and imaginary, which the locus $S_1' - kS_2' = 0$ may assume, follows. It is useful in suggesting geometrical constructions.

Here

$$S_1' = b_1(x^2+y^2) + c_1x + d_1y + a_1$$
$$S_2' = b_2(x^2+y^2) + c_2x + d_2y + a_2$$

Broadly speaking S_1' and S_2' may denote two straight lines ($b_1 = 0$, $b_2 = 0$) and finally two circles ($b_1 = 1$, $b_2 = 1$). In this latter case it will be more convenient by transforming to an origin at the centre of S_1' and axis of abscissas through centre of S_2', to replace S_1' and S_2' (when the distance between centres C_1C_2 is equal to a) by the forms

$$S_1 : x^2 + y^2 - r_1^2$$
$$S_2 : (x-a)^2 + y^2 - r_2^2$$

In the first eight cases below it is not difficult to determine the members of the system geometrically. In the remaining cases the geometrical constructions are for the most part cumbersome and it is desirable to determine the radical axis analytically. Its equation will be, in the new coordinates,

$$S_1 - S_2 = 0 \text{ or } 2ax = r_1^2 - r_2^2 + a^2 \tag{99}$$

and this will intersect the line of centres at a point O distant $\frac{a}{2} + \frac{r_1^2 - r_2^2}{2a}$ from the centre C_1 of S_1.

Evidently the radical axis is always real. The limit points P_1, P_2 of the system will be given by the intersection of a circle S, centre O and radius r, where

$$r^2 = \frac{a^2}{4} - \tfrac{1}{2}(r_1^2 + r_2^2) + (r_1^2 - r_2^2)^2 \div (2a) \tag{100}$$

with the line of centres, C_1C_2.

Hence the points P_1 and P_2 are real coincident, or imaginary according as r^2 is positive, zero or negative, *i.e.*, as O is exterior, on or interior to every circle of the system.

Two straight lines $b_1 = b_2 = 0$

(1) $c_1 = d_1 = c_2 = d_2 = 0$. Both at infinity. As the expressions are both constants there is no problem here.

(2) $c_1 = 0$, $d_1 = 0$, or else $c_2 = 0$, $d_2 = 0$. One only at infinity. The other members of the system are then parallel to that at finite distance.

(3) $d_1/c_1 = d_2/c_2$. Two parallel lines at a finite distance. The system consists of all lines parallel to them.

(4) $d_1/c_1 \neq d_2/c_2$. Two intersecting lines. The system consists of all lines through their intersection.

Straight line and a circle ($b_1 = 0$, $b_2 = 1$.)

(5) ($c_1 = 0$, $d_1 = 0$). Line at infinity and circle, (real, imaginary, or point). The system consists of all concentric circles (real, imaginary, or point).

(6) Line at finite distance and real circle which it intersects. The system consists of all circles through these points of intersection.

(7) Line at finite distance and real or point circle which it does not intersect. The system will consist of all the circles which taken with the given circle have the line as radical axis.

In Fig. 1, ROR' is the given line, DAF the given circle, centre C_1, C_1O perpendicular to ROR', OA tangent to DAF at A. The circle P_1EP_2 centre O determines the limit points P_1P_2 which are the point circles of the system. This circle P_1EP_2 is, further, orthogonal to every member of the system, some of which are shown in Fig. 1 and which of course have centres on the "line of centres" P_1OP_2.

(8) Line at finite distance and imaginary circle. In Fig. 1 ROR' is the given line, C_0 the centre of the imaginary circle and C_0Y the amplitude of the imaginary radius, C_0O being perpendicular and C_0Y parallel to ROR'. The circle, centre O, and radius OY determines P_1 and P_2 the point circles of the system and all others by the property of orthogonality as in Fig. 1.

Two Circles. $b_1 = b_2 = 1$.

(9) Two real circles intersecting in the points Q_1 and Q_2 (see Fig. 3). Here the point O is the midpoint of Q_1, Q_2, and r^2 is negative. All the circles of the system pass through Q_1 and Q_2 and have their centres on the line through O perpendicular to Q_1Q_2.

(10) Two real non-intersecting circles S_1S_2, external to each other as DAF, GBH in Fig. 1. The point O may be determined by the formula possibly more easily than by graphical means and the orthogonal circle is real. The members of the system have their centres on C_1C_2 and are orthogonal to S.

(11) Two real non-intersecting circles, S_1 being internal to S_2, as *KDL*, *GBH* in Fig. 1. This case is essentially the same as the preceding, number (10), and the point O and S may be determined as before.

(12) Two real circles touching each other at the point O (Fig. 2). The radical axis consists of the common tangent, and every circle of the system is tangent to it at the point O. The circle S is now a point circle, $r^2=0$.

(13) One real circle as *DAF* and one imaginary circle S_0, centre C_0 and amplitude of imaginary radius C_0Y, Fig. 1, where C_0Y is drawn perpendicular to C_0C_1 and Y lies on S.

The centre of S_0, namely C_0, may be outside on, or inside S_1, but in either case the radical axis is readily found by the formula or graphically and the circle S is always real.

(14) Two imaginary circles. Here both r_1^2 and r_2^2 are negative and r^2 is always positive.

19. Geometrical Solutions. Use of circular loci.

The preceding section contains a classification of loci corresponding to variable ratios of L_1 to L_2 according to the circular or linear characters of S_1 and S_2 corresponding to them. They correspond to the families of circles of which examples are to be seen in Figs. 1, 2, and 3. They are characterized by having a common radical axis associated with and determined by L_1 and L_2.

Cases O and R: Cases $L=0$, and $L_1-kL_2=0$ in which k is fixed, which correspond, to O and R of Sec. 17, may be represented geometrically by putting them in the form $S=0$ from which the centre and radius may be found as in Sec. 15 if it is a real circle, otherwise as a straight line by usual methods.

Section 17 I (A). Cases Q^3, Q^2O, QO^2: These may for the present purpose all be expressed as QO^2.

Each O will then define a circle (in general) which may be drawn. The intersection point or points will then give the values of u and v and thence all the ratios. The values are then given by Q, namely $L_1=q_1$ that is $S_1z=q_1$ or $z=q_1\div S_1$. Then q_1 being known and S_1 being a given function of u and v is known also for the intersection determined. Now that z is known the ratios found determine all the quantities.

Section 17 I (B). The type case of one-conditioned maximum QM_r viewed geometrically calls essentially for the maximum of a ratio to be determined, that is some limiting number for k in $S_2-kS_3=0$ is to be found. Such numbers, however, do not exist when S_2 and S_3 belong to the families of circles shown in Fig. 2, intersecting in two points, or in Fig. 3 with common point of tangency, but only in Fig. 1. In Figs. 2

and 3, k may have any value while in Fig. 1 the limit points P_1 and P_2 correspond to extreme values of k.

If then L_2 and L_3 are related as in Fig. 1, and external to each other the radical axis ROR' may be found since it bisects the common tangent and the two limit points are then on the line of centres at distance OA from O where A is a point of tangency on either circle. In any case the formulas of Sec. 18 may be used to locate the limit points and thus all ratios. Then $z=q_1 \div S_1$ will give all the quantities required.

An approximation that may be convenient in a case of two circles relatively small and moderately distant, of radii r_1 and r_2 and distance between centres d, is that measuring along the line of centres from the first centre the nearer limit-point is at the distance given by the continued fraction whose repeating period is $(r_1^2/d - r_2^2/d -)$.

Also if one be a circle of radius r and the other a straight line at distance p from the centre then the nearer limit point will be at distance $p-\sqrt{p^2-r^2}$ or if $r \div p = \sin\theta$ then at distance $2p\sin^2(\theta/2)$.

Section 17 I (C). Two-conditioned maximum. Type Case Q^2M_r. As before this means $L_1=q_1$, $L_2=q_2$ and $L_3 \div L_4$ to be maximum. Putting in the form QOM_r and leaving Q for final consideration, there remains OM_r. The equation O is $S_2q_1-S_1q_2=0$, or say $S_{21}=0$. M_r means $S_3-mS_4=0$ with m a maximum. Consequently the extreme members of the family S_3, S_4 are sought which are consistent with S_{21}: a geometrical problem of three circles.

If S_3 and S_4 are given such as to belong to the system of Fig. 3, and S_{21} is the circle T_1T_2 then the graphical problem is to draw a circle through Q_1 and Q_2 the points of intersection of S_3 and S_4 so as to be tangent to the circle T_1T_2. This can be done in two ways and hence T_1, T_2 found, thus determining the ratios and then as above the quantities.

Similarly if S_3 and S_4 belong to the system of Fig. 2, two circles of the system tangent to their radical axis at O and also to the circle T_1T_2 at T_1 and T_2 respectively may be found graphically.

If, however, S_3 and S_4 belong to the system of Fig. 1, as in Fig. 5, then taking them with the other circle, namely S_1-kS_2, pair by pair, they will have three radical axes meeting in a point, the radical centre. This point has the property that a circle having it for centre intersects orthogonally every member of each of the three systems of circles with these radical axes. Hence drawing tangents from the radical centre to the circle S_1-kS_2 the points (P_1, P_2) fulfill the required condition. The ratios are thus fixed and $z=q_1 \div S_1 = q_2 \div S_2$ then fixes the quantities.

In Section 16, II (D) (E) (F), cases QO, O^2 and O are mentioned of data insufficient per se to determine a definite solution. Some of these

arise from ratios; they may be QR, R^2 and R. Consider some of the principal ratio cases.

For transmission lines possibly the ratios arising from equivalent R, X, G, B, and Z at either end may not appear particularly natural, but the principles here developed apply to all circuits including those of communication engineering, so that these conditions are of importance. In any case the ideas are fundamental.

Other ratios of importance are efficiency, regulation, and power factor. Efficiency is either x'/x or x/x' according to circumstances. Regulation is neither z'/z nor z/z' but is definitely associated with these numbers, so that the regulation is constant when z'/z is. Power factor is given by y/x at one end and y'/x' at the other. These are the tangents of the phase angle and hence fix the power factor by the cosine. By using the principles stated it is possible to obtain very easily some of the fundamental loci. These are in general circles and belong to the system defined by the numerator and the denominator as illustrated in the "kite" framework of Fig. 4.

Regulation index z'/z: Circles centre A.

Efficiency index x'/x : $x=0$ is radical axis of the system.

$x'=0$ is shown with centre C. The loci for variable efficiency are circles with this radical axis and one limit point at $\sqrt{p^2-r^2}$ to the left of $x=0$, that is at distance $\sqrt{a_{33}{}^2-1} \div (2a_{32})$.

R at A: This is x/w: Radical axis $x=0$. Point circle $w=0$. Loci are circles tangent to $x=0$ at O.

X at A: This is y/w: Radical axis $y=0$. Point circle $w=0$. Loci are circles tangent to $y=0$ at O.

G at A: This is x/z: $x=0$ is infinite circle. $z=0$ circle at infinity. Loci are lines parallel to $x=0$ (as is evident).

B at A: this is y/z: $y=0$ is infinite circle. Loci are lines parallel to it.

Z at A: this is z/w: Loci concentric circles around O.

R at B end: this is x'/w': Point circle $w'=0$ is on circle $x'=0$. Loci are circles, centres on CB passing through B. DB is a member.

G at B: this is x'/z': Loci are circles, centres on CA passing through A. AD is a member.

X at B: this is y'/w': Loci are circles, centres on DB passing through B. CB is a member.

B at B: this is y'/z': Loci are circles, centres on DA passing through A. CA is a member.

Z at B: this is z'/w': Loci with AD as limit points and radical axis the perpendicular bisector of AD. This line also is a member.

This last diagram is the same as for lines of magnetic flux with A and B as parallel conductors.

Power factor at A given by $y/x = \tan\theta$. Loci are lines through O.

Power factor at B given by $y'/x' = \tan\theta$. Loci are arcs of the system through A and B. Relative phases of current and voltage at B are preserved by the directions of the tangents to these arcs at the point B.

20. Interconnection of algebraic and geometrical methods. It is not necessary to dwell on the fact that in practical problems according to circumstances one of these methods or the other may prove more convenient, nor that it is of advantage always to have methods as different as possible so as to make certain of the result by a check.

Section 17 I (B) Case QM_r: One-conditioned maximum. Here the algebraic solution is by M (2, 3) = 0 and the graphical by finding the limit points. They agree because the matrix gives equations which describe the line of centres and three circles intersecting this line in the limit-points, one circle being sufficient to define them.

Section 17 I (C) Case Q^2M_r: Two-conditioned maximum. Here $D\,(2, 3, 4)\; q_1 = D\,(1, 3, 4) q_2$ is the equation of the circle intersecting S_{21}, S_3 and S_4 orthogonally. By dispensing with the maximum condition and using this determinantal equation with the other algebraic conditions or else this circle with the geometrical conditions the same result should be obtained.

21. Examples.

(*a*) The voltage at Paugan Falls being 225.2 kv. (line) what are the conditions there when the loss on the line is the minimum possible?

First consider as a general circuit problem detached from its special features. It is a case of QM_q in Section 17 I (B) and is therefore solved by M (2, 1) = 0.

Here $L_1 = z' = q_1$ and $L_2 = x' - x$ is to be minimum.

Then M (2, 1) = 0 means

$$\begin{bmatrix} 1, & 0, & 0, & 0, \\ -b_{31} & -b_{32} & (1-b_{33}) & -b_{34} \\ w' & z' & -2x' & -2y' \end{bmatrix} = 0$$

That is

$$z' \div (-b_{32}) = -2x' \div (1-b_{33}) = 2y' \div b_{34}$$

or

$$z' \div a_{13} = x' \div (a_{33}-1) = y' \div a_{43}$$

is the solution of the *general circuit problem*. For the *problem proposed* $q_1 = 1.69$.

Hence

$x' = 0.001001$ or 10.01 kw. $y' = 0.431$ or 4310 kv-a.

$w' = 0.1122$ or 1122 amperes-squared that is 33.5 amperes.

The power factor would be 0.0023 (leading).

This problem might have been considered as determined by the centre of the circle $x'-x=0$, or $R\bar{w}+G\bar{z}=0$. As is evident from the latter form it is an imaginary circle but has a real centre.

(*b*) The voltage being 225.2 on the same line at the S-end and the power factor at the R-end being 0.85 (lagging) find the maximum power that can be delivered.

As a general circuit problem this may be put in the form

$$L_1=z'=q_1,\ L_2=x,\ L_3=y, \text{ where } y/x=\tan\theta$$

and $L_4=L_2$ to be maximum, a case of QRM_q of Sec. 17 I (C). The determinantal condition simplifies to D (1, 2, 3) $=0$ that is

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ w & z & -2x & -2y \end{vmatrix}=0$$

this reduces to $w=za_{11}\div a_{12}$. Let $a_{11}\div a_{12}=c^2$

Then

$$u^2+v^2=c^2,\ v/u=\tan\theta, \text{ hence } u=c\cos\theta,\ v=c\sin\theta$$

Then $\quad z=q_1\div S_1$ where

$\quad S_1=2a_{11}+ca_{13}\cos\theta+ca_{14}\sin\theta$

Thus $\quad x=uz=q_1c\cos\theta\div S_1$

$\quad y=vz=q_1c\sin\theta\div S_1$

In the special values of this problem $q_1=1.69$, $\cos\theta=0.85$ hence $\sin\theta=-0.52679$.

Then

$x=5.1125$ or 51125 kw. $\quad y=-3.169$ or 31690 kv-a. (lagging)

$z=0.4924$ corresponding to 121.5 kv. line volts.

In both this and the preceding example the results as worked out are for one phase both as to current and voltage, the data however are supposed given for line voltage.

22. In conclusion the author wishes to thank Mr. V. G. Smith and Mr. G. de B. Robinson for assistance in the preparation of this paper.

THE HEAT OUTPUT OF CONCEALED RADIATORS AT VARIOUS TEMPERATURES*

By E. A. Allcut [1]

Introduction

Most of the work already described in connection with concealed radiators refers to steam as the heating element. In Canada, however, the great majority of homes and small buildings are heated by hot water. Some time ago, the author made a series of tests on small radiators of various kinds, to determine by how much the heat output was reduced when using water at about 150° F., as compared with steam. The results obtained were lower than those expected and also varied somewhat with radiators of different form, so that the present series of tests was made for the purpose of obtaining some general information on this subject.

Radiators were selected of as many different forms as possible and were loaned by the makers for this work. The enclosures used were of the dimensions recommended by them.

Preliminary studies of the subject involved a consideration of the method of expressing the results. The heat output of a radiator can be established fairly definitely by the usual procedure of testing with steam or hot water. On the other hand, the object of installing a radiator is to provide heat for a room or building, and this involves not only the supply, but also the distribution of heat. Some manufacturers rate their radiators on the first basis and some on the second, and the situation is in urgent need of clarification as, for this reason, it is difficult to compare radiators of different design.

The author expresses no opinion as to which method is the better commercially, but came to the conclusion that the actual radiator output in B.T.U. per hour was the preferable medium for expressing the test results. The steam condensed, or quantity of water cooled, per hour indicates the actual amount of heat leaving the metallic surface, and previous experience had shown that this quantity was independent of changes in test conditions over comparatively wide ranges, so that similar results were obtained in rooms of different sizes, ceiling heights and window spaces. If this heat is not efficiently distributed, that is no fault of the radiator itself, but of other conditions. Given a definite

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[1]Professor of Mechanical Engineering.

weight of air leaving the radiator enclosure at a fixed temperature, the supply of heat to the room will be the same whatever type of radiator is inside the enclosure.

It is probable, also, that the floor dimensions of the room, the height of ceiling, the size and number of windows, direction, velocity and temperature of the air outside, all have a considerable influence on the distribution of heated air inside, thus introducing variables of unknown number and extent into the final results.

For these reasons, the results are expressed deliberately in terms of heat passing through the radiator heating surface, but readings of the velocities and temperatures at the stack outlet are also included.

Apparatus and Method of Testing

The tests were made in the Mechanical Engineering Laboratory of the University of Toronto, a shield of insulating material being placed at the back of the radiator enclosure. There was also a ceiling at a height of 9 feet from the floor level to give an air flow approximating to that obtained in practice. The layout is shown diagramatically in Fig. 1 and is further described in the following paragraphs.

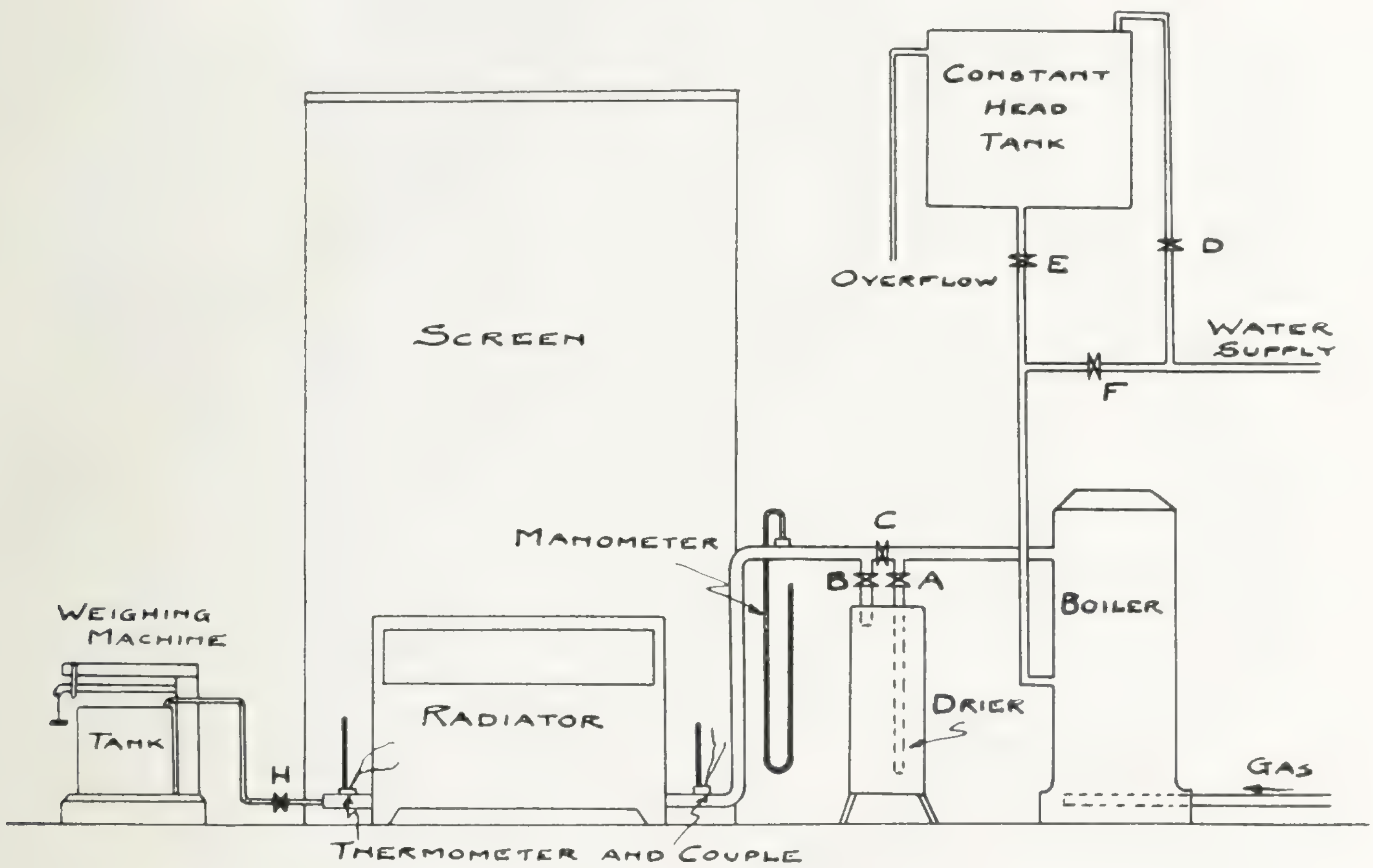
Each radiator was tested at several temperatures on steam and hot water, respectively. The heating element in both series of tests was a gas fired boiler, which could be regulated to supply the amount of heat necessary to maintain the required temperature difference between the radiator and the air entering the enclosure.

For the steam tests, the water was kept at about the centre of the boiler gauge glass and the steam was admitted to the separator by opening valves A and B and closing valve C. The water was drained from the separator at intervals during each test, so that dry steam alone was admitted to the radiator. All steam and hot water pipes were covered, to avoid radiation and condensation losses. This method was preferred to that of superheating the steam because of its greater simplicity and closer approximation to the conditions actually existing in radiator installations. Its accuracy has been proven by previous experience and by comparing the tests made in this way with results on the same or similar radiators tested by the superheated steam method in other laboratories. It was found that there was no appreciable difference in the results obtained.

The condensate was drained from the radiator at frequent intervals during each test, so that the heating surfaces were kept dry. It was then collected in a weighing tank, observations being taken every ten or fifteen minutes. In each case, no test was started until successive readings showed that a constant amount of steam was being condensed, so that conditions were steady. Each test was then run for 90 minutes

or more, as it was found by previous experience that this period was ample for the small radiators tested. The inlet and outlet temperatures were taken both by thermometers and thermocouples, and it was found generally that these agreed fairly well. The amount of disagreement is shown in the tables of test results. The steam pressure was indicated by a mercury manometer.

FIG. 1.



Considerable difficulty was experienced in making hot water tests, as it was by no means easy to maintain a constant flow of water through the radiator. Finally, the layout shown in Fig. 1 was adopted and was used throughout the tests. The same boiler was used as before, but it was now filled with water. The water was supplied from the city main to a constant head tank, by opening valves D and E and closing valve F. The water in the tank was kept up to the level of the overflow and the quantity supplied to the system was regulated by opening and closing valve H. This varied the temperature drop across the radiator, and at the same time preserved a full head of water on the system. The mean temperature was regulated by increasing or reducing the quantity of gas supplied to the burners. The water thus heated, left the boiler through valve C, which was now opened, valves A and B being closed, passed through the radiator and was measured by weighing. Thermocouples again were used in addition to the thermometers, and at first the bare wires were placed in the water stream, but very erratic results were obtained. It was found, after investigation, that these were due to

stray currents. The thermocouples were therefore placed in glass tubes filled with oil and it was found that with this method the inconsistencies disappeared.

The heat outputs for the steam tests were then plotted against the temperature differences between the steam and the air entering the radiator, and a curve was drawn through them of the form

$$\left(\frac{215\text{-}65}{\text{steam temperature}-\text{air temperature}}\right)^{1.3}$$

The hot water tests were plotted in a similar manner, the mean temperature of the radiator being considered to be the arithmetic mean between the inlet and outlet water temperatures. It is questionable whether this should be adopted in all instances, as in most cases of heat transmission the logarithmic mean is taken as the correct temperature, but the above method was employed in accordance with generally accepted practice. The difference between the arithmetic and logarithmic mean temperatures for the test conditions was generally less than one degree Fahrenheit.

In most radiators the hot water tests fell on a curve below that drawn for steam, and in such cases a similar curve was drawn through the points obtained by the hot water tests. The ratio of the heat output with hot water to that with steam at the standard temperature difference of 105° is given for each radiator. This indicates the extent to which the output on hot water falls short of that on steam with the same atmospheric conditions.

Form of Enclosure

The enlargement of the Royal Ontario Museum provided somewhat unusual conditions for enclosed radiators, the room being very large, the ceilings high and the windows nearly 12 feet wide. Stack heights of 8 feet were required to enable the air to be discharged above the specimen cases.

Experiments were made with the different types of radiator placed in an adjustable enclosure, so that heat outputs could be measured under different conditions, as follows:

(*a*) The height of the air inlet was reduced from 13 inches to 6 inches, the equivalent radiation being raised from 61.1 to 68.2 square feet, an increase of 11.6 per cent. Thus, it appears to be inadvisable to increase the air inlet opening beyond a certain limit.

(*b*) The original plans called for the air outlet opening in some cases to be arranged as in A H 2 (Fig. 2), the object being to cover the entire width with a screen of warm air. On the other hand, this increase of area decreases the outlet velocity of the air and therefore reduces its effectiveness.

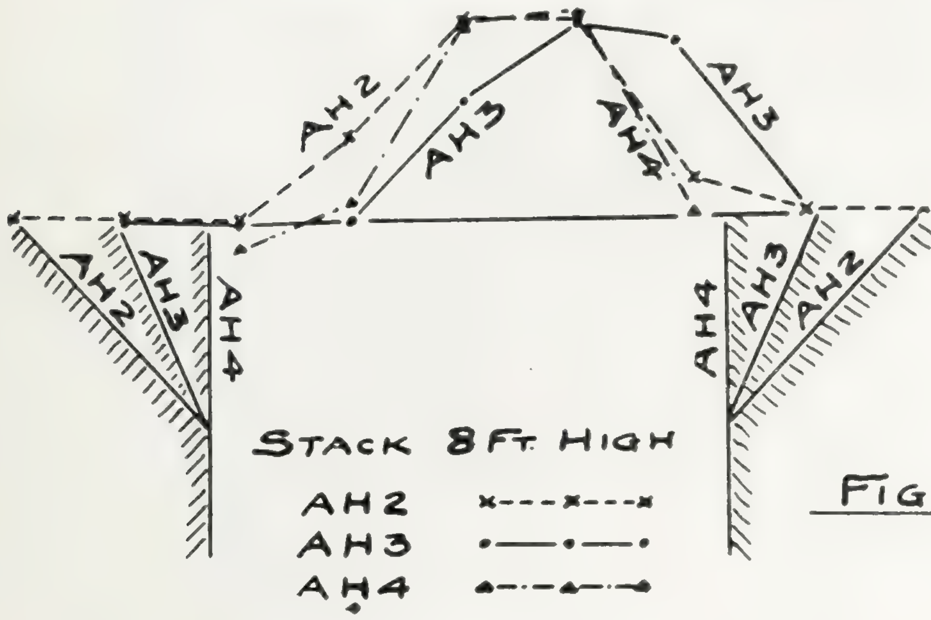


FIG 2

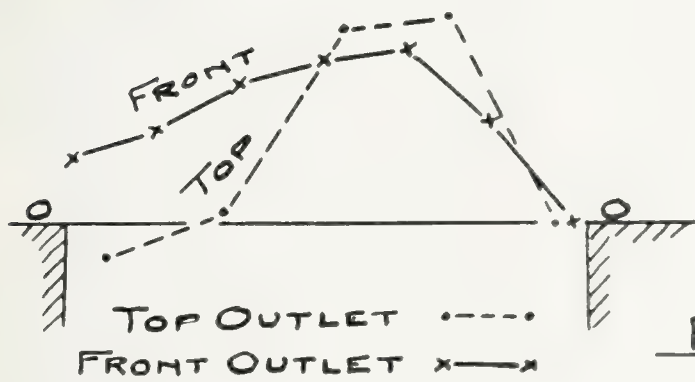


FIG 3

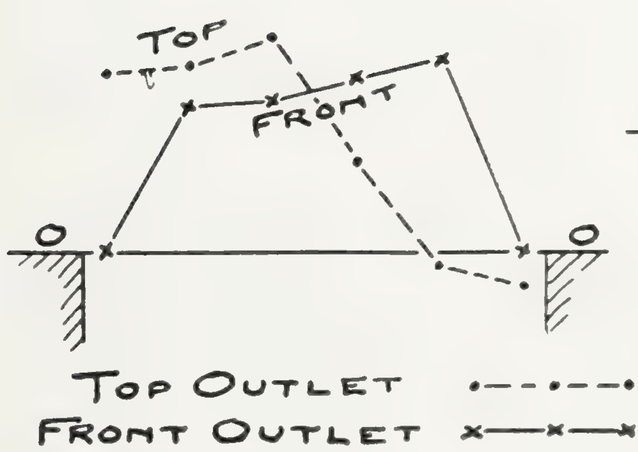


FIG 4

The results obtained with the top of the enclosure set at different angles are shown in Fig. 2. The air velocity curves show that there is practically no upward movement of air outside the vertical stack opening, and that on the left side there is actually a down draught. The equivalent radiations for the three tests were:

A H 2 (10 ft. 11 inches wide) 68.2 square feet (45°)
A H 3 (8 ft. 6 inches wide) 68.7 square feet (22.5°)
A H 4 (6 ft. 8½ inches wide) 71.4 square feet (Parallel)

Thus, the parallel stack is to be preferred.

These tests were all made on days when there was a south-west wind of 3 to 4 miles per hour, so that they are comparative.

(*c*) The uneven distribution of air in Fig. 2 attracted attention to the direction of air flow from the enclosure. It is evident that when the radiator is placed under a window, the downward current of cold air will tend to oppose the upward current of warm air from the enclosure. By placing a baffle over the top and directing the flow of air horizontally, this tendency is reduced and the following figures indicate that the improvement obtained from this change was about 10 per cent.

Test No.	Outlet	Equivalent Radiation square feet	Increased Output per cent.
A H 4	Top	71.4	....
A H 6	Front	78.4	9.8
A K 1	Top	84.2	....
A K 2	Front	92.1	9.4
A L 1	Top	84.6	....
A L 2	Front	93.5	10.5

It was noticed with the vertical outlet, that the air pulsated somewhat, so that correct readings of its velocity were difficult to obtain. These pulsations were almost entirely stopped when the horizontal outlet was substituted, the air being emitted in a steady stream. The distribution of air across the outlet with top and front outlets respectively, is shown in Figs. 3 and 4. These indicate a more uniform distribution for the front outlet, in addition to the steadier flow. The top outlet has a down draught on the left of Fig. 3 and the right of Fig. 4.

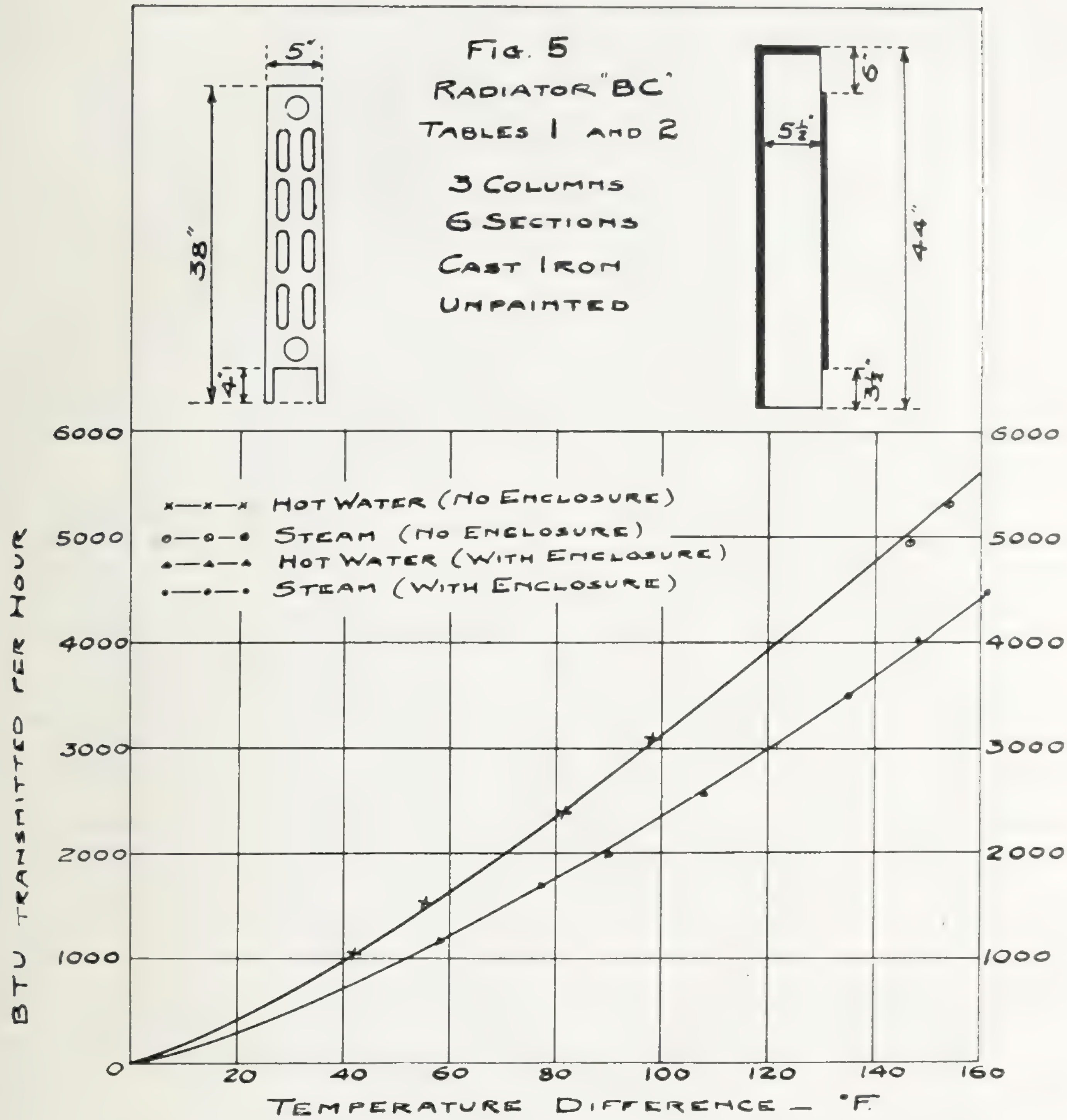
Steam and Hot Water Tests

These tests were made on small radiators of approximately 20 and 40 square feet rating, respectively, and of as many different forms as it was possible to get. There is no intention of comparing the relative merits of the different radiators, save insofar as their design and general dimensions affect the ratio of the output on hot water to that on steam.

The direct heating surface in each case, is considered to be that area which is in contact with steam or water on one side and air on the other. The rest of the measured area is indirect heating surface. Wherever practicable, the temperature of the direct heating surface and that of the indirect surface furthest from the heating medium were measured.

(1) *Radiators "B C" and "B D"*

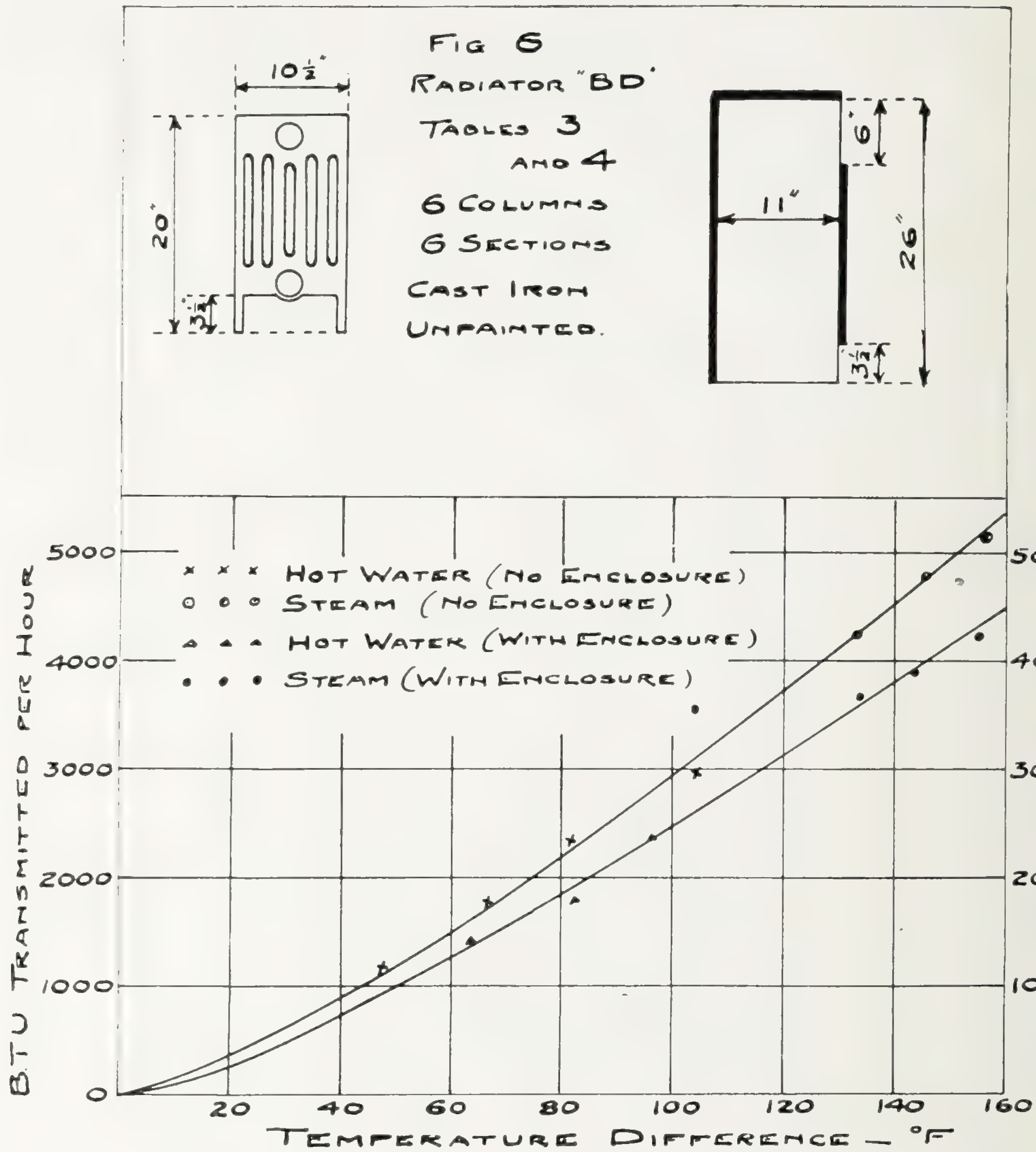
These were both of the ordinary cast iron type and were tested with steam and hot water, respectively, both bare and enclosed. Radiator



B C was tall and thin, and B D was low and wide. Both were unpainted and the results obtained are given on Figs. 5 and 6 and tables 1 to 4.

	B C	B D
Columns	3	6
Sections	6	6
Height—inches	38	20
Length—inches	15	15
Direct heating surface (sq. ft.)	18.7	17.6
Indirect " " (sq. ft.)	0	0
Air inlet area—sq. ft.	0.437	0.437
Air outlet area—sq. ft.	0.75	0.742

The curves show that whether bare or enclosed, the output on hot water, in both cases, follows the same curve as that for steam.



The following relationships are taken from the curves:

Radiator	B C		B D	
State	Bare	Enclosed	Bare	Enclosed
B.T.U. per hour at				
(*a*) 150° F. temp. diff.	5200	4000	4920	4120
(*b*) 105° F. temp. diff.	3300	2520	3100	2600
(*c*) 55° F. temp. diff.	1450	1080	1350	1110
$\frac{b}{a}$ per cent	63.5	63.0	63.0	63.1
$\frac{c}{a}$ per cent	27.9	27.0	27.2	26.9

The above temperature differences are selected because they correspond to those given in the new steam and hot water test codes.

The air velocities (feet per minute) given in table 4 are at the left, centre and right ends of the stack outlet, respectively, and indicate that

the highest velocities occur at the right, or entrance, side of the radiator (Fig. 1).

(2) *Radiators "A O" and "A X"*

These consisted of aluminum plates or fins, into which tubes were expanded, the whole assembly being painted black. The details were:

	A X	A O
Number of tubes	2	6
Diameter of tubes—inches	1½	1½
Length of tubes—inches	19½	19½
Number of fins	45	88
Area of each fin—inches	6¾×4	10¾×4
Direct heating surface—sq. ft.	1.2	3.6
Indirect heating surface—sq. ft.	14.6	45.7
Air inlet area—sq. ft.	0.482	0.482
Air outlet area—sq. ft.	0.878	0.878
Height of feet—inches	4½	4½
Thickness of fins—inches	.041	.041

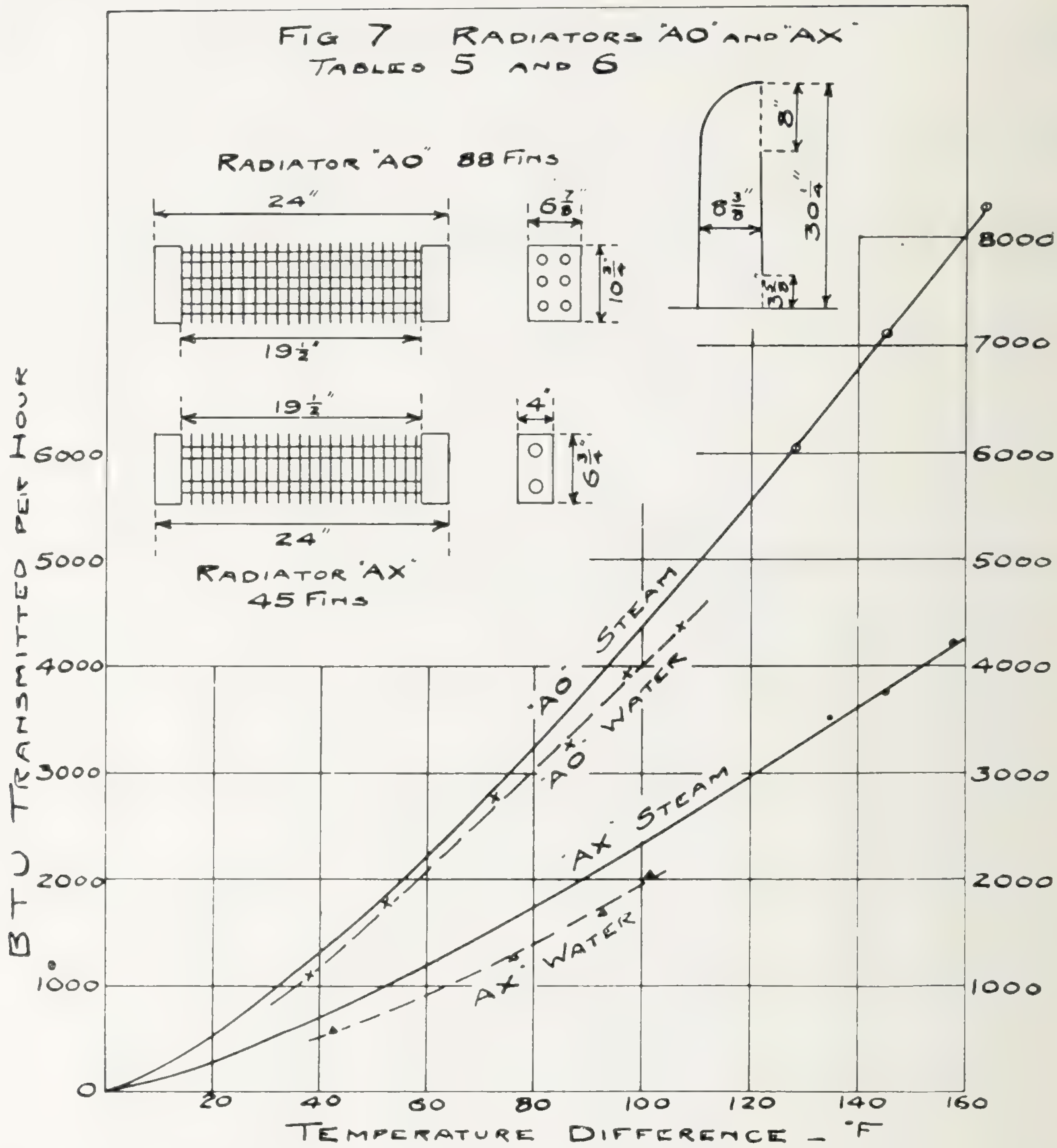
The same enclosure was used for both radiators.

The results obtained are given in tables 5 and 6 and are illustrated by Fig. 7. The following relationships are taken from the curves:

	A X	A O
B.T.U. per hour at		
(*a*) 150° F. temp. diff.	3920	7400
(*b*) 105° F. temp. diff.	1950	4250
(*c*) 55° F. temp. diff.	800	1850
$\frac{b}{a}$ per cent.	49.8	57.5
$\frac{c}{a}$ per cent.	20.4	25.0
Per cent. of steam curves at		
(*d*) 105° F. temp. diff.	79.6	91.4
(*e*) 55° F. temp. diff.	76.2	92.5

These figures show that the output on hot water at a temperature difference of 105° F. is from 10 to 20 per cent. less than that anticipated from the steam tests, and at the lower temperature the output may be as much as 24 per cent. lower.

The air velocities given in tables 5 and 6 were taken at the left, centre and right of the stack outlet, positions 1, 3 and 5 being at the top of the grille, and 2, 4 and 6 at the bottom. The zero figures given for the lower positions indicate in all cases that the velocities were too



low to work the anemometer. It is evident that most of the air comes out at the upper edge.

(3) *Radiators "A M" and "A N"*

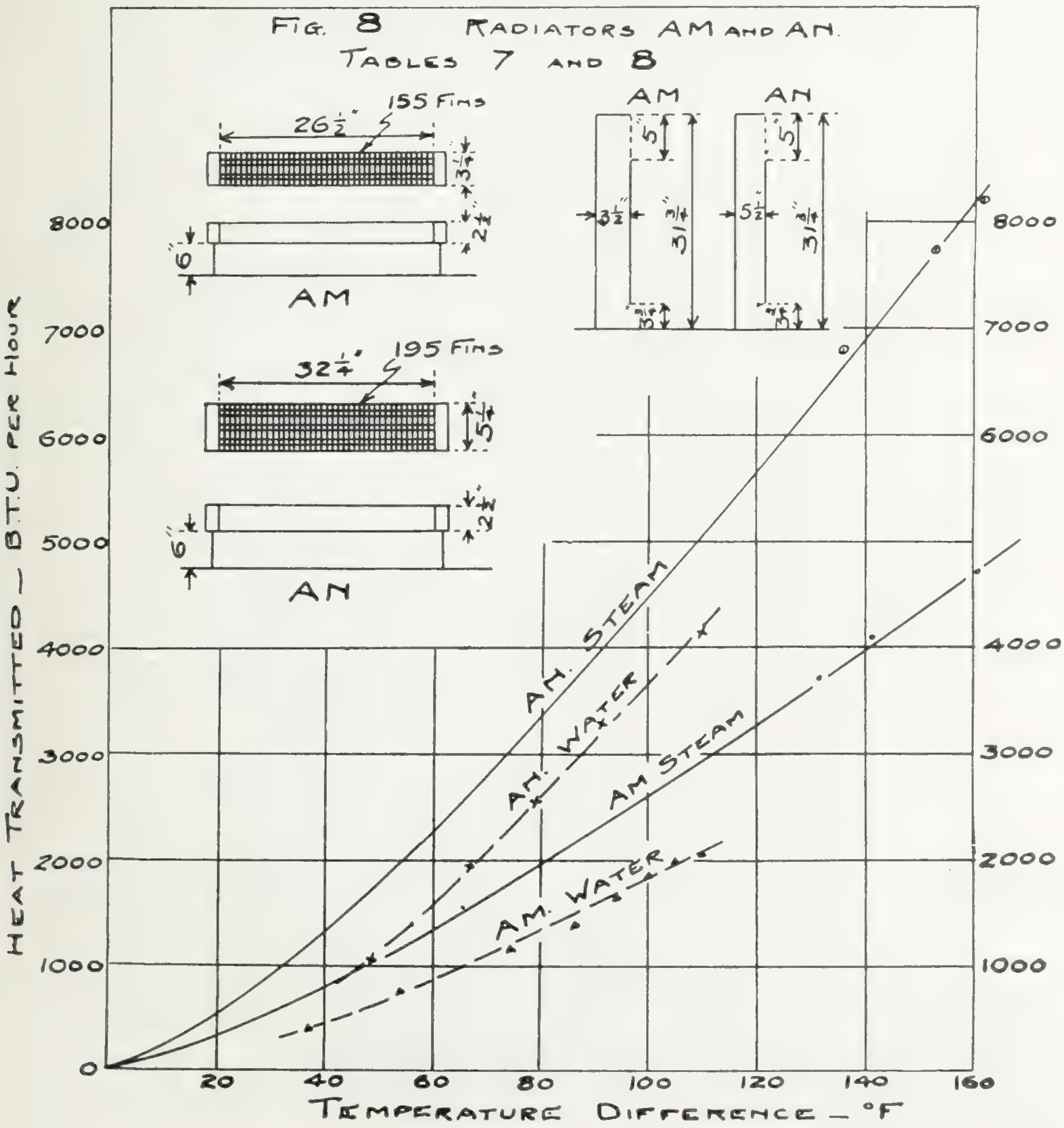
These were both composed of copper tubes expanded into aluminum fins. The latter were separated by means of spacers, which formed an integral part of each fin. The fins were smaller and were more closely spaced than in the previous case and were shrouded at the edges by thin metal strips. The finish was left bright. The dimensions were:

	A M	A N
Number of tubes	2	3
Diameter of spacers—inches	0.53	.53
Length of tubes—inches	26½	32¼
Number of fins	155	195

Area of each fin—inches	2×3 3/16	2×5 3/16
Direct heating surface—sq. ft.	0.95	1.40
Indirect heating surface	12.8	26.5
Air inlet area—sq. ft.	0.71	0.86
Air outlet area—sq. ft.	0.61	0.73
Thickness of fins—inches	.016	.016

The test results are given on Fig. 8 and in tables 7 and 8, and the following figures were obtained from the curves:

	A M	A N
B.T.U. per hour at		
(*a*) 150° F. temp. diff.	4320	7520
(*b*) 105° F. temp. diff.	1950	3920
(*c*) 55° F. temp. diff.	720	1350
$\frac{b}{a}$ per cent	45.2	52.1



$\frac{c}{a}$ per cent	16.7	18.0
Per cent. of steam curve at		
(*d*) 105° F. temp. diff	71.0	82.5
(*e*) 55° F. temp. diff	61.0	67.5

The loss of output in this case is greater than before, being 18 to 29 per cent. at 105° F. and 33 to 39 per cent. at 55° F.

The outlet grille was too narrow for double anemometer readings so that air velocities are given for the left, centre and right positions. The highest velocities were obtained in the centre. It should be noted that the outlet area of the enclosure was less than the inlet area.

(4) *Radiator "A Y"*

This was similar in form to radiators A M and A N, but was made entirely of cast iron, unpainted, the water or steam space being a rectangular cored hole passing through the centre of the casting, with fins radiating on each side. The following are the dimensions:

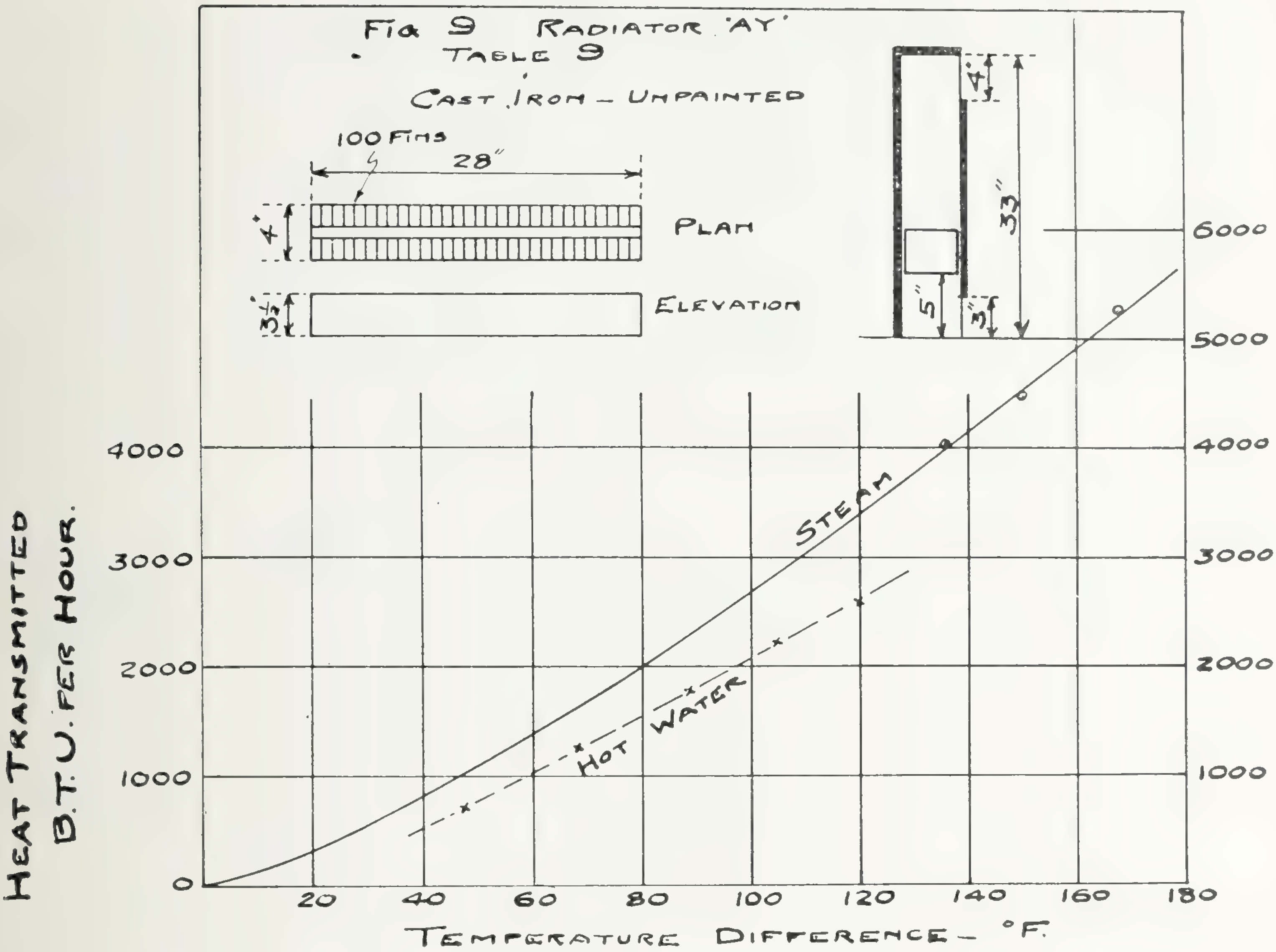
Length of radiator—inches	28
Number of fins	100
Size of fins—inches	$3\frac{3}{4} \times 3\frac{1}{2}$
Direct heating surface—sq. ft.	1.33
Indirect heating surface—sq. ft.	17.5
Air inlet area—sq. ft.	.630
Air outlet area—sq. ft.	0.845
Thickness of fins—inches	.063

The test results are given on Fig. 9 and in table 9.

The following figures were obtained from the curve:

B.T.U. per hour at	
(*a*) 150° F. temp. diff	4520
(*b*) 105° F. temp. diff	2200
(*c*) 55° F. temp. diff	890
$\frac{b}{a}$ per cent	48.7
$\frac{c}{a}$ per cent	19.7
Per cent. of steam curve at	
(*d*) 105° F. temp. diff	77.2
(*e*) 55° F. temp. diff	73.0

The losses in this case are 23 and 27 per cent. at 105° F. and 55° F. temperature differences, respectively. Other tests made on similar radiators with larger and smaller numbers of fins gave a similar relationship.



The air velocities, as before, were at the left, centre and right of the outlet, the highest velocities being obtained at the inlet end.

It was observed generally, that the higher radiators gave outputs on hot water more closely approximating to those on steam than did the lower ones. To check this, two radiators similar to A Y were arranged in parallel, one above the other, so that the steam or water entered at the right end of both radiators and came out on the left side. This increased the total output to 14.5 per cent. more than that of the single radiator A Y and made no difference to the ratio of hot water to steam output, which remained about 77 per cent. at a temperature difference of 105° F.

The single radiator A Y was then placed above and in series with the other two so that the steam or water entered at the right end of A Y, came out on the left and then traversed the other two in parallel, from left to right. The output was then 18.4 per cent. above that of the single radiator A Y, but the ratio of hot water to steam output was raised to 95 per cent.—a considerable improvement.

The next modification was to connect the radiators in such a way that there was a vertical bank of three radiators in the enclosure, all operating in series. This arrangement increased the output on steam

at 150° F. temperature difference by 4.5 per cent. as compared with the previous case. The transmission on hot water, however, was not increased, so that the output ratio was reduced to 90.2 per cent.

The final tests were made with two radiators similar to A Y arranged in series and placed one above the other. The heat transmission on steam was very slightly greater than that with the two radiators in parallel, but that on hot water was improved considerably. At a temperature difference of 105° F. the output on hot water was increased by 18 per cent., and the ratio of the heat transmitted with hot water to that with steam was raised from 77 per cent. to 93.2 per cent.

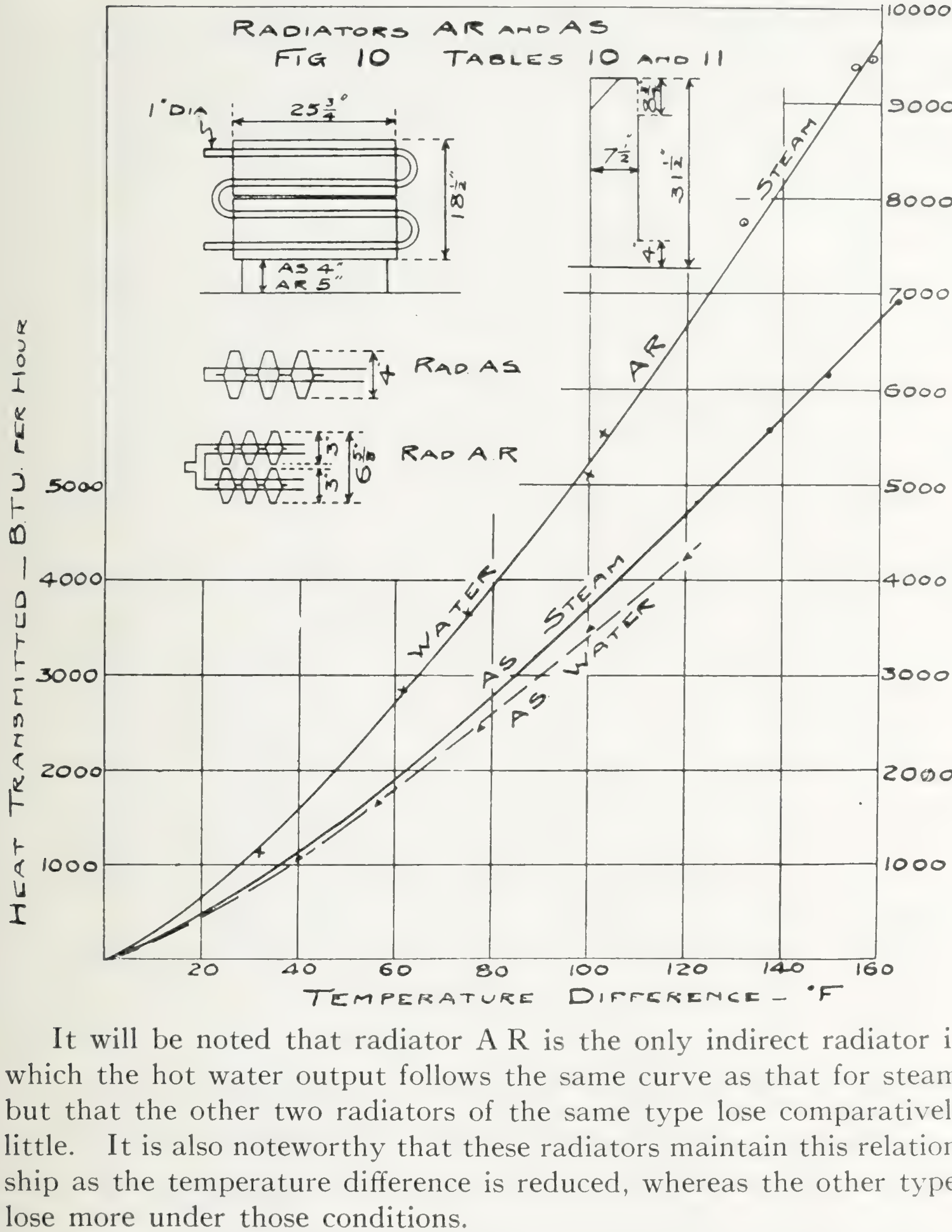
(5) *Radiators "A R", "A S" and "B B"*

These were all of similar construction, being composed of bent copper tubes with pressed sheets clamped and welded to them to form chimneys, as shown in Figs. 10 and 11. Radiators A R and B B were of similar ratings, but different heights, and this comparison was made to indicate the influence of overall dimensions, particularly height, on the relationship between outputs on steam and water. The finish in all cases was black and the dimensions were:

Radiator	A S	A R	B B
Diameter of tubes—inches	1.0	1.0	1.0
Height of chimneys—inches	18½	18½	16½
Number of sections	18	18	30
Rows of tubes	1	2	1
Direct heating surface—sq. ft.	2.83	5.7	4.15
Indirect heating surface—sq. ft.	41.13	64.8	59.34
Air inlet area—sq. ft.	1.10	1.10	1.74
Air outlet area—sq. ft.	1.69	1.69	2.43

The results obtained are given in Figs. 10 and 11 and tables 10 to 12, and the following have been taken from the curves:

	A S	A R	B B
B.T.U. per hour at			
(*a*) 150° F. temp. diff.	6120	8900	9150
(*b*) 105° F. temp. diff.	3600	5600	5250
(*c*) 55° F. temp. diff.	1600	2410	2300
$\frac{b}{a}$ per cent.	58.9	63.0	57.5
$\frac{c}{a}$ per cent.	26.2	27.0	25.2
Per cent. of steam curve at			
(*d*) 105° F. temp. diff.	93.5	100	91.3
(*e*) 55° F. temp. diff.	95.6	100	93.2



It will be noted that radiator A R is the only indirect radiator in which the hot water output follows the same curve as that for steam, but that the other two radiators of the same type lose comparatively little. It is also noteworthy that these radiators maintain this relationship as the temperature difference is reduced, whereas the other types lose more under those conditions.

The air velocity figures in tables 10 to 12, show that the velocity reaches a maximum in the centre of the air outlet. As in previous cases, the velocities at 2, 4 and 6 near the bottom of the air outlet are very small. A smaller enclosure could probably have been used with advantage in the case of radiator A S.

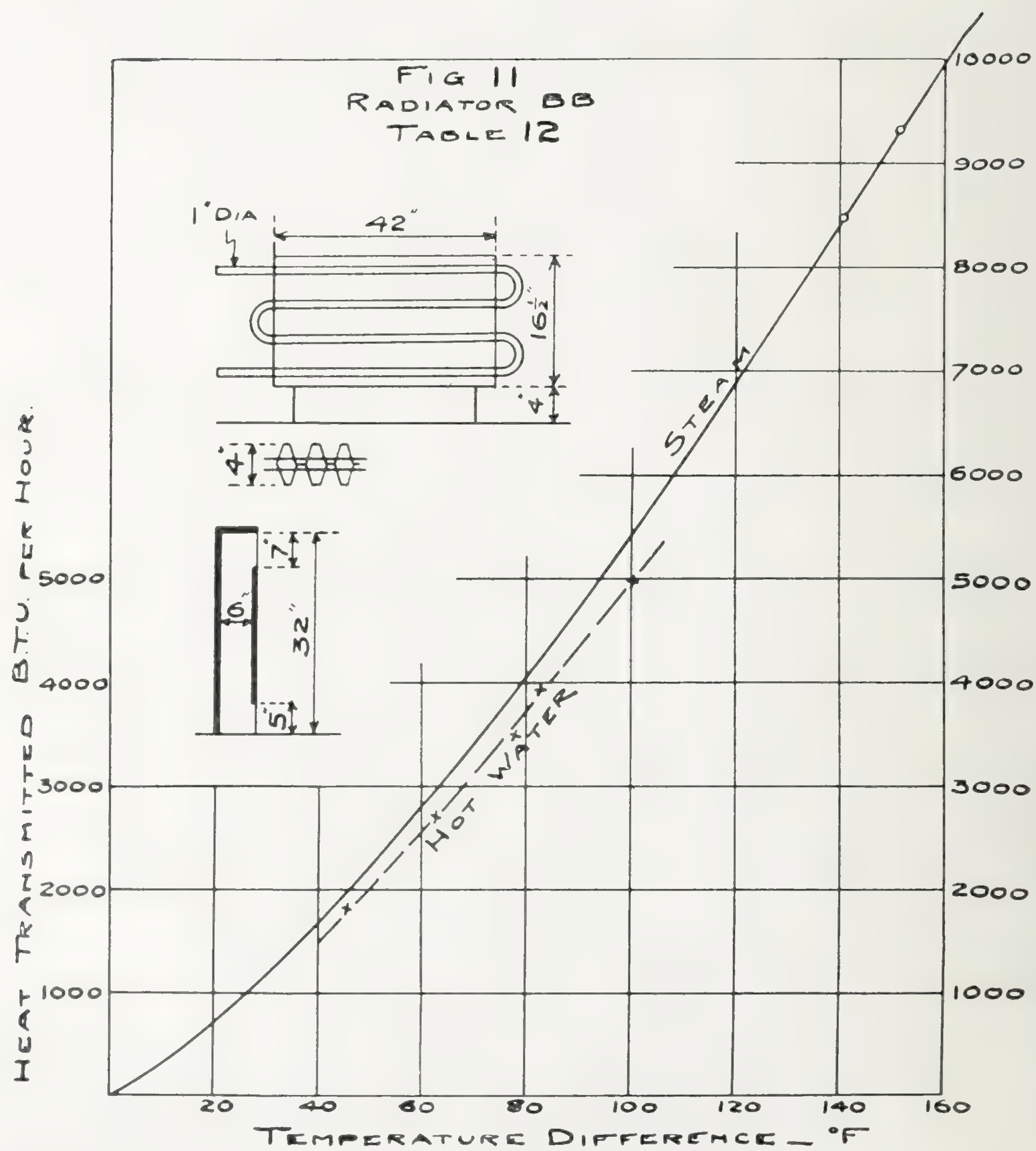
(6) *Radiators "B E" and "B F"*

These radiators consist of one and two rows, respectively, with 5 finned tubes in each, arranged diagonally in the enclosure, as shown in

Fig. 12. The peculiar form of the end castings is designed to permit easy removal of the radiator from the enclosure.

The fins consist of spirals of copper or brass surrounding the tubes, the whole of the surface being coated with lead or solder, making good metallic contact at the joints. The dimensions were:

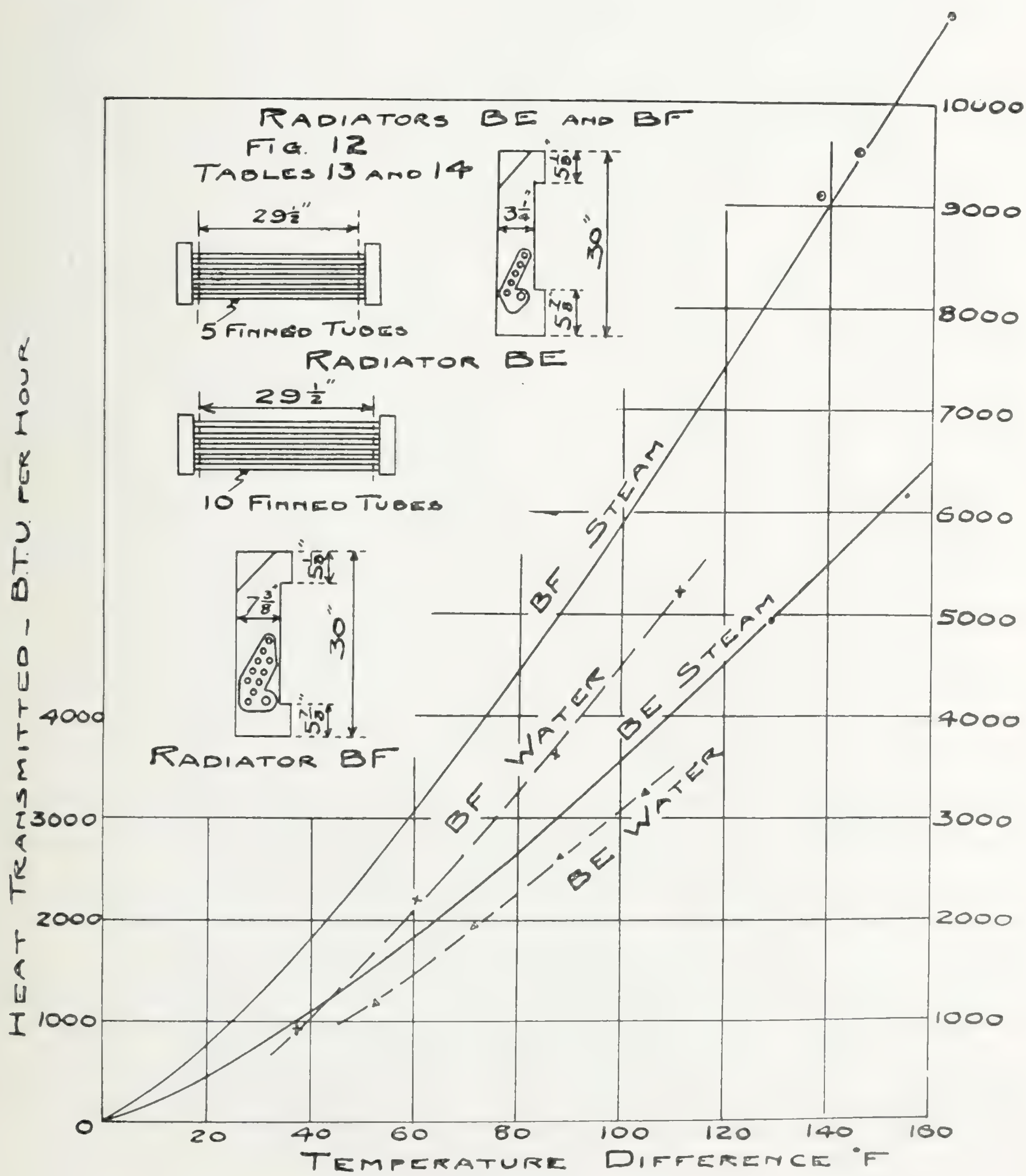
Radiator	B E	B F
No. of tubes	5	10
Diameter of tubes—inches	¾	¾
Length of finned surface—inches	29½	29½
Diameter over fins—inches	1½	1½
Direct heating surface—sq. ft.	3.5	6.2
Indirect heating surface—sq. ft.	16.5	32.9
Air inlet area—sq. ft.	1.21	1.21
Air outlet area—sq. ft.	1.10	1.10



The dampers supplied with the enclosures were kept fully open in all tests.

The results are given on Fig. 12 and in tables 13 and 14, but the following were taken from the curves:

Radiator	B E	B F
B.T.U. output at		
(*a*) 150° F. temp. diff.	6000	9880
(*b*) 105° F. temp. diff.	3250	4800
(*c*) 55° F. temp. diff.	1260	1820
$\frac{b}{a}$ per cent	54.2	48.6
$\frac{c}{a}$ per cent	21.0	18.2



Per cent. of steam curve at		
(*d*) 105° F. temp. diff.	86.2	77.5
(*e*) 55° F. temp. diff.	77.8	68.5

Conclusions

An analysis of the figures given in the preceding paragraphs gives no definite indication of the features of design that are necessary for obtaining the best results on hot water. The output ratio appears to be independent of the ratio of direct to indirect heating surface.

It is noticeable, however, that radiators such as A O, A S, A R, and B B, in which the air passes more than once over a direct heating surface give the highest output ratios. Also, the radiators with dull black finish appear to be better than those left in the bright condition, but the number and variety of experiments is not sufficient to prove this.

The velocities of water and steam, respectively, through the radiators were calculated. The former vary from 1.03 to 17.97 feet per minute at a temperature difference of 105° F., and the latter from 54.6 to 955 feet per minute. Again no relationship was observed between these velocities and the ratio of heat outputs.

The coefficients of heat transmission were then computed for each radiator, the figures for steam being plotted in Fig. 13 from 110° F. upwards, and for water from 110° F. downwards. This division is purely arbitrary but corresponds fairly well to the usual working range in each case. The variation in these coefficients is remarkable, ranging as it does from over 2.0 to less than 0.9 B.T.U. at the standard temperature difference of 150° F. It is also evident that the higher coefficients are associated with the greater differences between steam and hot water outputs and that radiators A O to A R (Fig. 13) having the higher output ratios, have very low heat transmissions.

The ordinary cast iron radiators B C and B D, when tested bare have almost exactly the same heat transmissions per square foot for one degree temperature difference, in spite of their different proportions, and are included for comparative purposes.

The results may, therefore, be summarized as follows:

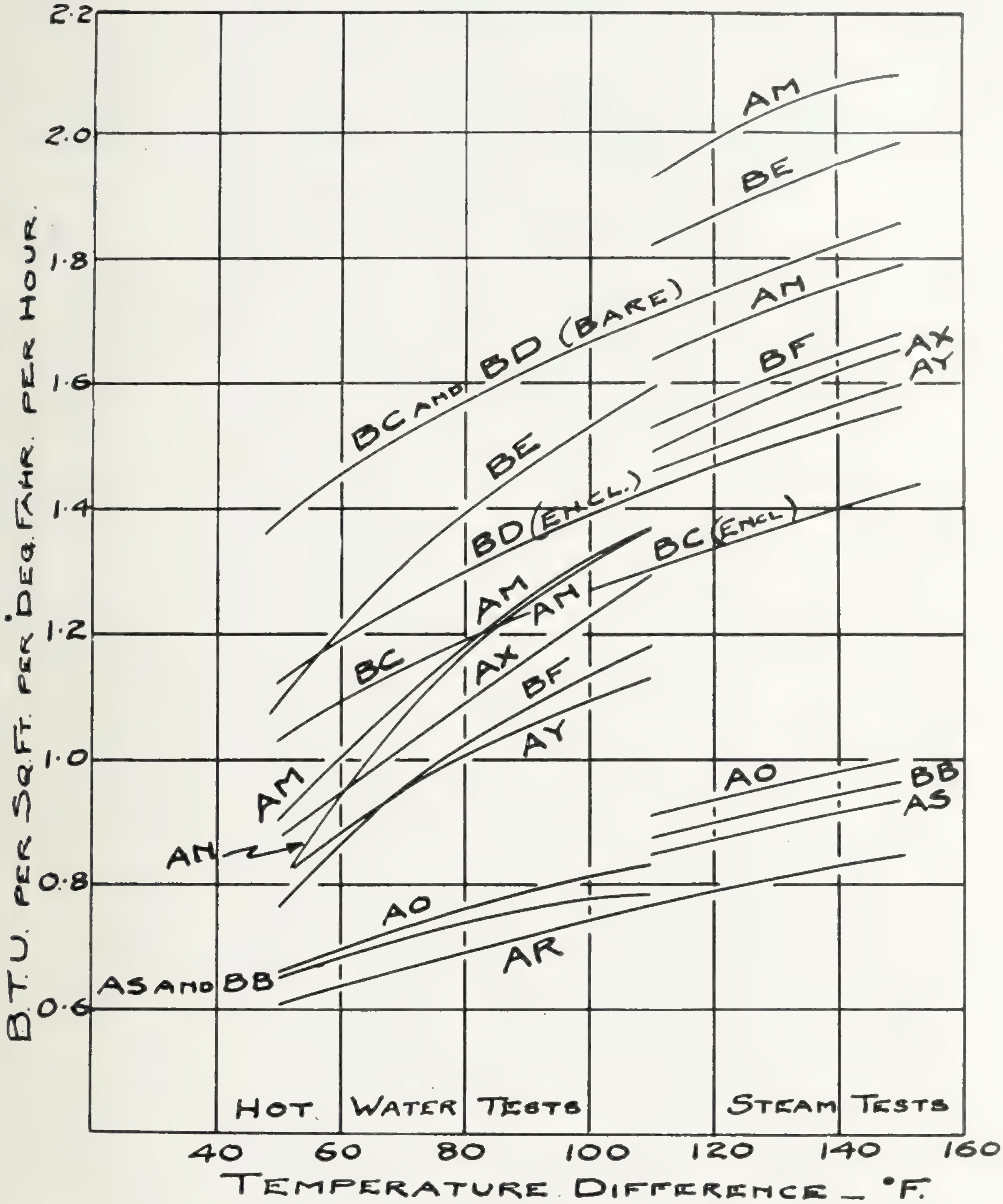
(1) The output of a concealed radiator varies with the area of the air inlet, which may be too large to give the best efficiency.

(2) The best form of stack is that with vertical sides. Increasing the area at the top may reduce the output considerably.

(3) When placed under a window, down draft and consequent inefficiency may be avoided by putting the air outlet in the front of the enclosure instead of at the top.

(4) The heat output with hot water is usually less than that expected

FIG. 13.



from steam tests. The amount of loss appears to depend more on the disposition of the direct heating surface than on the relative amounts of direct and indirect surface. It may also bear some relation to the finish of the surface, whether bright or dull.

(5) The distribution of air across the outlet is seldom uniform. In most cases the greatest velocity is obtained on that side of the radiator at which the steam or water enters. Most of the air is emitted from the top three or four inches of the outlet slot or grille.

(6) The differences between the temperatures of the direct and indirect heating surfaces were generally less than 1° F., except in the

case of A Y, where consistent differences of $1\frac{1}{2}$ to 2° F. were observed. It follows that if the direct heating surface is the more efficient, this is probably due to the fact that in most cases the air flow is perpendicular to the direct heating surface and parallel to the indirect heating surface.

(7) The coefficient of heat transmission varies considerably in concealed radiators, but generally the higher ratios of hot water to steam outputs are obtained with the surfaces that are lightly loaded from a heat transmission standpoint.

The author's thanks are due to Mr. E. B. Hymmen, who was largely responsible for the actual experimental work, and to Mr. D. T. Hewson, B.A.Sc., who assisted in this work.

TABLE I

RADIATOR B.C. (No enclosure)

Date—1931	June 30	June 30	July 6	July 3	July 3	July 4	July 6
Test No.	1	2	7	4	5	6	8
Barometer—in Hg	29.7	29.67	29.54	29.67	29.68	29.67	29.59
Humidity—per cent	72	65	61.5	71.5	71.5	67	72
Water used—lb/hour	5.14	4.58	5.57	109.8	74.6	50.7	143
Inlet water temp.—°F	...	...	...	176.5	150.3	132.5	195.7
Outlet water temp.—°F	...	...	...	154.8	129.8	112.2	174.2
Mean water temp.—°F	226.0	214.3	234.8	165.7	140.1	122.4	185.0
Temp. drop—°F	...	...	...	21.7	20.5	20.3	21.5
B.T.U./hour	4950	4450	5330	2380	1530	1030	3080
Mean air temp.—°F	82.2	85.7	80.7	84.1	84.9	81.0	86.9
Mean temp. diff.—°F	143.8	128.6	154.1	81.6	55.2	41.4	98.1
B.T.U./hr. (T. couples)	...	...	...	2430	1539	1005	2960

TABLE 2

RADIATOR B.C. (with enclosure)

Date—1931	Sept. 15	Sept. 15	Sept. 16	Sept. 16	Sept. 17	Sept. 17	Sept. 18
Number of test	1	2	3	4	5	6	7
Barometer—in Hg	29.7	29.73	29.93	29.87	29.55	29.6	29.83
Relative humidity—%	69	62	65	69.5	87	71	65
Waterused—lb/hour	4.6	3.55	4.14	135.6	80.8	108.9	55.1
Inlet water temp.—°F	...	...	...	193.9	162.8	177.7	139.8
Outlet water temp.—°F	...	...	...	175.2	142.1	159.7	119.3
Mean water temp.—°F	237.6	214.0	222.0	184.6	152.5	168.7	129.6
Temperature drop—°F	...	...	...	18.7	20.7	18.0	20.5
B.T.U. per hour	4400	3445	4010	2535	1674	1960	1130
Mean air temp.—°F	76.2	78.6	73.1	76.9	75.2	79.2	71.8
Air from radiator—°F	145	137	141.9	129.3	113.3	121.5	102.3
Mean temp. diff.—°F	161.4	78.6	148.9	107.7	77.3	89.5	57.8
Air velocity—ft/min. 1	130	114	115.5	87.5	62	73.5	48.5
Air velocity—ft/min. 2	155.5	138	152.5	120.5	89.5	104	69.8
Air velocity—ft/min. 3	121	114.5	117	90.5	74.5	85	56.5
B.T.U./hr. Th. couples)	...	...	...	2620	1755	2025	1130

TABLE 3

Radiator B.D. (no enclosure)

Date—1931	July 7	July 8	July 8	July 9	July 10	July 10	July 11
Test No.	1	3	4	5	7	8	9
Barometer—in. Hg	29.67	29.73	29.76	29.92	29.79	29.8	29.71
Relative humidity—%	69	61	62	58	62	60	75
Water used—lb/hour	84.8	126.3	110.3	4.95	5.35	4.35	52.3
Inlet water temp.—°F.	159.0	198.2	179.4	...	...	...	130.2
Outlet water temp.—°F.	138.0	175.0	158.2	...	...	...	108.1
Mean water temp.—°F.	148.5	186.6	168.8	225.6	236.2	214.5	119.2
Temp. drop—°F.	21.0	23.2	21.2	...	...	...	22.1
B.T.U./hour	1785	2980	2340	4770	5120	4220	1153
Mean air temp.	81.6	82.2	87.1	79.9	79.8	81.4	71.8
Mean temp. diff.	66.9	104.4	81.7	145.7	156.4	133.1	47.4
B.T.U./hour (T. couples)	1785	2890	2350	...	...	...	1125

TABLE 4

RADIATOR B.D. (with enclosure)

Date—1931	July 15	July 15	July 16	July 16	July 17	July 17
Test No.	1	2	3	4	5	6
Barometer—in. Hg	29.68	29.63	29.71	29.72	29.7[illegible]	29.66
Relative humidity—%	72	66	75	73	78	76
Water used—lb/hour	68.3	124.5	100.0	4.04	3.77	4.41
Inlet water temp.—°F	153.2	190.5	169.9	...	...	...
Outlet water temp.—°F	132.4	171.3	151.9	...	...	...
Mean water temp.—°F	142.8	180.9	160.9	225.5	214.3	236.1
Temperature drop—°F	20.8	19.2	18.0	...	...	...
B.T.U./hour	1423	2390	1800	3690	3660	4220
Mean air temp.—°F	79.1	84.3	78.6	82.0	80.5	81.0
Mean temp. diff.—°F	63.7	96.6	82.5	143.5	133.8	155.1
Air velocity—1	32.3	41	35	67	60	71.5
Air velocity—2	52	74	60	92.5	93	100
Air velocity—3	52	74	64	113	98	107.5
Air leaving rad.—°F	107.8	118.7	107.8	137.2	118.0	138.0
B.T.U./hr. (T. couples)	1335	2200	1660	...	...	...

TABLE 5

Radiator A.O.

Date—1931	June 16	June 17	June 17	June 18	June 18	June 19	June 19	June 20	June 22
Test No.	3	4	5	6	7	8	9	10	11
Barometer—in. Hg.	29.75	29.75	29.77	29.84	29.8	29.68	29.66	29.66	29.99
Relative humidity—%.	56	56	57	69.5	52	70	80	76	50
Water used—lb/hour.	164	139.6	89	46.5	209	199.2	6.21	7.37	8.65
Inlet water temp.—°F.	172.4	154.9	139.7	122.9	198.3	186.4	..	..	..
Outlet water temp.—°F.	152.5	135.1	119.7	99.8	177.5	166.7	..	..	..
Mean water temp.—°F.	162.4	145.0	129.7	111.4	187.9	176.6	213.4	224.6	236.4
Temperature drop—°F.	19.9	19.8	20.0	23.1	20.8	19.7	..	..	..
B.T.U./hour.	3265	2765	1780	1075	4350	3920	6030	7100	8270
Mean air temp.—°F.	75.6	72.5	77	73.2	80.6	79.2	85.1	79.1	72.9
Mean temp. diff.	86.8	72.5	52.7	38.2	107.3	97.4	128.3	145.5	163.5
Air from rad.	133.6	122.8	114.1	99.2	154.0	147.0	176.4	182.5	188.5
Air velocity—ft/min. 1.	57	49	33	0	63.5	60	84.5	..	..
Air velocity—ft/min. 3.	82	73	56	35	95	89.5	115	..	..
Air velocity—ft/min. 5.	73	68	52	31	90	87.5	108.5	..	..
Air velocity—ft/min. 2, 4, 6.	0	0	0	0	0	0	0	..	..
B.T.U./hr.(T. couples).	3360	2765	1790	1095	4540	4100	..	.	..

TABLE 6

RADIATOR A.X.

Date—1931	July 20	July 20	July 21	July 21	July 22	July 22	July 22
Test No.	1	2	3	4	5	6	7
Barometer—in. Hg	29.53	29.5	29.49	29.43	29.42	29.41	29.39
Relative humidity—%	81	83	78	76	57	57	65
Water used—lb/hour	65.6	33.3	85.2	74.6	4.4	3.98	3.61
Inlet water temp.—°F	166.2	130.6	199.9	185.0	...	...	...
Outlet water temp.—°F	146.5	112.9	178.5	162.8	...	...	...
Mean water temp.—°F	156.4	121.8	189.2	173.9	233.8	225.4	213.8
Temperature drop—°F	19.7	17.7	21.3	22.2	...	...	...
B.T.U./hour	1255	589	2030	1658	4210	3740	3503
Mean air temp.—°F	80.1	79.6	80.7	81.1	75.8	80.3	78.9
Mean temp. diff.—°F	76.3	42.2	108.5	92.8	158.0	145.1	134.9
Air from rad.—°F	102.7	92.5	113.6	109.2	131.2	131.6	127.1
Air velocity—ft/min. 1	33	0	53	42	89.6	73	73
Air velocity—ft/min. 3	47	0	68	64	109.5	93	93
Air velocity—ft/min. 5	42	0	53	55	101.0	80	72
Air velocity—ft/min. 2, 4, 6	0	0	0	0	0	0	0
B.T.U./hr. (T. couples)	1320	619	2210	...	...	...	...

*A zero figure for the air velocity indicates that the speed was too low for the anemometer to measure it.

TABLE 7

RADIATOR A.M.

Date—1931	May 13	May 14	May 14	May 15	May 15	May 16	May 18	May 19	May 19	May 20	May 21	May 21
Test No.	4	5	6	7	8	9	10	12	13	14	15	16
Barometer—in. Hg.	29.6	29.54	29.54	29.59	29.57	29.33	29.69	29.59	29.59	29.51	29.84	29.87
Relative humidity—%	56	42	40	49.5	48	53.0	43.0	56.0	52.0	61.0	43.0	40
Water used—lb/hr.	43.75	90	53.7	22.35	104.0	88.9	62.7	4.29	3.81	4.92	5.12	64.4
Inlet water temp.—°F.	141.6	196.7	165.1	124.4	201	186.9	173.9	...	...	...	...	180.0
Outlet water temp.—°F.	124.4	174.6	143.5	106.4	181.2	166.2	151.6	...	...	...	...	154.4
Mean water temp.—°F.	133	185.7	154.3	115.4	199.1	176.55	162.75	219.1	213	235.73	237	167.2
Temperature drop—°F.	17.2	22.1	21.6	18.0	19.8	20.7	22.3	...	...	...	...	25.6
B.T.U./hour	752	1990	1160	402	2065	1845	1398	4080	3700	4700	4900	1650
Mean air temperature—°F.	79.15	80.5	79.8	78.3	80.5	75.4	76.5	77.6	81.25	74.9	68.95	73.1
Mean temp. difference—°F.	53.85	105.2	74.5	37.1	110.6	101.15	86.25	141.5	131.75	160.83	168.05	94.1
Air from radiator	100.35	130	113.3	88.4	134.4	122.4	112.1	158	157.0	165.3	162.7	118
Air velocity—ft/min. 1	0	0	0	0	0	0	0	40.5	36.5	45	44.3	0
Air velocity—ft/min. 2	0	52	0	0	55.5	33.5	40	99.5	97.5	107.5	110	46.5
Air velocity—ft/min. 3	0	0	0	0	55	0	0	79	68.5	79.0	89.3	46.5
B.T.U./hour (T. couples)	775	2100	1225	...	2010	1994	1538	...	...	...	...	1720

TABLE 8

RADIATOR A.N.

Date—1931	June 5	June 5	June 6	June 12	June 12	June 13	June 15	June 15
Test No.	3	4	5	6	7	8	9	10
Barometer—in. Hg	29.66	29.71	29.8	29.85	29.85	29.78	29.76	29.77
Relative humidity—%	59	46	55	63	60	58	67	60
Water used—lb/hour	194.8	123	45.9	8.57	7.0	8.03	87.2	172.2
Inlet water temp.—°F	196.2	165.5	130.7	..	..	..	153.6	178.9
Outlet water temp.—°F	175.0	144.6	107.8	..	..	..	131.2	159.8
Mean water temp.—°F	185.6	154.1	119.3	237.8	213.6	226.2	142.4	169.4
Temperature drop—°F	21.2	20.9	22.9	..	..	..	22.4	19.1
B.T.U./hour	4130	2570	1052	8190	6800	7720	1955	3290
Mean air temp.—°F	75.7	76.2	69.8	75.2	77.4	73.2	75.1	78.2
Mean temp. diff.—°F	109.9	78.9	49.5	162.6	136.2	153.0	67.3	91.2
Air from rad.—°F	133.8	117.0	92.0	172.4	160.8	165.9	110.1	128.1
Air velocity—ft/min. 1	43.3	0	0	71	71	..	..	..
Air velocity—ft/min. 2	66	50	0	117.5	83	..	..	..
Air velocity—ft/min. 3	81	65.5	0	120.5	107	..	..	..
B.T.U./hr. (T. couples)	4320	2680	1090	..	..	..	1990	3290

TABLE 9

RADIATOR A.Y.

Date—1931	May 22	May 23	May 26	May 26	May 27	May 27	May 28	May 28
Test No.	1	2	3	4	5	6	7	8
Barometer—in. Hg.	29.98	29.73	29.85	29.88	30.02	29.99	29.93	29.88
Humidity—%	60	54	47	47	49	46	54	53
Water used—lb/hour	5.5	133	62.9	81.1	109.8	4.15	35.6	4.65
Inlet water temp.—°F.	..	200.4	152.1	180.1	188.9	..	129.2	..
Outlet water temp.—°F.	..	181.1	132.4	158.4	168.8	..	109.6	..
Mean water temp.—°F.	237.9	190.8	142.2	169.2	178.0	214.0	119.4	228.8
Temperature drop—°F.	..	19.3	19.7	21.7	20.1	..	19.6	..
B.T.U./hour	5270	2570	1240	1760	2210	4030	698	4470
Mean air temp.—°F.	70.0	72.8	74.1	80.3	73.3	77.9	71.8	79.1
Mean temp. diff.	167.8	118.0	68.1	88.9	104.7	136.1	47.6	149.7
Air from rad.—°F	155.8	119.5	97.5	115.8	114.1	150.1	89.7	156.4
Air velocity—ft/min. 1	44.8	0	0	0	0	34.5	0	56
Air velocity—ft/min. 2	113.0	31.5	32	45	52	109	0	102.5
Air velocity—ft/min. 3	112.0	85	67.8	85	97.5	113	52	109.5
B.T.U./hr. (T. couples)	..	..	1300	1790	2210	..	..	..

TABLE 10

RADIATOR A.R.

Date—1931	April 15	April 16	April 17	April 20	June 29	June 29	Feb. 18	Feb. 18
Test No.	3	4	5	6	7	8	1	1
Barometer—in. Hg	30.14	30.0	29.73	29.87	29.82	29.83	29.85	29.81
Relative humidity—%	..	..	..	..	60	63	48	60
Water used—lb/hour	264	138	179	55	7.89	9.9	9.75	250
Inlet water temp.—°F	181.3	145.8	160.4	117.5	..	..	..	174.4
Outlet water temp.—°F	162.1	125.3	140.0	97.4	..	..	..	152.7
Mean water temp.—°F	171.7	135.5	150.2	107.4	214.5	235.6	222.1	163.6
Temperature drop—°F	19.2	20.5	20.4	20.1	..	..	..	21.7
B.T.U./hour	5085	2835	3640	1109	7750	9470	9400	5540
Mean air temp.—°F	71.7	74.8	75.2	75.4	82.4	76.8	66.8	60.5
Mean temp. diff.—°F	100	60.7	75.0	32.0	132.1	158.8	155.3	103.1
Air from rad.—°F	108.8	103.8	119.3	.	150.8	163.1	158	120
Air velocity—ft/min. 1	81	70	75	32	69	61	..	..
Air velocity—ft/min. 2	0	0	0	0	0	0	..	..
Air velocity—ft/min. 3	100	73	87	33	111	96	..	..
Air velocity—ft/min. 4	0	0	0	0	0	0	..	..
Air velocity—ft/min. 5	88	63	68	31	52.5	86	..	..
Air velocity—ft/min. 6	0	0	0	0	0	0	..	..
B.T.U./hr. (Th. couples)	..	..	..	..	..	..	..	..

TABLE 11

RADIATOR A.S.

Date—1931	May 1	May 4	May 5	May 5	May 6	Sept. 1	Sept. 1	Sepr. 2
Test No.	2	3	4	5	6	7	8	9
Barometer—in. Hg	29.91	29.95	29.78	29.71	29.62	29.57	29.56	29.59
Relative humidity—%	52.0	67.5	59.70	43	67.5	64.0	69.0	68.0
Water used—lb/hr	216.0	146.9	97.1	122.1	52.3	7.25	5.7	6.33
Inlet water temp.—°F	202.1	189.3	142	165.4	126.4	237.6	214.0	222.2
Outlet water temp.—°F	182.3	165.4	124.8	145.5	106.1	..	..	..
Mean water temp.—°F	192.2	177.4	133.4	155.5	116.2	237.6	214.0	222.2
Temperature drop—°F	19.8	23.9	17.2	19.9	20.34	..	..	..
B.T.U./hour	4270	3510	1671	2430	1063	6920	5530	6110
Mean air temp.—°F	71.64	74.9	76.45	77.7	76.27	73.6	76.4	72.4
Mean temp. diff.—°F	120.6	102.5	56.95	77.8	40	164.0	137.6	149.8
Air from radiator—°F	96.6	100.6	88	93.8	84	157.1	144.1	146.9
Air velocity—ft/min. 1	55	68.5	0	34	0	88	70	76
Air velocity—ft/min. 3	94.5	97	40	47.5	0	141	125	130
Air velocity—ft/min. 5	57.5	0	0	0	0	100	78	86
Air velocity—ft/min. 2, 4, 6	0	0	0	0	0	0	0	0
B.T.U./hr. (Th. couples)	4450	3670	1770	2570	1075	..	..	..

TABLE 12

RADIATOR B.B.

Date—1931	June 23	June 23	June 26	June 24	June 24	June 25	June 25	June 26
Test No.	1	2	8	3	4	5	6	7
Barometer—in. Hg	29.78	29.82	29.84	29.94	29.94	29.51	29.85	29.82
Relative humidity—%	75	61.5	78	71.5	70	58	67	78
Water used—lb/hour	11.06	9.66	8.73	258	190	159.3	90.6	140.0
Inlet water temp.—°F	..	..	..	183.0	170.6	160.6	128.0	146.6
Outlet water temp.—°F	..	..	..	163.8	150.0	138.7	108.0	126.5
Mean water temp.—°F	236.2	223.5	213.7	173.4	160.3	149.7	118.0	130.6
Temperature drop—°F	..	..	..	19.2	20.6	21.9	20.0	20.1
B.T.U./hour	10590	9310	8470	4960	3915	3490	1812	2815
Mean air temp.—°F	68.4	71.7	73.4	73.2	76.97	71.9	72.5	73.9
Mean temp. diff.—°F	167.8	151.8	140.5	100.2	82.33	77.8	45.5	62.7
Air from radiator—°F	148.1	144.3	146.1	121.1	113.1	110.9	96.3	104.4
Air velocity—ft/min. 1	78	66.5	69	46.5	35	0	0	0
Air velocity—ft/min. 2	107	98	89	70.5	44.5	51	33.5	41
Air velocity—ft/min. 3	120	98.5	95	69	53.0	56	35.5	46
Air velocity—ft/min. 4	130.5	121.5	108	80	59	61	42	56.5
Air velocity—ft/min. 5	137.5	139.5	131	90	77.5	75	34	64
B.T.U./hr. (T. couples)	..	..	..	4820	4050	3160	1795	2770

TABLE 13

RADIATOR B.E.

Date—1931	July 25	July 27	July 27	July 28	July 28	July 29	July 29
Test No.	2	3	4	5	6	7	8
Barometer—in. Hg	29.78	29.74	29.72	29.68	29.65	29.57	29.56
Relative humidity—%	67	70	70	70.5	70	76	70
Water used—lb/hour	88.6	146.1	161.2	117.3	6.05	6.34	5.08
Inlet water temp.—°F	157.9	195.3	141.7	179.7	..	..	..
Outlet water temp.—°F	136.1	173.1	122.6	157.4	..	..	..
Mean water temp.—°F	147.0	184.2	132.2	168.6	228.8	236.5	214.0
Temperature drop—°F	21.8	22.2	19.1	22.3	..	..	..
B.T.U./hour	1953	3245	1169	2615	5820	6170	4930
Mean air temp.—°F	75.3	79.5	79.7	79.9	84.7	81.3	84.9
Mean temp. difference—°F	71.7	104.7	52.5	88.7	144.1	155.2	129.1
Air from radiator—°F	93.4	130.9	96.0	119.5	164.2	167.5	157.2
Air velocity—ft/min. 1	..	0	0	0	0	0	0
Air velocity—ft/min. 2	..	66.5	0	74.5	133	140	122.5
Air velocity—ft/min. 3	..	75	0	72.5	98	103.5	89.5
B.T.U./hour (Th. couples)	1980	3320	1163	2665	.	..	..

TABLE 14

RADIATOR B.F.

Date—1931	July 30	July 31	July 31	Aug. 28	Aug. 28	Aug. 29	Aug. 31
Test No.	1	2	3	4	5	6	7
Barometer—in. Hg	29.75	29.89	29.92	29.56	29.51	29.6	29.84
Relative humidity—%	60	64	53	75	75	55	60
Water used—lb/hour	170.2	105.6	9.88	11.35	9.35	236.5	48.0
Inlet water temp.—°F	182.9	146.0	...	...	...	196.5	115.5
Outlet water temp.—°F	161.6	125.3	...	...	...	174.5	96.6
Mean water temp.—°F	172.3	135.7	226.3	237.6	214.0	185.5	106.1
Temperature drop—°F	21.3	20.7	...	...	...	22.0	18.9
B.T.U./hour	3625	2185	9515	10,860	9070	5210	907
Mean air temp.—°F	85.0	75.1	80.6	74.5	76.1	74.0	68.8
Mean temp. diff.—°F	87.3	60.6	145.7	163.1	137.9	111.5	37.3
Air velocity—ft/min. 1	64	51.5	124.5	139.5	124	105	0
Air velocity—ft/min. 2	75.5	58.0	143.0	153.0	140.5	107	0
Air velocity—ft/min. 3	93	69.5	132	148.5	129.5	117.5	33
B.T.U./hr. (Th. couples)	3820	2125	...	...	...	5180	...

A FURTHER INVESTIGATION OF THE ELASTIC CATENARY AS APPLIED TO BASE MEASUREMENT

By Louis B. Stewart[1]

A former paper devoted to this subject was published in Bulletin No. 8 of the School of Engineering Research, in which equations were developed adapted for the reduction of base measurements with steel or invar tapes. In that paper it was assumed that a constant tension T is applied to the forward end of the tape in measuring a base, the effect of tension, however, being expressed in terms of a quantity T_m, the mean of the end tensions applied to the tape. A relation between T and T_m was derived, by which the latter may be computed when the former is given.

The base, it may be stated, was assumed to have been prepared for measurement by planting posts throughout its length, at intervals of about a tape-length apart, which have been carefully aligned and firmly driven into the ground, their heights being about 30 inches above the ground. To the top of each post a small plate of sheet metal is nailed on which a fine line is ruled at right angles to the direction of the base, thus giving a series of definite points between which the measurements may be made. In measuring the base the tape is stretched between each consecutive pair of posts, a constant tension being applied to its forward end, and is unsupported except at the ends, which rest lightly on the posts. The intervals between the zero marks on the tape and the lines on the posts are then read on scales engraved on the tape. The temperature of the tape, given by thermometers suspended near it and at intervals along its length, and the difference of level of the posts, found by spirit leveling, complete the field record.

In the reduction of the field measurements the quantity to be determined is the length of the projection of the tape on a horizontal plane, when it is stretched in the manner just described. This reduction involves the application of four corrections to the standard length of the tape: for temperature, tension, sag, and grade. The practice has been to apply these corrections separately by the aid of four distinct expressions, and in so doing certain assumptions are made which are only approximately true; *viz.*, that the tension is the same at all points of the tape, and is unchanged when the tape is inclined; that the weight of the tape per unit of length is constant throughout; and that the sag of the tape is the same when it is inclined as when its ends are at the

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same level. The errors due to these assumptions are doubtless very small, but in the method developed in the paper above referred to they are avoided altogether by deriving an expression for x, the required horizontal projection of the tape-length, in terms of its standard length, its coefficient of elastic extension, the tension T_m, and the difference of level of its extremities. This expression thus applies the last three of the corrections above enumerated in one operation; the correction for temperature is applied by means of a differential expression which gives the change in x due to a small change in the length of the tape. The effect of tension in stretching the tape is the integrated effect of the tension—assumed to vary continuously from point to point—upon each infinitesimal portion of the tape.

In the present paper the principal departure from the method above outlined consists in expressing the tension in terms of τ, the constant horizontal component of the tension at any point of the tape. This quantity appears therefore instead of T_m in the final expression for x, which by this exchange is placed in a more concise and convenient form, and one better adapted for numerical computation. This would be of little service, however, if there were not some ready method for determining τ. A form of tension apparatus is therefore described for which the value of τ is constant irrespective of the slope of the tape. The value of that quantity may then readily be determined once for all, and regarded as the constant of the apparatus.

GENERAL EQUATIONS

In Fig. 1, AB represents a tape, stretched between two posts as in measuring a base, the tensions T_1 and T_2 being applied at its extremities.

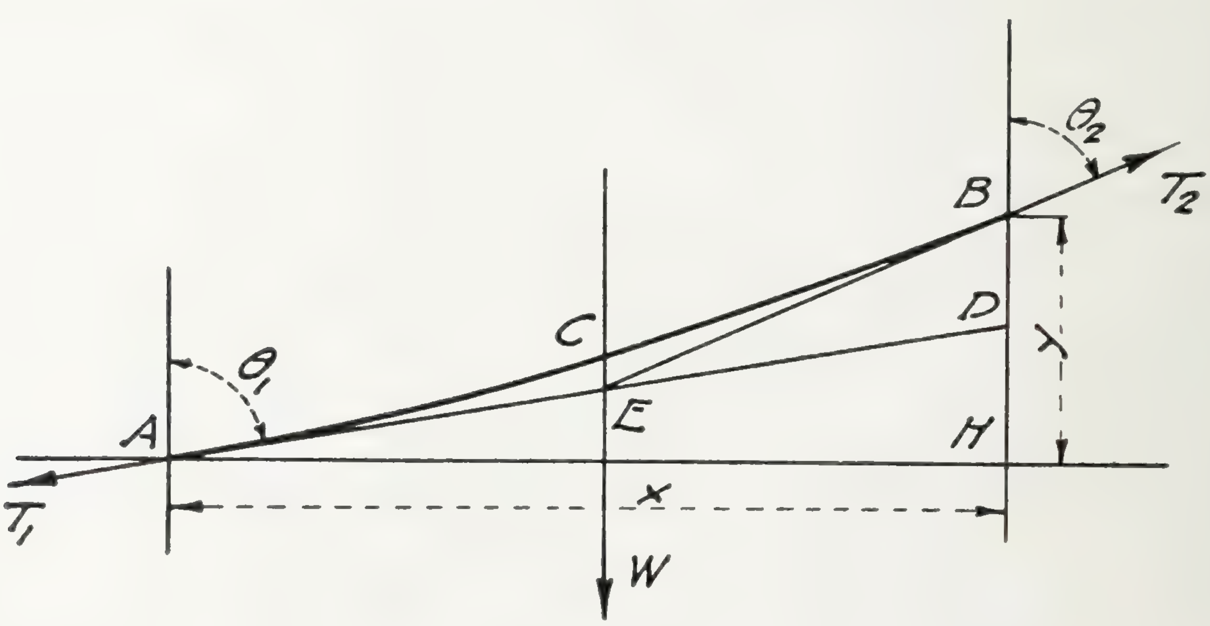


FIG. 1

Let θ_1 and θ_2 denote the angles which the tape at A and B, respectively, makes with the vertical direction, and x and y the rectangular coordi-

nates of B, referred to A as origin. The y coordinate is given by the field measurements; x is the quantity to be determined. The tape is in equilibrium under the action of the tensions T_1 and T_2, and the weight of the tape W acting vertically through its centre of gravity C. The directions of these three forces therefore pass through some point E. The sides of the triangle BDE are parallel to, and, taken clockwise, in the same sense as, the three forces $T_1 T_2$ and W, so that we have at once the two equations,

$$T_1 = W \frac{\sin \theta_2}{\sin (\theta_1 - \theta_2)}, \tag{1}$$

$$T_2 = W \frac{\sin \theta_1}{\sin (\theta_1 - \theta_2)}, \tag{2}$$

From these we have the relation,

$$T_1 \sin \theta_1 = T_2 \sin \theta_2; \tag{3}$$

so that, if T denotes the tension at any point of the tape, and θ the angle which the direction of the tape at that point makes with the vertical, we should then obtain in the same manner the equation,

$$T_1 \sin \theta_1 = T \sin \theta;$$

whence it follows that the horizontal component of the tension at any point of the tape is constant. Denoting it by τ we have,

$$\tau = T \sin \theta = W \frac{\sin \theta_1 \sin \theta_2}{\sin (\theta_1 - \theta_2)}, \tag{4}$$

or,

$$\tau = \frac{W}{\cot \theta_2 - \cot \theta_1} \tag{5}$$

Derivation of expressions for the coordinates x and y.

From (1) and (2) we have,

$$T_2 - T_1 = W \frac{\sin \theta_1 - \sin \theta_2}{\sin (\theta_1 - \theta_2)}, \tag{6}$$

which gives, when applied to an element of the curve,

$$dT = dW \frac{\sin (\theta + d\theta) - \sin\theta}{\sin d\theta},$$

$$= w.ds \frac{d \sin \theta}{d\theta},$$

in which ds denotes the length of the element of the curve, and w the weight of a unit of length; therefore,

$$dT = w.ds.\cos \theta = w.dy \tag{7}$$

To find an expression for w, let

s_0 denote the natural or unstretched length of the tape,

w_0 the corresponding weight of a unit of length, and

e the extension of a unit of length due to a unit tension;

then, when subjected to the tension T, the length ds_0 becomes, by Hooke's law,

$$ds_0(1+eT);$$

so that we have,

$$w_0 ds_0 = w . ds_0(1+eT);$$

whence
$$w = \frac{w_0}{1+eT}. \tag{8}$$

Equation (7) then becomes,

$$dT = \frac{w_0 dy}{1+eT},$$

or,
$$w_0 dy = (1+eT)dT.$$

Therefore, integrating, we have,

$$\begin{aligned} w_0 y &= \int_{T_1}^{T_2} (1+eT)dT, \\ &= T_2 - T_1 + \tfrac{1}{2}e(T_2^2 - T_1^2), \\ &= (T_2 - T_1)\left\{1 + \tfrac{1}{2}e(T_2 + T_1)\right\}, \\ &= (T_2 - T_1)(1 + eT_m), \end{aligned}$$

T_m denoting the mean of the end tensions; then

$$y = \frac{1+eT_m}{w_0}(T_2 - T_1) = \frac{T_2 - T_1}{w_m}, \tag{9}$$

by (8), w_m denoting the weight of a unit of length of the tape when it is subjected to the tension T_m.

Again, if an element ds_0 of the tape be stretched by the tension T, its length becomes $ds_0(1+eT)$; and the length of its projection on a horizontal plane is,

$$\begin{aligned} ds_0(1+eT)\sin\theta &= \sin\theta ds_0 + eT\sin\theta ds_0; \\ &= \sin\theta ds_0 + e\tau ds_0; \end{aligned}$$

$$\therefore \qquad x = \int \sin\theta ds_0 + e\tau s_0,$$

the integration being taken between proper limits. But,

$$\begin{aligned} ds_0 &= \frac{ds}{1+eT} = \frac{dy}{(1+eT)\cos\theta}, \\ &= \frac{dT}{w(1+eT)\cos\theta}, \end{aligned}$$

by (7), $$=\frac{dT}{w_0 \cos\theta},$$

by (8). Also, $$T=\frac{\tau}{\sin\theta}$$

$$\therefore \qquad \frac{dT}{d\theta}=-\frac{\tau\cos\theta}{\sin^2\theta},$$

and $\therefore$ $$ds_0=-\frac{\tau\cos\theta d\theta}{\sin^2\theta\, w_0\cos\theta}=-\frac{\tau d\theta}{w_0\sin^2\theta}.$$

Therefore, $$x=\frac{\tau}{w_0}\int_{\theta_2}^{\theta_1}\frac{d\theta}{\sin\theta}+e\tau s_0$$

and $$x=\frac{\tau}{w_0}\log\frac{\tan\frac{1}{2}\theta_1}{\tan\frac{1}{2}\theta_2}+e\tau s_0 \tag{10}$$

This equation for x is not suitable for numerical computation; we proceed therefore to place it in a more practical form. Writing,

$$m=\frac{\tan\frac{1}{2}\theta_1}{\tan\frac{1}{2}\theta_2}, \tag{11}$$

and expanding, we have,

$$\log m=2\left\{\frac{m-1}{m+1}+\tfrac{1}{3}\left(\frac{m-1}{m+1}\right)^3+\tfrac{1}{5}\left(\frac{m-1}{m+1}\right)^5+\right\},$$

and then writing, $$M=\frac{m-1}{m+1}, \tag{12}$$

we have,

$$x=\frac{2\tau}{w_0}\left(M+\tfrac{1}{3}M^3+\tfrac{1}{5}M^5+\right)+e\tau s_0, \tag{13}$$

or $$x=s_0\frac{2\tau}{W}\left(M+\tfrac{1}{3}M^3+\tfrac{1}{5}M^5+\right)+e\tau s_0. \tag{14}$$

The quantity M may be expressed in terms of other quantities. Thus, we have,

$$M=\frac{\tan\frac{1}{2}\theta_1-\tan\frac{1}{2}\theta_2}{\tan\frac{1}{2}\theta_1+\tan\frac{1}{2}\theta_2}=\frac{\sin\frac{1}{2}(\theta_1-\theta_2)}{\sin\frac{1}{2}(\theta_1+\theta_2)}. \tag{15}$$

Also, by (1) and (2),

$$T_1+T_2=W\frac{\sin\theta_1+\sin\theta_2}{\sin(\theta_1-\theta_2)}=W\frac{\sin\frac{1}{2}(\theta_1+\theta_2)}{\sin\frac{1}{2}(\theta_1-\theta_2)}=\frac{W}{M};$$

$$\therefore \qquad M=\frac{W}{2T_m}. \tag{16}$$

In order to eliminate either T_m or τ from equation (14) it is necessary to find a relation between these two quantities. The form of that equation suggests that probably the preferable course is to eliminate the former quantity.

We have, by (4) and (16),

$$\frac{\tau}{T_m}=\frac{2\sin\theta_1\sin\theta_2}{\sin\theta_1+\sin\theta_2}.$$

Also, by (9),

$$w_m y=T_2-T_1=W\frac{\sin\theta_1-\sin\theta_2}{\sin(\theta_1-\theta_2)},$$

or

$$\frac{y}{s_m}=\frac{\sin\theta_1-\sin\theta_2}{\sin(\theta_1-\theta_2)}=\frac{\cos\frac{1}{2}(\theta_1+\theta_2)}{\cos\frac{1}{2}(\theta_1-\theta_2)};$$

in which s_m denotes the length of the tape when subjected, throughout its length, to the tension T_m,

or,

$$s_m=s_0(1+eT_m).$$

Therefore,

$$1-\frac{y^2}{s^2_m}=\frac{\cos^2\frac{1}{2}(\theta_1-\theta_2)-\cos^2\frac{1}{2}(\theta_1+\theta_2)}{\cos^2\frac{1}{2}(\theta_1-\theta_2)}$$

$$=\frac{4\cos\frac{1}{2}\theta_1\cos\frac{1}{2}\theta_2\sin\frac{1}{2}\theta_1\sin\frac{1}{2}\theta_2}{\cos^2\frac{1}{2}(\theta_1-\theta_2)}$$

$$=\frac{\sin\theta_1\sin\theta_2}{\cos^2\frac{1}{2}(\theta_1-\theta_2)}.$$

Also, by (15),

$$1-M^2=\frac{\sin^2\frac{1}{2}(\theta_1+\theta_2)-\sin^2\frac{1}{2}(\theta_1-\theta_2)}{\sin^2\frac{1}{2}(\theta_1+\theta_2)}$$

$$=\frac{4\sin\frac{1}{2}\theta_1\cos\frac{1}{2}\theta_2\cos\frac{1}{2}\theta_1\sin\frac{1}{2}\theta_2}{\sin^2\frac{1}{2}(\theta_1+\theta_2)}$$

$$=\frac{\sin\theta_1\sin\theta_2}{\sin^2\frac{1}{2}(\theta_1+\theta_2)}.$$

Therefore,

$$\left(1-\frac{y^2}{s^2_m}\right)(1-M^2)=\frac{\sin^2\theta_1\sin^2\theta_2}{\sin^2\frac{1}{2}(\theta_1+\theta_2)\cos^2\frac{1}{2}(\theta_1-\theta_2)},$$

$$=\frac{4\sin^2\theta_1\sin^2\theta_2}{(\sin\theta_1+\sin\theta_2)^2},$$

$$=\frac{\tau^2}{T_m^{\,2}};$$

$$\therefore \qquad \frac{\tau}{T_m} = \left\{\left(1-\frac{y^2}{s^2_m}\right)(1-M^2)\right\}^{\frac{1}{2}} \tag{17}$$

The next step is to eliminate T_m from this equation by means of (16). We thus find,

$$\tau^2 = \frac{W^2}{4M^2}\left\{1-\frac{y^2}{s_0^2\left(1+e\frac{W}{2M}\right)^2}\right\}(1-M^2). \tag{18}$$

Then, clearing of fractions and re-arranging the terms we find the following quartic in M:

$$\left(1+\frac{4\tau^2}{W^2}-\frac{y^2}{s_0^2}\right)M^4+eW\left(1+\frac{4\tau^2}{W^2}\right)M^3$$

$$-\left(1-\frac{y^2}{s_0^2}-\tfrac{1}{4}e^2W^2-e^2\tau^2\right)M^2-eWM-\tfrac{1}{4}e^2W^2=0. \tag{19}$$

This equation cannot have more than one positive root.

An approximate solution of the problem may be obtained as follows: Returning to equation (18), and writing it,

$$\frac{4\tau^2}{W^2} = \left\{1-\frac{y^2}{s_0^2\left(1+e\frac{W}{2M}\right)^2}\right\}\frac{1-M^2}{M^2},$$

or

$$\frac{M^2}{1-M^2} = \frac{W^2}{4\tau^2}\left\{1-\frac{y^2}{s_0^2}\left(1-e\frac{W}{M}\right)\right\},$$

—by expanding, and omitting the square and higher powers of e—or,

$$\frac{M^2}{1-M^2} = \frac{W^2}{4\tau^2}\left(1-\frac{y^2}{s_0^2}\right)+e\frac{W^3}{4\tau^2M}\cdot\frac{y^2}{s_0^2},$$

$$= M'^2+e\frac{W^3}{4\tau^2M}\cdot\frac{y^2}{s_0^2}, \tag{20}$$

where,

$$M'^2 = \frac{W^2}{4\tau^2}\left(1-\frac{y^2}{s_0^2}\right) = \frac{W^2}{4\tau^2}\cdot\frac{x'^2}{s_0^2} \tag{21}$$

and

$$x' = (s_0^2-y^2)^{\frac{1}{2}}. \tag{22}$$

Then, writing v^2 for the right-hand member of (20), we have,

$$M^2(1-M^2)^{-1} = M^2+M^4+M^6+ = v^2.$$

Reversing this series, we find,

$$M^2 = v^2-v^4+v^6.$$

whence
$$M = v(1 - v^2 + v^4)^{\frac{1}{2}}$$
$$= v\left\{1 - (\tfrac{1}{2}v^2 - \tfrac{3}{8}v^4)\right\}, \tag{23}$$
omitting higher powers of v. We find also,
$$M^3 = v^3(1 - \tfrac{3}{2}v^2),$$
$$M^5 = v^5(1 - \,.\,.\,).$$
Then, substituting in (14) and reducing, it becomes,
$$x = s_0 \frac{2\tau}{W}(v - \tfrac{1}{6}\,v^3 + \tfrac{3}{40}\,v^5) + e\tau s_0.$$
But, by (20),
$$v^2 = M'^2\left(1 + e\frac{W^3}{4\tau^2 M M'^2}\cdot\frac{y^2}{s_0{}^2}\right),$$
or
$$v = M'\left(1 + \tfrac{1}{2}e\frac{W^3}{4\tau^2 M M'^2}\cdot\frac{y^2}{s_0{}^2}\right).$$
Therefore,
$$v^3 = M'^3\left(1 + \tfrac{3}{2}e\,\frac{W^3}{4\tau^2 M M'^2}\cdot\frac{y^2}{s_0{}^2}\right),$$
$$v^5 = M'^5\left(1 + \tfrac{5}{2}e\,\frac{W^3}{4\tau^2 M M'^2}\cdot\frac{y^2}{s_0{}^2}\right).$$
We therefore find,
$$x = s_0\frac{2\tau}{W}\left\{M' - \tfrac{1}{6}M'^3 + \tfrac{3}{40}M'^5 - \right.$$
$$\left. + \tfrac{1}{2}e\,\frac{W^3}{4\tau^2 M}\cdot\frac{y^2}{s_0{}^2}\left(\frac{1}{M'} - \tfrac{1}{2}M' + \tfrac{3}{8}\,M'^3\right)\right\} + e\tau s_0.$$
Also, by (23),
$$\frac{1}{M} = \frac{1}{v}\left\{1 - (\tfrac{1}{2}\,v^2 - \tfrac{3}{8}\,v^4)\right\}^{-1}$$
$$= \frac{1}{v}\,(1 + \tfrac{1}{2}v^2 - \tfrac{1}{8}v^4),$$
omitting higher powers of v; therefore,
$$\frac{1}{M} = \frac{1}{M'}\,(1 + \tfrac{1}{2}M'^2 - \tfrac{1}{8}M'^4),$$
omitting terms containing e. Therefore,
$$x = s_0\frac{2\tau}{W}\left\{M' - \tfrac{1}{6}M'^3 + \tfrac{3}{40}M'^5 - \right.$$
$$\left. + \tfrac{1}{2}e\,\frac{W^3}{4\tau^2 M'}\cdot\frac{y^2}{s_0{}^2}\left(\frac{1}{M'} - \tfrac{1}{2}M' + \tfrac{3}{8}M'^3\right)\left(1 + \tfrac{1}{2}M'^2 - \tfrac{1}{8}M'^4\right)\right\} + e\tau s_0,$$

$$=s_0\frac{2\tau}{W}\left(M'-\tfrac{1}{6}M'^3+\tfrac{3}{40}M'^5+\tfrac{1}{2}e\frac{W^3}{4\tau^2M'^2}\cdot\frac{y^2}{s_0^2}\right)+e\tau s_0.$$

Then, substituting the value of M' given by (21), this becomes,

$$x=x'-\tfrac{1}{24}\frac{W^2x'^3}{\tau^2s_0^2}+\tfrac{3}{640}\frac{W^4x'^5}{\tau^4s_0^4}+e\tau s_0\frac{y^2}{x'^2}+e\tau s_0,$$

or

$$x=x'\left(1-\tfrac{1}{24}\frac{W^2x'^2}{\tau^2s_0^2}+\tfrac{3}{640}\frac{W^4x'^4}{\tau^4s_0^4}\right)+e\tau\frac{s_0^3}{x'^2}. \tag{24}$$

This may also be written,

$$x=x'\left\{1-\tfrac{1}{24}Ax'^2+\tfrac{3}{640}(Ax'^2)^2\right\}+\frac{B}{x'^2} \tag{25}$$

in which,

$$A=\frac{W^2}{\tau^2s_0^2},\quad B=e\tau s_0^3,$$

$$x'=(s_0^2-y^2)^{\frac{1}{2}}.$$

The quantity x' may be expanded in series. Thus,

$$x'=s_0\left(1-\frac{y^2}{s_0^2}\right)^{\frac{1}{2}}$$

$$=s_0\left\{1-\tfrac{1}{2}\frac{y^2}{s_0^2}-\frac{1}{2^2\,\underline{|2}}\left(\frac{y^2}{s_0^2}\right)^2-\frac{1.3}{2^3\,\underline{|3}}\left(\frac{y^2}{s_0^2}\right)^3\right.$$

$$\left.-\frac{1.3.5}{2^4\,\underline{|4}}\left(\frac{y^2}{s_0^2}\right)^4-\frac{1.3.5.7}{2^5\,\underline{|5}}\left(\frac{y^2}{s_0^2}\right)^5-\right\} \tag{26}$$

$$=s_0\left\{1-\tfrac{1}{2}\left(\frac{y}{s_0}\right)^2-\tfrac{1}{8}\left(\frac{y}{s_0}\right)^4-\tfrac{1}{16}\left(\frac{y}{s_0}\right)^6-\right\} \tag{27}$$

Equation (27) contains a sufficient number of terms for any ordinary case that may occur. Thus, if $s_0=100$ and $y=5$ the result is as follows:

$$\begin{aligned}x'=&100\\&-0.125\\&-0.000078125\\&-0.000000098\\=&99.874921777\end{aligned}$$

Again, if $s_0=100$ and $y=10$, we have,

$$\begin{aligned}x'=&100\\&-0.5\\&-0.00125\\&-0.00000625\\=&99.49874375\end{aligned}$$

In this second example the last digit is slightly in error.

The value of x' may be tabulated in the form,

$$s_0 - x' = s_0 \left\{ \tfrac{1}{2}\left(\frac{y}{s_0}\right)^2 + \tfrac{1}{8}\left(\frac{y}{s_0}\right)^4 + \tfrac{1}{16}\left(\frac{y}{s_0}\right)^6 + \right\}$$

If $y=0$ in equation (24), or if the ends of the tape are at the same level, it takes the form:

$$x = s_0\left(1 - \tfrac{1}{24}\frac{W^2}{\tau^2} + \tfrac{3}{640}\frac{W^4}{\tau^4}\right) + e\tau s_0.$$

In the former paper, to which reference has been made, four numerical examples are worked out—p. 365—in order to show the application of the formulæ there given, and the results are carried correctly to the ninth decimal place. Equation (25), above, will now be applied to two of those examples in turn in order to test the precision of that equation.

Example 1. $s_0 = 100$ ft. $W = 1.2$ lbs.
$y = 5$ ft. $T = 25$ lbs.
$e = 0.00000911$

T is here the tension applied to the forward end of the tape. We find,

$$\tau = 24.9315883195$$
$$x' = 99.8749217772$$

so that,

$$\begin{aligned} x &= 99.8749217772 \\ &\quad -0.0096165826 \\ &\quad +0.0000025000 \\ &\quad +0.0227696000 \\ &= 99.8880772946 \end{aligned}$$

Former value: $x = 99.888077278$
Difference $= 0.000000017$

Example 2. $s_0 = 100$ ft. $W = 1.2$ lb
$y = 10$ ft. $T = 25$ lbs.
$e = 0.00000911$

We find, $\tau = 24.8078749766$
$x' = 99.4987437107$

so that,

$$\begin{aligned} x &= 99.4987437107 \\ &\quad - 0,0096033980 \\ &\quad + 0.0000025026 \\ &\quad + 0.022828258 \\ &= 99.511971074 \end{aligned}$$

Former value: x 99.511970994
Difference $= 0.000000080$

These examples show conclusively that a sufficient number of terms have been retained in equation (25) to insure numerical results precise enough to meet all possible requirements.

There remain two corrections that must be applied to x, *viz.*, the corrections for temperature and change in the force of gravity with latitude. The latter correction must be applied if the latitude of the station of standardization of the tape differs from that of the base.

Correction for temperature

Writing equation (24) in the form:

$$x = x'\left(1 - A'\frac{x'^2}{s_0^2} + B'\frac{x'^4}{s_0^4} + C'\frac{s_0^3}{x'^3}\right);$$

we must find from this the effect upon x of a small change in s_0. Differentiating, we have

$$\begin{aligned}\frac{dx}{ds_0} = \frac{dx'}{ds_0}&\left(1 - A'\frac{x'^2}{s_0^2} + B'\frac{x'^4}{s_0^4} + C'\frac{s_0^3}{x'^3}\right)\\ +x'&\left\{-\frac{A'}{s_0^4}\left(2s_0^2x'\frac{dx'}{ds_0} - 2x'^2s_0\right)\right.\\ &+\frac{B'}{s_0^8}\left(4s_0^4x'^3\frac{dx'}{ds_0} - 4x'^4s_0^3\right)\\ &\left.+\frac{C'}{x'^6}\left(3x'^3s_0^2 - 3s_0^3x'^2\frac{dx'}{ds_0}\right)\right\}\end{aligned}$$

But $$\frac{dx'}{ds_0} = s_0(s_0^2 - y^2)^{-\frac{1}{2}} = \frac{s_0}{x'};$$

therefore we have after reduction

$$\frac{dx}{ds_0} = \frac{s_0x}{x'^2} + x'\left(-2A'\frac{y^2}{s_0^3} + 4B'\frac{x'^2y^2}{s_0^5} - 3C'\frac{s_0^2y^2}{x'^5}\right) \tag{28}$$

in which $$A' = \tfrac{1}{24}\frac{W^2}{T^2},\quad B' = \tfrac{3}{640}\frac{W^4}{T^4},\quad C' = eT.$$

Assuming, now, that $ds_0 = 0.02$ ft.—which is the expansion of a 100-ft. steel tape due to a change of temperature of about 30° F.—, $y = 10$ feet, and the remaining data as in the above examples, the value of dx is as follows:

$$\begin{aligned}dx &= 0.020103431\\ &\quad -0.00000003880\\ &\quad +0.00000000002\\ &\quad -0.00000013835\\ &= 0.020103254\end{aligned}$$

As the bracketed terms of (28), in this extreme case, amount to less than 0.0000002 ft., we may therefore neglect them and write,

$$dx = \frac{s_0 x}{x'^2}\, ds_0. \tag{29}$$

Then, as

$$ds_0 = a s_0 (t - t_0),$$

in which,

a denotes the coefficient of expansion of the tape,

t the temperature during a measurement, and

t_0 the temperature at which the length of the tape is standard;

therefore the correction for temperature:

$$c_t = a x (t - t_0) \frac{s_0^2}{x'^2}. \tag{30}$$

The error caused by regarding as infinitesimals the change in the length of the tape due to a change of temperature, and its consequent effect upon x, may be shown to amount to only a unit in the seventh decimal place, in the last example.

In preparing the base for measurement, in the manner described at the beginning of this paper, the metal plates should be set at such a distance apart that the algebraic sum of the scale readings on the lines ruled on them, when measuring the base, may be as small as possible. The sum of the scale readings may be combined with the expansion of the tape due to temperature, and the whole taken as the value of ds_0 in (29).

Correction for change in the force of gravity.

Let,

ϕ_1 denote the latitude of the station at which the tape was standardized,

ϕ that of the base,

h_1 and h the heights of the two stations, respectively, above sea level,

g_1 and g the values of the force of gravity at sea level at the two stations, and

g_0 its value at sea level at the equator.

Then, omitting the Bouguer correction, Bowie's formula may be written, for the two stations,

$$\left.\begin{aligned} g_1 &= g_0\left(1+0.005294 \sin^2\phi_1 - 2\frac{h_1}{\rho_1}\right) \\ g &= g_0\left(1+0.005294 \sin^2\phi - 2\frac{h}{\rho}\right) \end{aligned}\right\} \tag{31}$$

ρ_1 and ρ denoting the radii of curvature of the earth at the two stations. Also,

$$g_0 = 978.039$$

Referring now to equation (24), it appears that as W and τ are changed in the same proportion by any change in g, therefore W/τ remains constant. That τ and P and therefore τ and W, vary proportionally is shown by equation (33), below. The last term of (24) is therefore the only one that is affected by a variation of gravity. The resulting correction to x then is,

$$c_g = e\frac{s_0^3}{x'^2} \cdot \Delta\tau.$$

Also $$\frac{\Delta\tau}{\tau} = \frac{\Delta g}{g_1}, \quad \text{or } \Delta\tau = \tau\frac{\Delta g}{g_1}$$

$$\therefore \qquad c_g = e\tau\frac{s_0^3}{x'^2} \cdot \frac{\Delta g}{g_1}, \tag{32}$$

$\Delta g(=g-g_1)$ being found by equations (31).

A form of apparatus will now be described, by which the tension may be applied to the tape, and which will insure τ remaining constant whatever the slope of the tape may be. The principle of the apparatus is shown in Fig. 2. ABC represents a bent lever, weighted at A. The tape is attached at C, and the pull is applied at B, and is sufficient to

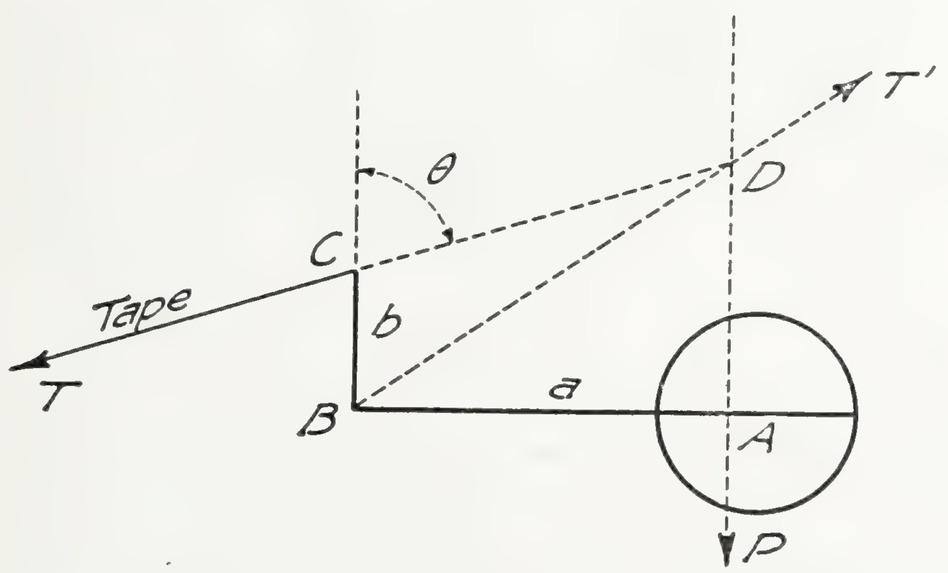


FIG. 2

bring the arm AB to the horizontal position, or, to be exact, to bring BC to the vertical position, as indicated by a level attached to AB. The three forces acting on the lever: the tension T of the tape, the weight P of the lever, and the pull T' applied to it, being in equilibrium, their directions will pass through some point D. Denoting the lengths of

AB and BC by a and b, respectively, and taking moments about B, we have,

$$Tb \sin \theta = aP$$

or

$$\tau = \frac{a}{b} P. \tag{33}$$

τ is therefore constant, while T and T' increase with the slope of the tape. The ratio $a : b$ may have any convenient value, say, $3 : 1$.

A proposed design for an apparatus embodying the above principle is shown in Fig. 3. ABC is the bent lever, L the level attached to it.

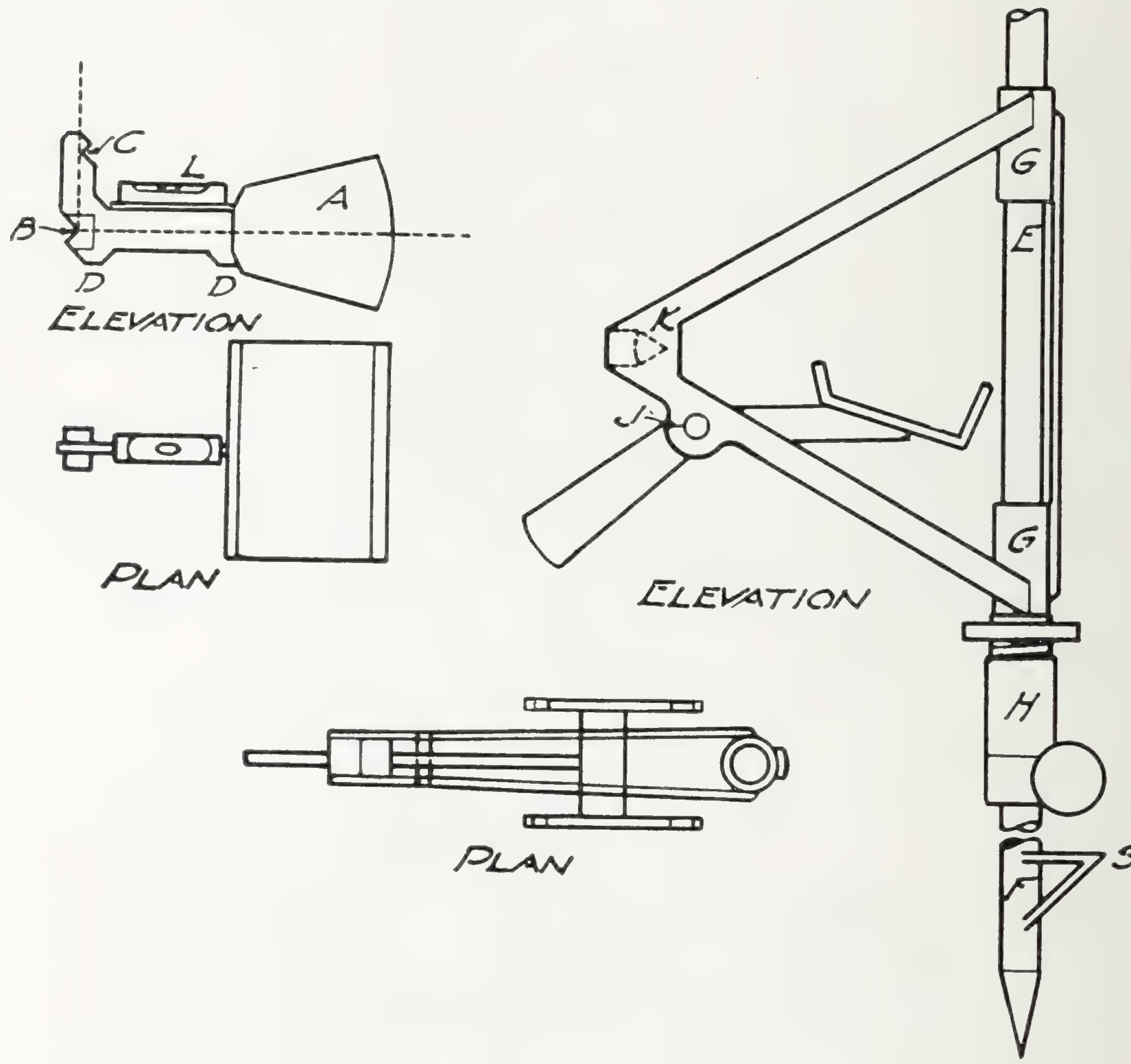


FIG. 3

At DD are two finished surfaces, portions of a plane which is perpendicular to BC, and therefore parallel to the axis of the level when it is in adjustment; their use is in adjusting the level by reversing on two fixed supports (*e.g.*, the heads of two nails driven into a stump). At B is a V cut in a piece of hard material attached to the lever, that receives the knife edge K when in position. EF is a pole made of steel tubing, about six feet in length, terminating in a steel point. with a stirrup

at S by which it may be pushed into the ground. The knife edge K is held between the two parts of a double triangular frame GKG, G and G being two collars sliding freely on the pole EF and thus allowing the frame to be placed at any desired height. H is a clamping collar, detached from G, that is designed to hold the frame at the required height; it is provided with a slow motion attachment by which the final adjustment for height may be made. A lever is shown, pivoted at J, by which the bent lever may be raised into position, brought against the knife edge, and left there supported by the tension of the tape, and afterwards lowered from the knife edge when that tension is withdrawn.

The method of using the apparatus in the field is illustrated in Fig. 4, which requires little explanation. EP is a guy cord, which may be lengthened or shortened in the usual manner at M, and which holds the pole EF in a vertical position. S is a cord, by drawing steadily on which the bubble of the level may be brought to its central position. By means of a rough model of the above apparatus the writer found

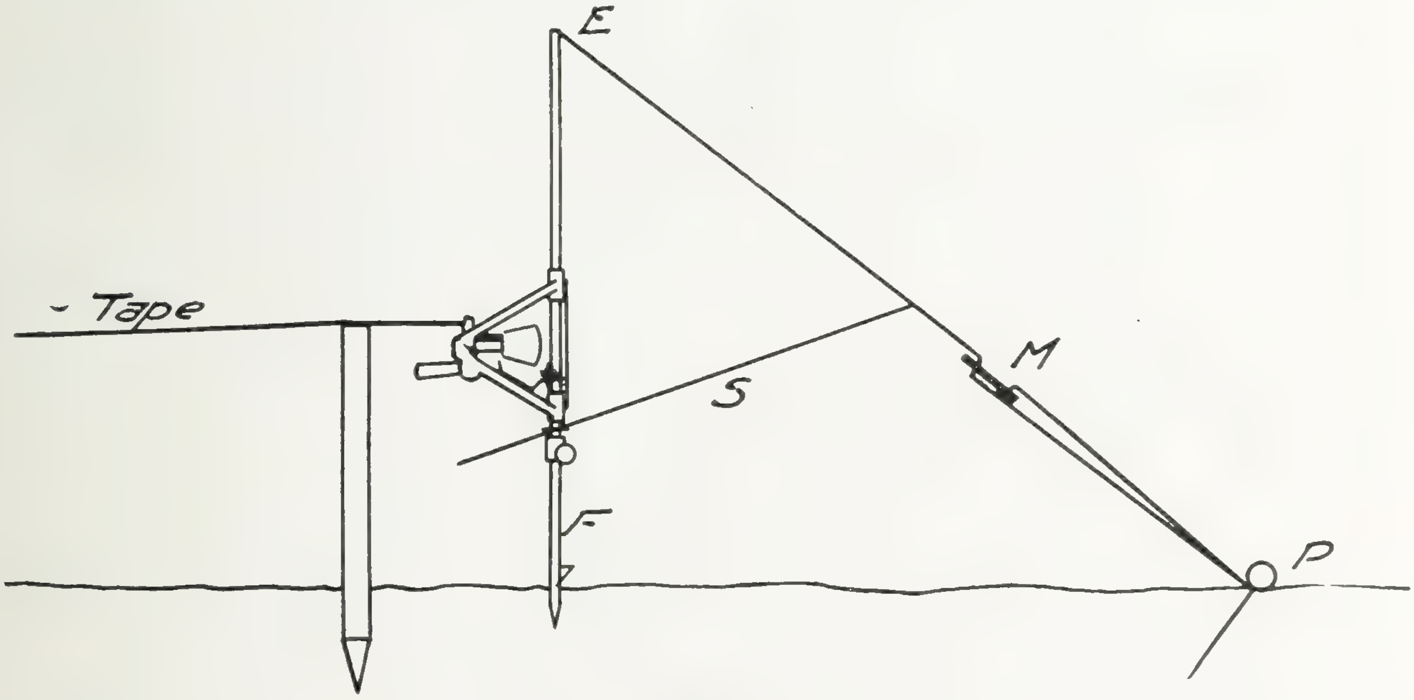


FIG. 4

that in a steady atmosphere there is no difficulty in holding the level bubble within $0'.2$ of its correct position; and it will be shown presently that an error of this magnitude will have no appreciable effect upon x. The longitudinal curvature of the level vial may be such that 1 mm. of its length corresponds to $1'$ of arc.

The effect of a small inclination of BC (Fig. 2) to the vertical may be determined as follows: Denoting the inclination by $\Delta\theta$ we have, as in deriving equation (33),

$$Tb \sin (\theta \pm \Delta\theta) = Pa \cos \Delta\theta.$$

Then expanding, and neglecting higher powe s of $\Delta\theta$ than the second, this becomes,

$$Tb\{\sin\theta(1-\tfrac{1}{2}\Delta\theta^2)\pm\Delta\theta\cos\theta\}=Pa(1-\tfrac{1}{2}\Delta\theta^2),$$

or,
$$T\sin\theta\{(1-\tfrac{1}{2}\Delta\theta^2)\pm\Delta\theta\cot\theta\}=\frac{a}{b}P(1-\tfrac{1}{2}\Delta\theta^2),$$

or,
$$\left(\tau-\frac{a}{b}P\right)(1-\tfrac{1}{2}\Delta\theta^2)=\mp\tau.\Delta\theta\cot\theta,$$

or,
$$\tau=\frac{a}{b}P\mp\tau.\Delta\theta\cot\theta\,(1+\tfrac{1}{2}\Delta\theta^2),$$

nearly. Comparing this with (33) it appears that the error in τ due to $\Delta\theta$ is,

$$\Delta\tau=\tau.\Delta\theta\cot\theta\,(1+\tfrac{1}{2}\Delta\theta^2); \tag{34}$$

or, neglecting $\Delta\theta^2$ which is inappreciable, we have,

$$\frac{\Delta\tau}{\tau}=\Delta\theta\cot\theta. \tag{35}$$

The effect of this on x may be found by differentiating equation (24), thus,

$$\frac{dx}{d\tau}=\frac{1}{24}\frac{W^2x'^3}{s_0^2}\cdot\frac{2\tau}{\tau^4}-\frac{3}{640}\frac{W^4x'^5}{s_0^4}\cdot\frac{4\tau^3}{\tau^8}+e\frac{s_0^3}{x'^2}.$$

Then, denoting the last three terms of (24) by A_2, A_3, and A_4, respectively, this may be written,

$$dx=(2A_2-4A_3+A_4)\frac{d\tau}{\tau}.$$

The second term in this equation is found to be negligible, so that we have finally,

$$dx=(2A_2+A_4)\Delta\theta\cot\theta. \tag{36}$$

Proceeding now to apply equation (36) to each of the two numerical examples given above, assuming $\Delta\theta=1'$, we first find θ by means of the equation:

$$\sin\theta=\frac{\tau}{T}.$$

For the first example we find,

$$\theta=85°\ 45'\ 36.''87,$$

and
$$\begin{aligned}dx&=\ \ 0.000000415\\&+\ 0.000000491\\&=\ \ 0.000000906\ .\end{aligned}$$

Also, for the second example,

$$\theta = 82^\circ\ 53'\ 31.''92$$

and

$$\begin{aligned} dx &= \ 0.000000697 \\ &+ \ 0.000000828 \\ &= \ 0.000001525. \end{aligned}$$

The conclusion to be drawn from these results and the experimental fact stated above, is that the error due to this cause should not exceed two or three units in the seventh decimal place.

The rear end of the tape may be held in a manner similar to that of the forward end, but without the use of a tension lever. The appliance there will therefore consist of a steel pole, with its guy and draw cords, its clamp and slow motion cylinder, the latter carrying a hook to which the tape may be attached. The observer at that end of the tape, by pulling steadily on the draw cord, may bring the zero mark of the tape into coincidence with the mark on the post, and hold it there while the forward observer is reading his scale.

The method of base measurement developed in this paper appears then to be capable of meeting all reasonable requirements. The suggested form for the tension apparatus is simple in design, efficient, and precise in its results, and may readily be kept in adjustment. Also, the introduction of the quantity τ into the expression for x makes it possible to place that expression in a simpler and more precise form than any that have come to the writer's notice.

"COLONIAL DOMESTIC ARCHITECTURE OF THE SEVENTEENTH AND EIGHTEENTH CENTURIES IN CANADA AND THE UNITED STATES"

By E. S. S. Armitage-Stanley

(Under the Direction of the Staff in the School of Architecture)

Social and political disturbances and unrest have at all times in the World's history been the causes of emigration and colonial expansion. The ancient Grecian and Roman cities spread their politics, art and culture in Asian territories, through the medium of their far-flung colonies. Colonial expansion through centuries languished until the revival of Spanish and Portuguese trade, which fostered the adventurous instincts of the later explorers, Cabot, Cartier, Frobisher and Raleigh who led many of the dissatisfied spirits to the perils of the New World. The High Church policies under the Stuart kings, in England, caused Non-Conformists to flee the country, finally colonizing the North and South Atlantic Seaboard of North America. Religious persecutions on the continent drove many Swedes, Germans and Dutch to American shores. They formed colonies at New Amsterdam, Delaware, Interior Pennsylvania and Georgia. The leaders, receiving large grants of primitive land, were required to colonize them.

The early colonists, landing on a hostile and inhospitable shore as that of Massachusetts, in the winter of 1620, found themselves lacking to a great extent, the essentials of contemporary civilized life which they had left behind them in the mother countries. The first essential was shelter—domiciles to protect the travel-worn men, women and children from the elements. Modern research has found that, in contradiction to the idea of log huts chinked with clay, the early primitive dwelling was the conical, rude, tepee or hut, covered with branches, turf and rushes. This was later improved by the introduction of the ridge pole, supported on forked sticks at the ends. Sod-covered branches were laid slopingly against the ridge pole. John Smith writes of the first church at Jamestown, Virginia, being of this type. The early Massachusetts colonists used a hillside burrow roofed by branches and sods, with the primitive fireplace a hollow against the hillside. In Pennsylvania, similar shelters were formed by digging several feet into the ground and walling above the grade with sods of earth and brush. The roofs were built of branches or split logs covered with sods and

bark. This type persisted in remote localities until the middle of the 18th century, despite the efforts of the authorities to have it abandoned. A few years later the cottages were often composed of clay-daubed wattles, placed between posts driven into the ground. Storms and winds played havoc with, and fire took toll of, these flimsy structures.

The next consideration after the supplies of food, to augment the stores which they had brought with them had been procured, and their protection from the elements assured, was that of protection from hostile neighbours, the Indians, and predatory animals. A fort, as the centre of communal life, and to which they could flee in time of danger, was usually built on some commanding eminence, as that of Fort Hill at Plymouth, or the Citadel at Quebec. This was built of trunks of trees or heavy planks standing vertically like palisades, reminiscent of the early Saxon fortifications in England. Each settler's home was in itself a fort, surrounded by its own allotment of garden, the whole enclosed by the strong upright palisades noted by Governor Bradford in his diary of the Plymouth Colony in 1620.

In Smith's *History of New Jersey* we find English settlers using for their houses, the method of placing heavy planks in the ground and plastering them within to keep out the cold. For walling, the early settlers in Canada used the tree trunks which were cut from the primeval forest. They chinked the interstices with clay. This was the Ontario custom in the 18th and 19th centuries and in Champlain's *Habitation de Quebec* (1608). This method seems to have been quite unknown to the early Colonist of New England. The Swedes, in their Delaware Colony of 1638, seem to have brought the tradition of horizontal logs as walling from their native land where it was common construction. This usage was ideally suited for the prevailing conditions in a heavily forested country where the land had to be cleared before the crops could be planted. By 1654 the primitive shelters of the Massachusetts Bay Colony had, by great industry, given place almost everywhere, to better built wooden homes in which the colonists were more secure, and in which the rude comforts of the time were more in evidence.

In England, from early times, oak was used almost entirely, due to its profusion and low cost, even where stone was most abundant. The building of the English Navy, by royal decree in the 17th century, prohibited the use of oak for any purpose except ship-building. London's Great Fire of 1666 showed the grave danger of wooden buildings. From that time building in wood seems to have languished, and was superseded by the use of the more permanent stone and brick. The English colonists in the New World carried with them a tradition of wood framing, which they had seen in their English homeland. The Virginia colonists, under the able leadership of Sir Thomas Dale in 1611, had

erected at Jamestown—by 1614—two rows of frame houses "of two storeys with an upper garrett". The newly-founded town of Henrico, had three streets of frame houses, which by 1617, on the arrival of the Deputy-Governor Argall, seemed to be in a bad condition necessitating yearly repairs. The damp climate and neglect, caused the total disappearance of the 17th century buildings in that locality.

The Plymouth Colony, according to *Young's Chronicles of Massachusetts*, was more forward in building. By 1624, carpenters had reared a large house; while by 1629, there were "half a score of houses, and a large one for the Governor". The framing for Governor Winthrop's house was first cut in Charlestown and removed to Boston in 1631. The first houses were built for the people of quality, who were the leaders of the expeditions, the councillors, and ministers of the Gospel.

As a rule, all the early New England houses were similar in design as the various localities were settled from the original Plymouth Colony. The Connecticut settlers, locating around Hartford in 1636, were followed closely by those of New Haven in 1638 and New London in 1648. The building ideas of these colonists had common characteristics, their artisans and carpenters bringing with them the traditional building methods which they had learned during their Old Country apprenticeships. The small differences lie in the matter of local details.

PLANS

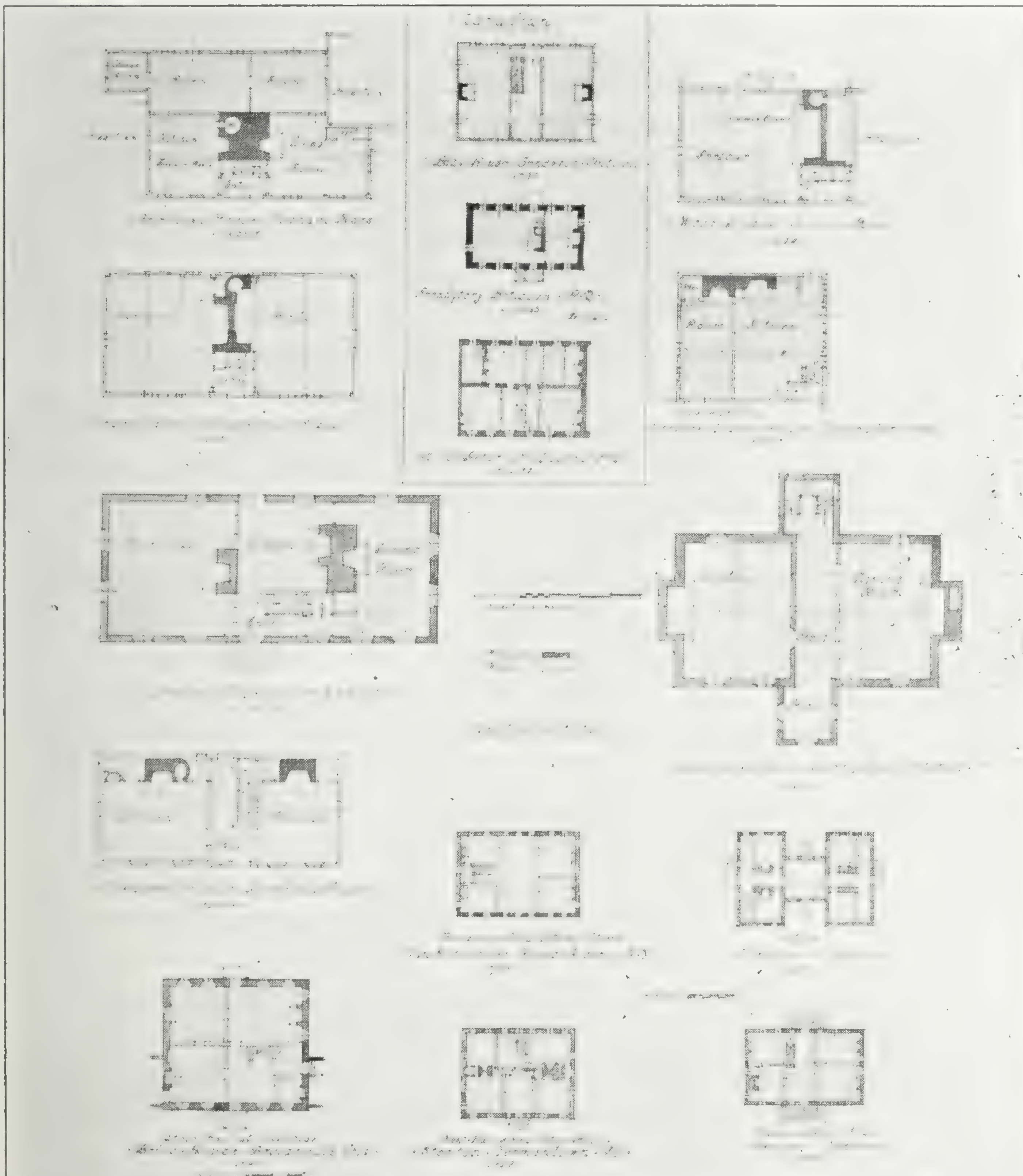
The early 17th century houses were in general but one room deep and two and a half storeys in height—with the addition of a stone-walled cellar, sometimes framed like the rest of the house. The chimneys were in the centre of the building with the timbers framing into the brick stack. Sometimes they were placed at one or both ends and were of wood daubed with clay, as in the early types in the Rhode Island area. This latter practice seems to have disappeared early, due to the fire hazard, although ancient thatched houses in England still have low thatched chimneys on a wooden frame-work. Governor Bradford in his Diary for 1639, noted the sale of a frame house with chimney to be set up and "thacked" (thatched) at Yarmouth, Massachusetts.

Of the early houses which remain, we are able to trace the development of the component parts from the one roomed type to the multiple roomed mansion. The simplest type of one room on each floor started with the large chimney at one end, as in the Ward House at Salem, Mass. (sheet No. 1) which although a year later than the Capen House, was of the earlier type. As the need for additional accommodation arose, an extra room was added at one end, forming the two-roomed type with the chimney and fireplace in the centre, as the Capen House

at Topsfield. This left room for an entry at the front entrance door, where the steep winding staircase was placed against the central chimney stack. To the left, the room was generally called the "parlour" with the "parlour chamber" above. To the right, was the "hall" with its corresponding second floor chamber. Further accommodation was gained by means of the rear "lean-to", low in the second storey, as in the "Scotch-Boardman" house, built in 1651 to house some of the 10,000 Scotch prisoners captured by Cromwell, at the Battle of Dunbar, the previous September. They had been sent to New England as indentured servants to work in the iron mines of Saugus, Massachusetts. A narrow extra staircase was added in the lean-to rooms, which were reached through the rooms in front. The Fairbanks house at Dedham, Massachusetts, although built in 1636-8, had five original and distinct rooms, grouped around the central chimney. Later additions added to the picturesque group. The similar two room type was early in Canada, where the Quebec habitants carried out their traditional French building in local stone, using the larger room, as at the Batiscan Presbytery, 1665 (sheet No. 1) as living-room, and the smaller as parlour, with sleeping accommodation in the second or attic storey. The Hancock-Clarke house at Lexington, built in 1698, shows the two adjoining rooms with separate fireplaces, at the same end, instead of being back to back, as heretofore.

The Dutch house plan in the Hudson River area was the simple long rectangular one with two rooms on the ground floor. The cooking and the life of the house was carried on in the kitchen. The best parlour was used for the storage of china and furniture. The wide central hall gave access to the rooms and the fireplaces were placed on the gable ends. When more space was required, additional ells, usually of one storey, were built as wings to the main house, as at the Bergen homestead at Flatlands, N.Y. To evade the tax on two-storey buildings the thrifty Dutch farmers built a storey and a half, as the Hoffman House (1650) at Kingston-on-Hudson. The upper storey was a model of compactness, due to the need for storage space. Cupboards were built even under the eaves. Bedrooms, usually in the second storey, were sometimes placed behind the kitchen, which then would be relegated to the additional ell, the living room taking its place. The simple stairway, often boxed to conserve the heat, was plain and unimportant and led to the secondary attic rooms used as sleeping quarters for the children and servants.

An early plan (1660) in the South is that of "Bacon's Castle", Surry County, Virginia, which is unusual in that the central hall, separating the parlour and dining room with their end fireplaces, is prolonged into two bays, one a stair hall, and the other an entrance porch. Warren's house, Smith's Fort, Virginia, has a similar plan but without the bays.



Colonial Domestic Architecture of 17th & 18th Centuries

1

This is a prototype of the central hall plan, which by 1700, was becoming more prevalent, although the earlier plan continued to be built well into the latter years of the 18th century. The Townsend House at Lynnfield (sheet No. 1), was an early 18th century example showing the adjacent rooms separated by a stair hall. Here the separate chimney stacks were placed at the rear, not at the ends. This plan was becoming more prevalent and after the United Empire Loyalist migration to Canada in 1783, was used in Ontario homes as in the Bâby House at Sandwich, built in 1790 (sheet No. 1).

The early frame houses of Delaware, which passed from Swedish control under Peter Minuet in 1638, to the rule of the Dutch Governor of New Amsterdam in 1654, and back to the English in 1664, have disappeared. In Pennsylvania, where Penn, in 1628, under his charter from James, Duke of York, founded Philadelphia, all the older houses have also disappeared. One of the early remaining habitations in that region is the simple brick "Old Dutch House" of New Castle, Delaware, with a two room plan and central chimney. Penn's "Letitia" House at Philadelphia, 1682, was of this type with the main entrance door opening into the front room which was traversed in reaching the room in the rear. Penn's more elaborate brick "State House" built about fifteen years later, had two projecting wings and several rooms. "Wyck" at Germantown, built of irregular trap rock, plastered over, was originally two separate houses with a driveway between. This gap was filled in, making a wide connecting hall between the one-roomed building to the right and the older three-roomed one to the left. Stonework of the "Wyck" type reached its zenith at "Wynnesty", Pa., 1689, where the stones were large with irregular joints. Projecting brick band courses above the first floor windows marked the floor of the second storey.

In the New York region, Hurley, settled as "Nieuw Dorp" by the Dutch in 1660, has several small interesting houses of the "farmhouse" type, which has become well known under the title of "Dutch Colonial". They are one storey in height with low overhanging eaves on the front and rear. The eaves on the front were slightly curved to form a porch. The gables were peaked and very flat cornices were used. These were really a cover mould under the projecting roof boarding as shown in the Shenks-Crook house at Flatlands. Before the close of the 17th century, the well known gambrel roof, known in New England by 1670, had become the prominent feature of these houses. The roofs were invariably covered with hand-split pine shingles. The Payne and Mulford Houses at East Hampton, Long Island, where the New England influence was paramount, would be quite at home in Massachusetts, with their long, low, unbroken sweeps of roof and the central chimneys.

By the end of the 17th century, the Indians were becoming settled, due to the missionaries and the power of the white settlers' arms. Prosperity was increasing, with an enlarged export trade to foreign countries. Slavery was an established custom. In the Southern States it was almost the sole labour on the large estates. Wealth had increased and the standard of living was higher. This resulted in the earlier type of domestic buildings losing their Puritan austerity and increasing the decoration and the number of rooms. The merchant princes of the seaport towns were not content with their grandparents' simple abodes but need must build their own homes more pretentious. "King" Hooper of Marblehead, Massachusetts, as well as Governor Byrd of "Westover" in Virginia, and the Brewtons of Charleston, S.C., held the same ideas of more comfortably housing themselves. During the opening years of the 18th century, an added impetus was given to domestic work by the publication in England, of numerous books on architectural practice, giving plates of plans, elevations and details, and the correct proportions of their component parts. The influence of Inigo Jones and Wren was paramount in England and designers published their works in book form. Campbell's *Vitruvius Britannicus* appeared in 1717. Stephen Primatt published his *City and Country Purchaser and Builder* in 1667, and Joseph Moxon his *Art of House Carpentry* in 1694, followed by the *Mechanical Exercises* in 1700. Copies of these folios and plates found their way to the colonies and were extensively used. The master carpenters of the more rural districts evolved charming and well proportioned buildings by incorporating their own ideas with the published data.

The earlier 17th century plan persisted in the small houses for a considerable time in rural districts, as in the Thompson or Count Rumford House at Woburn, Mass. (1714); the Holabird House in Falls Village, Connecticut, the Richards House at Litchfield, Connecticut, (which still retained the slight second storey overhang); and in the Samuel Porter House at South Hadley, Massachusetts. The removal of the chimneys from the centre to the rear of the room gives us the plan of the Townsend House at Lynnfield, Mass. (sheet No. 1), with its central stair hall. In the Shortt House at Newbury, Mass. (1717), the chimneys were placed on the gable ends. Although the Ayrault House at Newport, R.I., built by 1739, continued the old plan, we find at Graeme Park, Pa., built by John Kirk in 1721, a central stack and another nearer the gable, giving a three roomed plan, each room entered from the entry stair hall. This shows the increasing tendency to have the various rooms entered from halls or corridors, not as in the previous planning, entered from the room in front. The Governor's Palace, Williamsburg, Virginia (1705-6), has a more elaborate plan, with a

reception hall and rooms on either side. The main and subsidiary staircases are relegated to the rear, instead of being placed in the central hall.

The most common type with four rooms to a floor and with the central stair hall, is exemplified in the Vassal-Craigie-Longfellow House in Cambridge; "Westover", Va. (sheet No. 1); Gunston Hall, Va.; the Brewton House, Charleston, S.C.; the Hancock House, Boston, Mass., and the Jeremiah Lee House, Marblehead (1768).

Houses of the "H" form, which is a survival of Elizabethan planning, as at "Tuckahoe", Va. (which is of wood), have a wide central hall and wings on either end, with two rooms and a stair hall in each wing. "Stratford", Westmoreland County, Va. (sheet No. 1), has the same plan but has only one storey with a high basement. The "Mulberry", Goose Creek, S.C., built between 1708-25 has the central rectangular block, with a curiously roofed square room at each of the four corners.

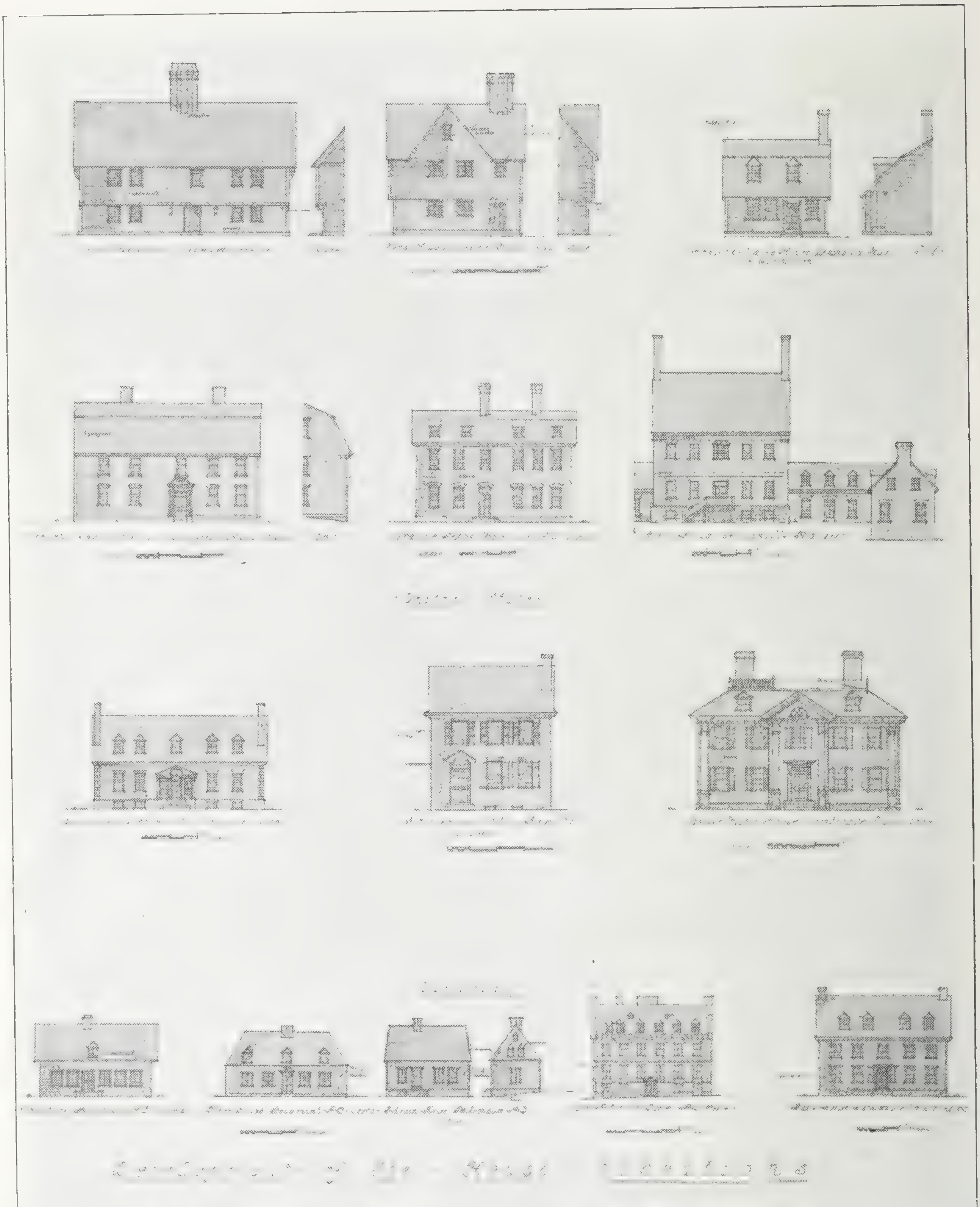
A further type more typical of the southern work, was that in which we have the stair hall placed to one side of the entrance. "Kenmore", at Fredericksburg, Va.; "Shirley" nearby, and the Brice House at Annapolis, Md. (sheet No. 1), show this feature. In "Rosewell", Virginia, and the Moffat-Todd house at Portsmouth, N.H., the stair hall is much longer than the other rooms. Again, we find "Mt. Airy", Richmond County, Virginia; "Whitehall", Maryland; and the Van Rensellaer House at Albany (sheet No. 1), with broad central halls the full depth of the houses, and with the stair halls to one side. "Whitehall" is unique in that the stair halls, of two storey heights, at the extreme ends of the wings, are reached through the side rooms.

Formality was one of the essentials of 18th century planning in the South, whether it was symmetry in fenestration, chimneys, or the placing of the outbuildings, which in the Brice House at Annapolis (sheets No. 1 and No. 2), and "Whitehall" are connected by lateral passages to the main house. The outbuildings housed the servants and the estate offices. "Westover" and "Carter's Grove" in Virginia, show the simple plan of having the outbuildings at the side, away from, and in line with the main block. The Governor's Palace at Williamsburg, was symmetrically placed in relation to the square offices on the front corners. The offices were connected by screen walls to the main building. "Stratford" has the outbuildings placed at the four corners, away from the main portion. "Mt. Airy" and "Mt. Pleasant" at Philadelphia, have them in front, connected in the former by curved passages. The Brice House has the rectangular outbuildings, each connected by a passage and rooms to the main house but placed on a lower level beyond the line of the main building (sheet No. 2). In Canada, the simple central hall type with the fireplaces on the gable ends sufficed for the

smaller buildings, as in the Bâby House at Sandwich, Ontario, built in 1790. The stone "Poplar Hall" at Prescott, Ontario, later by five years, shows the same plan, while the more sophisticated house on St. Peter Street, Quebec, was a rich merchant's city home. This contained a multiplicity of rooms, with fireplaces on the gable ends as well as toward the centre (sheets No. 1 and No. 2).

PORCHES

Projecting porches were not uncommon on the front of the houses. In Shaw's *Description of Boston*, there is a print of the Bridgham House, later called Julien's Restaurant, which shows a projecting entrance porch and roof gables. Governor Winthrop's inventory of 1649 mentions a porch chamber. Mention of a porch is also made in a Hartford inventory of 1680. The second story of the "House of Seven Gables" or Turner House (1670), at Salem (sheet No. 2), was in reality an enlarged porch. A brick porch with a chamber above is a feature of the brick Pierce-Little house at Newbury, Mass. A suggestion of a porch in a Farmington Conn. house built in 1690, is the semicircular pediment placed above the door and in line with the second floor overhang. The pedimented doorway of the Miller House on Long Island (1700), has the door recessed while the Doak House at Marblehead shows the curved pediment projecting considerably. This hood was then pulled forward to make a porch as in the Miller House at Byfield, Mass. (c 1710), while the door behind remained ornamented. A simple semi-circular hood on cantilevered brackets is used over the entrance to the Moravian brothers' house at Salem, N.C., and again in the Potts House at Valley Forge, Pa. (sheet No. 2). A flat porch effect is most happy in the Arnold House at Weymouth, Mass., which shows the curved central architrave. "Lord" Timothy Dexter's porch at Newburyport shows rusticated columns and pilasters supporting the architrave and the balustrade above. In South Hadley, the Squire Bowdoin house (sheet No. 5), shows a charmingly simple four columned porch with a bowed centre in the cornice. The Lee Mansion at Marblehead has a simple pedimented porch with Ionic columns. The Treadwell House at Portsmouth shows a charming shallow porch with coupled columns, while in the same town the Governor Langdon house, built in 1768, has a very ornamental half octagonal porch with fluted corner columns. At "Mt. Pleasant", Philadelphia (1761), the Roman Doric porch (sheet No. 5) makes a massively fitting entrance, while the porch at Gunston Hall (sheet No. 2) follows a more delicate Palladian motif. An enclosed porch or vestibule with arched side windows and a complete "order" ending in a pediment, is used on the old Swift House at Andover.



COLONIAL DOMESTIC ARCHITECTURE OF 17th & 18th CENTURIES

Here the pilaster and arched treatment is repeated as the motif on the shed-wing of the house.

In Rhode Island, the Capt. George Benson house has a sturdy round-headed pediment and architrave supported by heavy Doric columns. Providence houses are rather heavy in detail when compared with similar Massachusetts types. Going further south into the German and Moravian settlements of Pennsylvania and Delaware, we find the Old Grey House at Nazareth, Pa., with a simple, sturdy, charming, latticed entrance porch. Simplicity throughout this district was the keynote, emphasizing the simple life of the settlers. In the south, porches were in general use, mostly of simple rectangular form, supported by the usual round or square corner columns, with pilasters at the wall. Usually the pediments were triangular as in the Ridout house at Annapolis which is charming with its Doric simplicity. "Acton", Anne-Arundel County, Va., was more delicate in detail with coupled columns at the corner. Gunston Hall has the rear portico octagonal with arched openings between the angled corner pilasters, all in delicate Doric detail. A most charming porch is that at No. 711 Prince St., Alexandria, Va., where the delicate tapering, fluted, columns carry an open columned frieze. New Bern, N.C., gives us charming examples in the simple Taylor house and the old Groendyke-Hargett place with its simple modillions. The gem of the neighbourhood is the Smallwood-Jones house with its chaste yet decorative classical ornamentation.

A development of the porch was the piazza which was used very early on the Hudson Valley farmhouses of New York and New Jersey. The simple cornices on the front and rear—more important architecturally—were pulled forward, as in Quebec, to protect the mortar in the face of the building. Then posts were added to support the wider cornice overhang, as at the Shenks-Crook house and the Terhuen (Terhune) house at Hackensack, N.J., built in 1670. The porches on the Philipse Manor House are simple protective coverings for the entrances, not elongations of the roof line. In Ontario the 18th century houses, as the William Hand homestead (1780) at Sandwich, have the roof elongated to form the porch. The two storey Munro House at the Long Point settlement of the United Empire Loyalists, had the porch supported by square columns. The ends were boarded in, as was the custom in that neighbourhood, presumably to act as a wind break. Piazzas were usually placed at the ends as in the Royall House at Medford, and in the Ellsworth House at Windsor, Conn., where the columns extend the full height under the roof of the ell extension. The Dutch farmhouses as that of the Dyckman family, New York, have the roof with a distinct "bellcast" projected forward across the front of the building and supported by columns. This was the usual Quebec

method of forming a piazza and dates from the 17th century as in the stone Repentigny Manor house, built in 1672, at St. Henri de Mascouche and also in the Georges Larue house at St. Jean on the Isle d'Orleans (1678-80). In Long Island, the Roe House at Patchogue, shows this Dutch influence. In New Jersey, the Lincoln House at Hackensack, has the full height portico with free standing columns. The front piazza was common on the small farmhouses of New Jersey. Further South, we find a columned piazza, forming enclosed porches at either end of a house at Alexandria, Va., "Beverly" on the Pocomoke River, in Maryland, has the piazza developed into a two storey pedimented portico with a second floor balustrated gallery. In Charleston, South Carolina, two storeyed piazzas were built on the garden side of the house, and one entered through the main entrance door from the street into the lower storey. The Maginault house (1770) shows this, as also does the entrance on a King Street house, where the piazza end is wood instead of the usual brick. The Miles Brewton (Pringle) house, built in 1765-9, shows the full two storeyed, columned and pedimented piazza.

CONSTRUCTION

The construction of the existing early houses was of white pine timbers, hewn from the solid logs (although occasionally the framing of additional rooms was done in oak or hemlock) which were cut, tenoned, and framed on the ground; then, as in the rural districts of the present day when a barn is being "raised", each side of the framing was hoisted into position, with the sills on the foundations of stone and clay mortar. They were then tied into position by wooden pegs. The main roof timbers were framed on, and the purlins, usually about $3'' \times 4''$, were notched into position. The wall studs, usually about $3\frac{1}{2}'' \times 4''$ in size, were placed vertically between the corner posts, and notched into the "girts" which were the main structural beams around the outside of the rooms. The openings for the stairs and chimney were framed. Then across the centre of the room ceilings, generally from the fireplace girts to the outside girts, the large "summer beams" were fixed. In Massachusetts the summer beams ran parallel to the fireplace, in Connecticut, at right angles. This was repeated on the second floor, the plain or chamfered girts extending beyond the first floor framing, forming the well-known "overhang" which varies in projection. In the "Scotch-Boardman" house it is $17\frac{1}{2}''$, while in the Hyland-Wildman house the chamfered girts overhang slightly on the exterior. This house has shouldered corner posts and girts each side of the door which are carved as brackets. This overhang was a structural detail inherited from contemporary English architecture, not, as has been so often erroneously

stated, as a vantage point in protecting the main entrance doorway from Indian attacks. The posts at the front corners and at the middle chimney girts, were ornamented with carved drops, as in the Capen house. Here too, shaped brackets were carried from the projecting girts at the side of the entrance door. This bracket was repeated in the south gable under the third floor overhang. Carved shoulders on the corner posts are a feature of the Bray House at West Gloucester. The Low House at Wenham and the Corbett House at Ipswich have the overhang completely around the buildings. The latter has a slight gable overhang as well, similar to that on the Capen House.

Between the framing the soft local bricks were set up in clay mortar and covered with a clay plaster. (Lime mortar was later secured from burnt oyster shells.) This was not very conducive to durability under the fierce New England storms and the colonists found that the exterior plaster coating disintegrated, necessitating an exterior covering of narrow pine clapboards, often an inch in thickness, nailed to the studs. A few of the later houses, as that of the Winslow's, at Marshfield, Mass., have been found to have the walls packed with a certain kind of seaweed, non-inflammable and insulating. Seaweeds between the studs, and clapboards over the exterior clay and hay filling, were used in part of the Corwin House. The exposed surfaces of the clapboarding decreased in size toward the top, usually from 6″ to $3\frac{1}{2}$″ as on the Capen House. Around the grade line, forming a base, was a wide, thick skirting board, present in the Capen, Ward and "Scotch-Boardman" houses, but absent in the Whipple and Hancock-Clarke houses. (Count de Puisaye's House at Niagara-on-the-Lake, Ontario, built in 1799, was an early New England clapboard type, with one front gable overhang, and wide skirting board above the grade.) Vertical strips received the clapboards on the corners. Hand-wrought split pine shingles, were also used as a covering, and frequently, as on the Bray House, to accentuate the gables.

Snow's *History of Boston* gives us an interesting description of the wall finish of the "Old Feather Store" built in 1680, which had a rough-cast plaster exterior, with broken glass instead of pebbles. The ornamentation consisted of squares, diamonds, and "flowers-de-luce" worked in the plaster, similar to the plaster parging of the English cottages. This might have arisen from the tradition of geometric damascening found so frequently in English heraldic work of earlier periods.

The "Nieuw Amsterdam" settlement by the Dutch antedated by some few years, that of the New England Colonies. The Dutch colonists brought with them the ideas of the traditional brick buildings with crow-stepped gables, so prevalent in Holland. Those shown in old prints have

all disappeared due to the rapid expansion of Manhattan Island. The examples of early buildings which remain to us, are not of brick but of stone and wood, the "farmhouse" type, scattered along the valleys of the Hudson and its tributary streams. They were built of local materials, varying in their location; stone along the Hudson and wood on Long Island. In New York and on Staten Island (where the Lake Tyson House at "Nieuw Dorp" is an uncommon example of an early shingled house in that locality) whitewashed or stuccoed stone was used. The New Jersey fields offered an unfailing supply of red sandstone of glacial origin, which was used in the main walls of the lower storeys, the gable ends and the wings being built of wood. So often the mortar, which was of inferior quality, caused trouble through disintegration. This was remedied by projecting the eaves beyond the building—reminiscent of the early Quebec houses.

The Dutch houses of the frame type were similar to those of New England, in being of the stalwart framing construction of heavy girts and posts. The earliest walls were built of thick planks, set vertically edge to edge, and the interiors roughly adzed to give a mortar clinch. Shingles were then nailed on the exterior. Later brick, small stones, and clay were used as in-filling between the posts. The sill was laid on a masonry foundation, but the broad "watertable" board was missing as the shingles or clapboards came directly to the grade line. The shingles were wider and with more exposure to the weather than in New Englnad. Frequently, the front and rear walls of the stone built houses were of dressed ashlar stone with corner quoins, while the ends would be ordinary field stone. Brick quoins were used in the Manor House at Croton-on-Hudson while the Philipse Manor House is entirely of brick. The stone walls under the front piazzas were often covered with white-washed stucco to help the lighting of the rooms behind.

Further south, Virginia, named in 1584, was first permanently settled by John Smith. At Jamestown were erected many of the earlier dwellings of which S. H. Yonge has identified the brick foundations which are all that remain of the wooden buildings. The earliest wooden house remaining is "Bond Castle" on Chesapeake Bay. The southern planters and proprietors, as their wealth increased, replaced their wooden houses of "wattle and daub" construction by the more up-to-date ones of brick. Neglect and the passing of the houses from the owners use to that of slave quarters continued the destruction of these earlier habitations. The impossibility of procuring suitable lime for mortar retarded the building of brick houses. Stone was lacking in the Virginia Peninsula, and local brickmaking took place as early as 1653. In the *New Life of Virginia*, 1612, the houses at Henrico had the first floor of brick with a high basement, while the existing brick

"Bacon's Castle" in Surry Co., Virginia, built in 1660, was very reminiscent of contemporary English work with the stepped gables and diagonally placed chimneys. The heavy beam construction was still used in the roof and basement, as well as the interior partitions and floors. As noted before, Bond Castle is of the traditional heavy beam construction similar to the north and with a clapboard covering and the overhang on the second storey gable extension.

The theory has been advanced that the English bond in brickwork was in general use before 1710 and Flemish bond thereafter. In Jamestown, English bond is found in the church (c1639-47) and in the foundations of some of the excavated houses (c1666) but in the Warren House of earlier date (1651-2), Flemish bond was used. The oldest house in Yorkstown, Va. (date unknown but before 1700) is of brick with a high basement. Two deep end chimneys project about three and one-half feet. The base and chimneys are all stretchers and the first floor to the eaves has alternating headers and stretchers.

The construction of the early Canadian Colonial house in Quebec was almost wholly of the stone wall type. The rough field stones were roughly dressed and built with thick mortar joints. The walls were thick, occasionally three feet, giving very deep reveals on the interior. Stone was used throughout the century, although the early Jesuit house at Sillery (1637-8) was stone on the front and rear but with wooden clapboarded ends and annex. In 1650 the Denechaud Manor at Berthier-en-bas was built with wide vertical wood sheeting and the Gagnox House at Louisville (c 1660), was covered with horizontal clapboards. The Larue house (1678-80) has a plastered front and rear with vertical sheeting. By the end of the century the stonework is plastered, as on the Chevalier House at Cape-Sante (1696); whitewashed as the Turgeon House at Beaumont (c 1695); or both, as on the Girardin House at Beauport (c 1690). Some old houses at Murray Bay have the projecting second storey on a corbelled wooden construction.

The earlier rural houses of the 18th century, used the local materials at hand for their construction. In New England, where white pine and oak were plentiful, we find the vast majority of the houses of wood, of the simple "five-windowed" type as the Townsend-Sweetser house at Lynnfield (sheet No. 2), although many of the city residences and public buildings, as the State House and Fanueil Hall at Boston, were of brick. The McPhedris house at Portsmouth, N.H. (1728); the Hancock and the Pinckney Houses at Boston; as well as numerous others there and in Salem, were among the larger brick houses built. In the smaller centres and the seacoast towns, wood was the prevailing material. The Royall house at Medford, "King" Hooper House at Marblehead, and the Vassal House at Cambridge are fine wooden

examples of the larger type. Many of the wooden houses, particularly where the chimneys were on the gables, had brick ends with wooden front and rear to offset the fire hazard. The Ackerman-Brinckerhoff, part of the Vreeland House, and the Board-Zabriskie house at Hackensack, New Jersey, show skillful blending of wood and the dressed New Jersey sandstone, side by side with full wood houses of the Jan Ditmars type, built in 1768. This is also true of nearly all the New York and Long Island houses as in the stone and the wooden Dyckman House of 1787 and the larger wooden Morris-Jumel Mansion in New York City. Stone seems to have been more prevalent in the rural districts of Pennsylvania. In the Moravian settlements we have the delightful Sisters' House at Bethlehem, the Manor House and the brick "Nazareth Hall" built in 1785, at Nazareth. Graeme Park at Horsham, Pa., of three storeys, is built of stone, as is also the Potts House at Valley Forge (sheet No. 2), and the more pretentious "Cliveden", Pa., built after 1763. An interesting feature may be noted, that frequently the relieving arches in the stone wall over doors and windows, were of brick. The farmhouses in this locality were almost always built of stone.

The further south we come into the Delaware settlements, brick is used more extensively, as for example the George Read House at New-Castle, built in 1791 and the Amstel house of 1730. The Kensey-Johns House (1790) is a small gem of brick building, with a contemporary wooden wing. In the Lower Delaware Valley, between New Jersey and Delaware, a region of stone and brick, the prosperous farmers built of wood. "The Willows" at Gloucester, N.J., and houses at Bordenstown built in 1740, are of wood, but "brick-paned", *i.e.*, lined with brick between the studding.

The apex of brick building was reached in the more Southern colonies of Virginia, Maryland and the Carolinas, where Flemish bond was used extensively, although not entirely. Dressings of harder and different coloured brick used to accentuate the openings, were found at "Mulberry", "Rosewell" and "Stratford". Moulded bricks were sometimes used for watertables and cornices as in the Brewton House at Charleston. Doorways in brick were built in Christ Church at Lancaster (1728). Stucco over brick and rubble stone offset the dampness due to condensation, as at "Mt. Pleasant", where the stucco is coursed to resemble ashlar.

"Carter's Grove" (1751), "Monticello" (1781), "Westover" (1726) and Gunston Hall (1758), (sheet No. 2), were notable brick examples of the Virginian plantations. Maryland had its "Beverley" (1774); and Lord Baltimore's shooting lodge or the Dower House, Prince George County. Among those in the charming town of Annapolis, is the well-known Brice House, built in 1740 (sheet no. 2). Here the facade is laid entirely

in brick headers whereas the wings are in the usual Flemish bond. In St. James Church, Goose Creek, S.C., (1711), the brickwork is covered with stucco. The Ralph Izard House (1757) shows the stone quoins and string course projecting beyond the stuccoed brick walls.

Travelling northward to Canada, we find that in Quebec the stone and wood tradition is strongly entrenched and most interchangeable. The majority of the houses in the rural districts have thick stone walls, sometimes almost smothered in mortar; and again regular ashlar is the predominant wall construction of the ground floor. Repeated coats of whitewash, forming a protective covering, give a delightful play of light and shade. The wood house is also omnipresent, two of which are shown on sheet No. 2. The first, the Poirè House at Beaumont, Quebec, was built in 1795—with the vertical sheeted exterior walls. The contemporary Sylvain House at Bellechase, shows wide horizontal sheeting on the front and rear, but narrow clapboards on the ends, all above a high stone base. In the old Mission House at Sandwich, Ontario, built in 1747, we have the French-Canadian type, similar to the Sylvain House. In the towns, the sophisticated St. Peter Street House at Quebec, is in regular coursed ashlar.

The 18th century houses were usually built with two storeys, but as time went on, we note an increasing number of houses adding the third or attic storey, especially in the larger cities, as in Quebec, (here the St. Peter Street House, Sheet No. 2, is four storeys) where the land congestion was more acute. Boston, shortly after the opening of the 18th century, had houses of "brick three storeys high, with a garret, a flat roof and a balustrade" as the Hutchinson House, with its pilastered front and three full storeys. In 1773, Isaac Royall, at Medford, raised the older Ussher House, to three full storeys. The Brice House and the Chase House, both at Annapolis, the Hall House at Medford (1760) and the Jeremiah Lee House at Marblehead (1768), were among those built with three storeys from the beginning. Later the Bull House, at Newport, the Morris-Jumel Mansion at New York and Governor Tryon's Palace at New Bern, N.C. (1767-80), "Rosewell", Va. (1730), and the Ralph Izard House, as well as the Blake Houses at Charleston, had three floors. The attic was usually lighted by dormers and frequently, as in the "King" Hooper House at Marblehead, the second floor contained a large ballroom.

The Bâby House at Sandwich (sheet No. 2), filled between the studding with mortar and bricks, had a stone basement. The Moy House at Windsor, built after 1776, is covered with clapboards, while in the same locality the William Hand homestead (1780) is similar. The Munro House (1796) of the Long Point Settlement, used wide

horizontal clapboards instead of the narrow ones. These types of buildings could be seen everywhere in New England. However, a few among them, notably "Poplar Hall" at Prescott, built in 1795, are of irregular coursed ashlar with stone trimmings. The Col. Smith House, at Niagara-on-the-Lake (1793), was symmetrical and stuccoed, with rusticated corner pilasters. A one storey "lean-to", at each end, was an unusual feature

INTERIORS

The interiors of the early United States examples which remain to us, show the rough hewn constructive girts exposed in the corners, and on the ceilings. They were plain, chamfered or ornamented in varying degrees of simple carving, never to any great degree of elaboration. The sill was exposed around the room, forming a baseboard and the brick in-filling was usually plastered, although in the "Scotch-Boardman" house the bricks in the second storey were left exposed. The fireplace ends of the rooms were lined with vertical boards resting on the sill or passing before it, with the edges widely or narrowly "thumb-moulded" and fitting into one another. The hall in the Ward House shows horizontal moulded wood lining which is uncommon. The corner posts were sometimes carved and shouldered, sloping in from the girt to the sill. Moulded lining alone was used for the interior partitions, which consequently were very thin, although the lining was frequently nailed on both sides of the stud partitions. The flooring was wide pine boarding, sanded or scrubbed. Time and constant polishing have given to the woodwork that satisfyingly, subdued, brownish tone against which the furniture to-day shows to its best advantage. The second storey floors were laid directly over the usual $2\frac{1}{2}'' \times 3\frac{1}{4}''$ joists, about 18″ on centres. Frequently the 17th century ceilings are plastered or white-washed between the joists, or else wood lined. Clay and chopped straw were used as a "sound deadener" between the ceiling and the floor above. Over the fireplaces plaster was used on the chimney breasts as in the Boardman House (sheet No. 3). Sometimes, as in the Capen House (on the same sheet) a plaster cove projected to the huge summer beam which passed before the fireplace. The doors are merely "cut-outs" in the vertical lining with battens (sometimes diagonal in a "Z") across the rear. The width of the door usually spanned several moulded boards although a vertical joint would sometimes be placed in the centre of a board. A later type (both are found in the "Scotch-Boardman" House) had the simple panels with the thumb-moulds and plain fielding similar to the panel over the fireplace shelf in the Hancock-Clarke house (sheet No. 3). The panelling became more sophisticated,

and was mostly restricted to the fireplace end with the mouldings of the heavy bolection type.

The 18th century developed the interiors of the houses to a greater degree perhaps, than the exterior. The increase in the number of rooms brought with it the elaboration of the interiors which resulted in panelled and papered walls. The more important rooms were panelled, showing an increase of the fieldings with mouldings simpler than the earlier heavy type. The panelling was usually restricted to one or two sides of the room with special emphasis placed on the fireplace end. In the Webb House at Wethersfield, Conn., and at Graeme Park (sheet No. 3), the fireplace was flanked by simple Doric pilasters and two round-headed, transomed panelled doors. Usually the panelling was symmetrical and a door on one side would be balanced by a cupboard on the other side. The size of the panels varied. Sometimes the lower panel would be the same width as the larger upper panel as in Graeme Park, or at the 1735 Holabird House, where two long narrow panels were placed above a square panel. In the early Thompson House at Woburn, Mass. (1714), the panels were of a more equal size.

Corner cupboards with glass or panelled doors were frequently used to balance corner fireplaces. A cupboard with a shell top is seldom placed in the side return as it is in the fireplace of the Pepperal House at Kittery, Maine. A panelled dado, which was commonly used in the stair halls, spread to the more important rooms where a moulded dado cap separated the panelling above and below. In the Van Courtland House, New York City, built in 1748, plaster, possibly papered at the time, appears above the dado cap (sheet No. 3). The dado cap butted, or was returned on itself, at the window and door architraves, as shown in the Brewton-Sawter House at Charleston (1795), and in the panelling of the 1720 "House of Seven Gables" at Salem (sheet No. 3). The dado cap as in the Brice House was frequently carried through as, and on a level with, the window stool and apron.

Pilasters were used as fireplace, door or window elaboration (House of Seven Gables and Holabird House, sheet No. 3), although at "Stratford" and at "Marmion", Virginia, they were used as a wall treatment. Engaged columns occasionally, as at the Dalton House at Newburyport, replace pilasters. Free-standing columns are found at "Cliveden" (1763) and at the Chase House at Annapolis. The introduction of wall paper restricted the woodwork to a low dado, with or without panels. The St. Peter Street house at Quebec (sheet No. 4) shows a distinct French influence in the curved panelling and the small pilasters over the mantel shelf, as well as in the low panelled dado, surmounted by the attenuated panels.

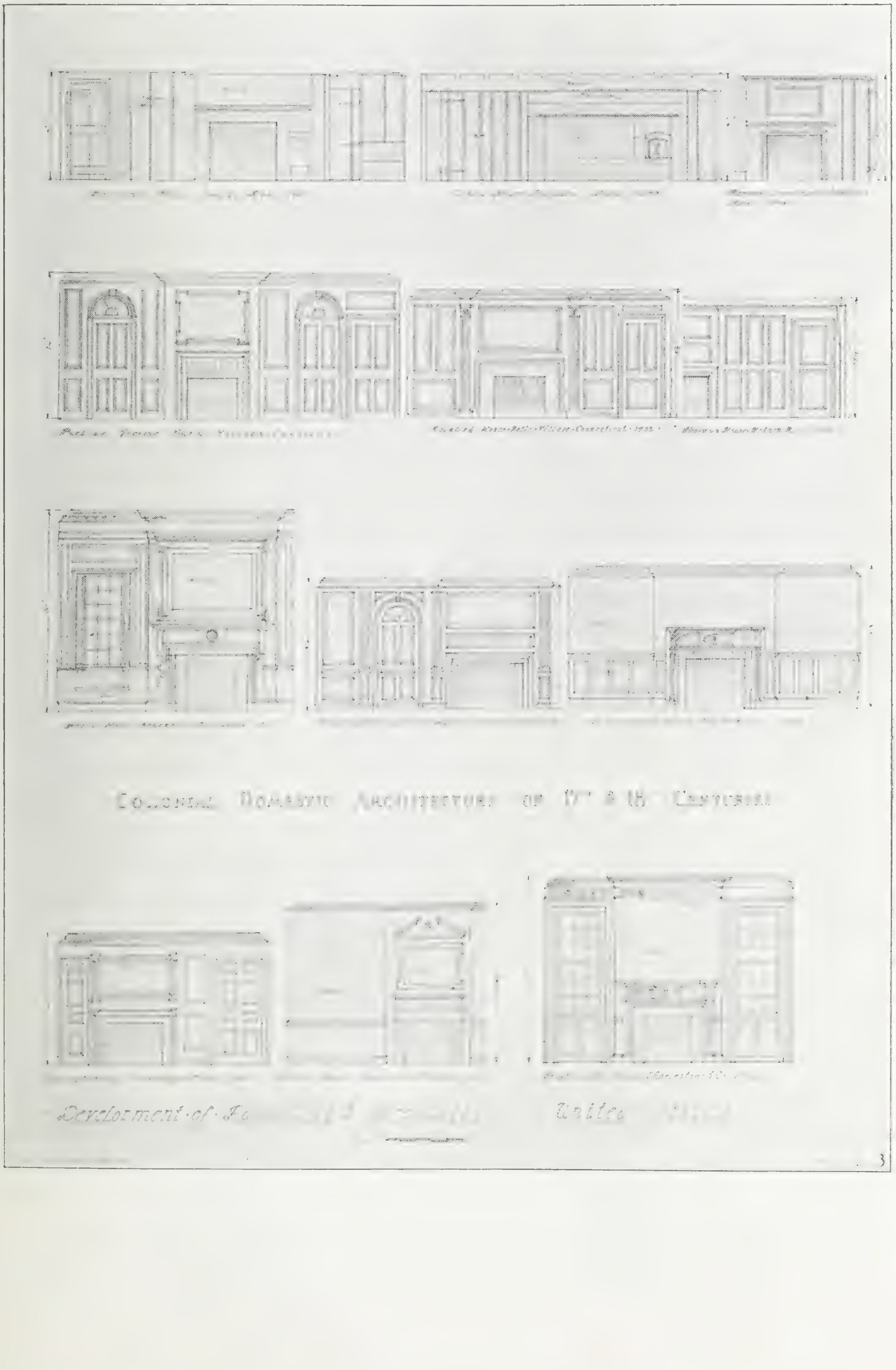
CEILING HEIGHTS

These varied with the building itself, and depended on the locality, the prosperity of the owner, and whether it was a farmhouse, a wealthy gentleman's country home or his town house. The period, too, added inches to the heights of the rooms. The earlier houses are found to have the lower ceilings, although in the same house, the heights may vary, due perchance to a change of owner before the completion of the house, or to additions years later when the newest ideas would be incorporated. New Englanders seemed to prefer the lower heights which gave to their buildings that low, snug, belonging-to-the-soil feeling which we find in the Capen, Ward and Whipple houses. The ground floors were usually higher than the second floors. The third or attic floors were the full roof height and sometimes were very low, as in part of the House of Seven Gables (6′-1″). The Potter Mansion from East Brookfield, which now graces the Eastern States Exposition Grounds at Springfield, Massachusetts, had the lowest kitchen which I have measured. The original house (c1740) was relocated and surrounded by 1790 additions, the old living room becoming the flagstoned kitchen, with a 6′ 0″ ceiling.

The usual heights were between 7′ 0″ and 8′ 0″ for the ground floors and usually less for the second floors. The town mansions had the rooms built in proportion to their size. The large ballrooms in the mansions of the wealthy merchants of the Atlantic Seaboard, were finely proportioned.

Following is a list of varied room heights in a few houses. Some of these are shown on sheets No. 3 and No. 4. The heights given are for the first or ground and second floors.

"Scotch-Boardman" House	1651	—7′ 1″ and 7′ 5″.
Bacon's Castle	1660	—8′ 0¾″ and 9′ 4″.
Early Quebec House	c1665	—7′ 10″ ....
Capen House	1683	—6′ 10 and 7′ 0″.
John Ward House	1684	—7′ 9½″ and 8′ 4″.
Buckman Tavern	1690	—8′ 0″ and 7′ 10¼″.
Hancock-Clarke House	1692	—6′ 9½″ ....
Thompson-Rumford House	1714	—7′ 8½″ and 7′ 8½″.
Adams House	1716	—7′ 6″ and 8′ 2″.
House of Seven Gables	1720	—8′ 6″ and 7′ 6½″.
Graeme Park	1721–2	—9′ 6″ and 9′ 5″.
Holabird House	1735	—8′ 5½″ and 7′ 5½″.
Brice House	1740	—13′ 1″........
House at Williamsburg, Va.	c1745	—9′ 9½″ and 6′ 6″.



Van Cortlandt House	1748	—9′ 3″ ....
Vassal-Craigie House	1759	—10′ 10¾″ and 9′ 1¾″.
Potts House	1760	—9′ 2½″ and 8′ 2½″.
Morris-Jumel Mansion	1765	—11′ 1½″ and 9′ 0″.
Marrett House	1765	—8′ 7″ and 8′ 3½″.
92 St. Peter Street	1781-4	—........ 10′ 2″.
Kensey-Johns House	1790	—10′ 4½″ ....
Bâby House	c1790	—8′ 6″ ....
Brewton-Sawter House	1795	—11′ 10½″

Interior doors and windows and simple recesses in the panelling, as in the Brice House dining-room (sheet No. 3), were later very much ornamented. Here the moulded door architrave is topped by a broken pediment. It also surrounds the windows and carries out the fireplace motifs. The Cowles-Lewis house at Farmington, Conn., has the doors and windows surrounded by an eared architrave and full pilastered order with modillions. This enframement is similar to that at Gunston Hall and the Miles-Brewton House at Charleston, the latter of which has also consoles supporting the door cornices. Panelled interior shutters were used extensively in the deep window reveals, the fielded panel of which usually had a very simple moulding. "Homewood", in Maryland, has elaborate, double-panelled doors with leaded glass side-lights and fanlight. The cornices of simple mouldings, as in Graeme Park, became more elaborate in Carter's Grove (1751) and most ornate in the Lee House (1768) and the Cowles and Brewton-Sawter Houses. The last, shows the carving and fluting used on a smaller scale in the mantel and dado.

FIREPLACES

The life of the early settlers revolved around the fireplaces, in the living room-kitchens. Built on stone or brick foundations, the fireplace of generous width, was in most cases, deep. This depth varied from 2′ 2″ to 2′ 8″, the rear corners being rounded in the older examples. The ovens, about two feet in diameter, were placed on the side or in the corner. They had their own small flues to carry away smoke or odours and the arched or square entrances were closed by handled wood or iron doors. Many fireplaces have a curious shallow recess under the firebeam, presumably for pipes or glasses of mulling liquors. Other shallow recesses in the back of the fireplaces were used for keeping cooked foods warm. High in the throat was placed the heavy trammel bar, of 3″ diameter green oak, from which the pot hooks hung. In later examples a round or square iron bar was used. In the Thomas Lee House at East Lyme, Conn., the shallow stone fireplace has an iron

crane swung at the side. In the Burnham House at Ipswich, the very wide brick and plaster fireplace has a forty-five degree splay to the rear wall, giving it a shallow depth. To counteract this, and to carry all the smoke up the flue, a four-inch recessed shelf, of a width varying with the fireplace opening, is carried at the rear into the smoke chamber. This recess usually starts about 2′ 4″ from the hearth. It is absent in the Boardman and Ward houses. It is present in the first floor fireplaces of the Parson Capen House (sheet No. 3) but absent in those of the second floor. The hearths are usually stone or brick and the outer hearth, about one to two feet in depth, extends to the full width of, and sometimes beyond, the fire opening.

The firebeams or lintels, were of heavy hewn oak timbers, either simply chamfered, or, as in the Burnham House at Ipswich, richly carved in a dentilled effect. The wall over the fireplace usually was a continuation of the moulded wall lining, or else was plastered in a cove or splay to meet the chamfered chimney girt, or the summer beam overhead. The brick sides of the fireplace exterior were frequently plastered and often had a wide wooden bolection-mould surround on which the lining above and on the sides terminated. The splay-back of the chimney breast often allowed, as in the later 18th century work, a hidden cupboard behind the panelling, useful for valuables or wines. The fireplace of the New York houses of this period, particularly the one in the kitchen-living room, which was used for cooking as well as the ordinary life of the house, was of large size. It was often six or seven feet in width and two and a half feet in depth. The "Dutch" ovens were placed at the side or rear. There was little pretence of an overmantel, although the walls adjacent were filled with cupboards for storage purposes.

A feature typical to Quebec was the oven opening on the outside of the chimney stack as in the Gascon House at St. Francis-de-Sales. Bread ovens were frequently placed away from the house and have been in continual use since, as the traveller through rural Quebec will not fail to notice.

The 18th century fire openings were reduced in size in comparison to the cavernous ones of the preceding century. They were treated as architectural adjuncts, being framed with imported marble facings as in the Brice, Johns and Brewton Houses, (sheet No. 3). They were then surrounded by moulded frames which were frequently heavy wood or marble bolection moulds as in the Holabird House and the "House of Seven Gables". Above this, panelling would be employed, as in the Lee House at Marblehead and as at Graeme Park (sheet No. 3). A moulded shelf was used above the mould at "Carter's Grove" (1751) and "Tuckahoe" (1730). Frequently pilasters and entablatures frame the

fireplace. This was unusual before 1750, but almost universal in the latter half of the century, reaching its zenith in the decorative Brewton-Sawter house. In many cases shelves were added at a later period, as at "Tuckahoe" and "Stenton". Console brackets were frequently used under the corniced shelf as in the Brice House (sheet No. 3). Simple shelves, moulded or plain, were early used in Quebec and Ontario, as at the Bâby House. More decoration was introduced by pilasters and fluting, particularly in the city houses, until complete orders were the vogue. The decoration on the whole was much simpler in Canada than in the prototypes to the South of the border.

CHIMNEYS

The chimneys, at first of wood and clay or thatch over a wood frame, were soon replaced by local brick. In the Ellery house at East Gloucester, Mass., the brick chimney above the roof is very simple, almost square in plan, and plastered. This is unusual in comparison with the usual rectangular type. Simple projecting brick caps were used. At the bottom, slightly below the roof ridge on the slope, a brick drip course which carried the water to the side of the roof, was used as flashing was unknown. This is quite well shown in the Capen House (sheet No. 2) and in the Corbett House at Ipswich which was built during the second half of the 17th century. As additions and "lean-to's" were added, the additional flues were carried up against the original chimney, forming an interesting, clustered, brick mass as in the Capen and Ward houses. The Scotch-Boardman House chimney shows added narrow pilasters on the front which are corbelled back to the main chimney below the roof. The Corbett House shows a similar feature in which the corbelling is not concealed. Below the drip course the chimneys were often plastered as on the Capen House (sheet No. 3). The stone Whitfield House at Guildford, Connecticut and the Tate House at New Castle, Delaware, have huge stepped brick chimneys on the gable ends. The Peirce-Little House at Newbury, Mass., is an early example of a brick and stone house with a brick stepped chimney on the end gable. The chimneys on the Warren House (1651) in Virginia have simple brick caps, while "Bacon's Castle" (1660) as well as "Fairfield" (1692) have clustered diagonally placed chimneys with wide corbelled caps. Chimney pots are occasionally used as on the Tufts House at Medford, Mass. (1667-80).

The brick chimneys of the Hudson Valley houses were usually on the gable ends, although in parts of Long Island, where the settlement was divided between the Dutch in 1635 and the English in 1740, we find houses as the Payne and Mulford ones, which followed the New

England plan with the central chimney and simple projecting brick capping. Stone, although used extensively for the farmhouse walls, was rarely used for the chimneys which were almost invariably of brick. As flashing between the chimney and roof was a difficult proposition due to the scarcity of sheet metal, we find the projecting drip course used, as in New England, on the Lake Tyson house at Nieuw Dorp, Staten Island. The increase in the number of fireplaces in the 18th century resulted in the flues being brought together in one or more chimneys which were placed on, or towards, the gable ends,—very rarely in the centre of the building. The chimneys were still elaborate in plan, particularly T shaped as at Graeme Park (sheet No. 3) and on the Hancock House in Boston (1737) and the Van Cortland House in New York (1784). Sometimes they were ornamented as shown on the McPhedris house at Portsmouth and at "Cliveden", Pa. (1763). Later, the rectangular form was used almost invariably with a simple corbelled-out brick cap. When stone is used, the caps range from simple flat blocks to classical mouldings which are found on such houses as "Rosewell" and "Monticello" in Virginia. An interesting thing about the Quebec houses (the Ontario ones were usually the rectangular simple capped type) is, that they were very low—rarely more than three feet above the roof. They were almost invariably square and built of stone. When two appear on the gable ends to give a symmetrical effect, quite frequently one is false. Frequently too, an addition to the building may throw the one gable chimney toward the centre of the whole house. The Poiré house (1795) at Beaumont, Quebec (sheet No. 2), shows the low simple capless type. The city houses usually had high rectangular chimneys above the parapets or ridges, unsymmetrically placed and with simple Gothic stone moulded caps. More rural houses as that of Gouin-Bureau, (c1669) at St. Anne-de-la-Perade, have ornamental carved caps and drip moulds above the ridge. Chimneys were placed infrequently astride the ridge but pierced the roof anywhere.

ROOFS

Thatch seems to have been the earliest roof covering of the 17th century but the frequent fires caused its popularity to wane and give place to shingles, tile and slate, often insecurely fastened. The 17th century roofs were steeply pitched, often broken in the sweep due to added lean-to's, as in the Scotch-Boardman house. The shingles were nailed to the vertical or horizontal boarding, which in turn was fastened to the purlins and heavy roof framing. Owing to the scarcity of metal flashing, we find the long slope of the roof unbroken by the dormers which appear to have been later additions to provide lights for the

attic rooms which were now used as bedrooms for the children and servants, as well as for storage. The dormers on the Dutch Philipse Manor House (1682) are of unusual type, being casement sash divided by a mullion. The Lefferts house (1675) shows elaborate pilastered and fan-lighted dormers which are very obviously later. The roof was the usual ridged gable of forty-five degree slope.

Gables frequently take the place of dormers as these gave greater room and a greater exterior picturesqueness, of which the House of Seven Gables and the Ward House (sheet No. 2), both at Salem, are well known examples. The Bridgeham House of Boston and the Old Feather Store, in a gable of which latter house is the date 1680, were also well known from drawings, although now unfortunately destroyed. Gabled dormers are used in the 1698 Hancock-Clarke House at Lexington. Shed dormers appear at Graeme Park and in the later 18th century. Gabled and hipped dormers were very prevalent for attic rooms.

New England seaport towns developed a special roof addition, the cupola, generally lighting the stair halls below. According to tradition, the railed promenade of the "Captain's Walk" was used as a lookout for the returning ships. The late 17th century Province House at Boston (1676-9) shows a large octagonal cupola spired by an Indian with his bow and arrow. The McPhedris House (1728) and the Warner House (1723), both at Portsmouth, N.H., show simple hexagonal cupolas on the flat railed roof decking. "Shirley Place", at Roxbury, Mass. boasts a large pilastered and balustraded cupola, showing a spire and louvred shutters on the windows. By 1768 the Lee House at Marblehead boasted an ornate octagonal cupola, with round-headed windows and curved roof. Cupolas were not used in the south to the extent that they were in New England. A double-decked cupola, however, is used on the Town Hall at New Castle, Delaware, and "Rosewell" Virginia has a simple, square, double-windowed example.

Monitor roofs with wide low windows, gave to the attic storey an added height for storage spaces. The Winslow House at Plymouth, Mass., built in 1753, has a good example of this. They became increasingly common in the latter half of the 18th and the early part of the 19th century. In New England the "Gable-hip" roof was prevalent. As its name implies, it was a combination of the roof hips on the lower end wings butting into the gable of the main house. A distinctive Cape Cod roof is the "Rainbow" type in which there is apparent a distinctive convexity of the roof silhouette.

In the 18th century the gable roofs were flattened at the summit forming the gambrel type which increased the attic space, as at Graeme Park (1721) and the Hancock House (1733). Quite frequently, as in the McPhedris House (1728), the chimney gable received the roof, the

upper deck being protected by a balustrading. The gambrel is sometimes carried completely around the building, as at the "Mulberry" (1708-25). This is called the "Jerkin-head". Pitched roofs varied in slope between thirty degrees and forty-five degrees. Hipped roofs with the same cornice level all around and with steep ridges are noted at "Westover" (1726); part of "Rosewell" (1730); "Carter's Grove" (1751); and the Vassal House (1759). Frequently the ridge was cut off, forming a flat deck, where the size of the building would bring the roof too high. This was the case at "Stenton" (1728). After 1750 the deck was usual in houses such as "Mount Pleasant" at Philadelphia, the Chase House at Annapolis, the Morris House at New York and the Brewton House in Charleston. Balustrades bordering the flat deck appear after 1728, in the McPhedris, the Hancock (1737) and the Pickman (1750) houses, where it returns across the ends. "Shirley Place", Roxbury (1746), has the Mansard roof balustraded, as is also the case at the Vassal House, Cambridge (1759), the Hooper House, Danvers, and the Morris-Jumel House (1765). Charleston houses, as a rule, have very low hipped roofs of about 30 degree slopes. The Izard House (1757) has a small deck on top.

The roof was the main distinguishing feature of the early domestic architecture of Quebec. Roofs can be placed roughly in several classes, which are not, however, restricted to any one period or date. The builders carried the older traditions of one locality into newer regions. Usually shingles were employed but many were later covered with "fer-blanc" or tinplate. The gabled roof, is exemplified in the oldest settlements on the Isle d'Orleans. Here the houses show the heavy stone walls carried up into the gable, with attic windows breaking the wall surface as in the Batiscan Presbytery (1665) (sheet No. 2). However, there are numerous examples of the stonework stopping at the level of the eaves and with the triangular gable built up in wood, as in the house at Sillery. Occasionally this portion was clapboarded as on the Repentigny Manor at St. Henri-de-Mascouche. The gabled roof in many places has the bell-cast cantilevered beyond the roof and supported by a porch gallery. A St. Rose house has an unusual bay projecting beyond the wall and roofed by the overhang. Frequently the rear overhang projected less than that on the front. The Forget House (1694) at St. Francis-de-Sales has overhangs of equal depth. The roof gable at the apex frequently projected beyond the main roof forming a bonnet to the attic ventilators.

The steep hipped roof in the interesting, ancient example on the Gaudias Sylvain House at St. Michel-de-Bellechase, (sheet No. 2), has superimposed, steeply-roofed, casement dormers on the hipped gable. The hips are sloped at varying degrees. On the Sylvain House the

slope is only slight, while steeper on the Pouliot House in the same locality of St. Jean. On the latter the shingles are carried around the roof corners without ridge boards.

Then we have the hipped roof with bell-cast projecting beyond the wall line, at the front or rear only. In the Taschereau and Silly Manor houses the wide overhang is carried around, the projection being reduced on the sides.

An unique type of the Mansard roof is that on the Bouliane house, built in 1725, at La Malbaie. The projecting bell-cast around the entire house runs at about a sixty-degree slope to a point above the dormers where it sharply cuts back at a very flat angle to the ridge. The superimposed attics at the Chateau Bellevue at St. Joachim, increase the height of the hipped roof.

The town houses, owing to congested small lots usually sandwiched between other buildings, had the gables carried up above the walls as stone parapets. The roofs themselves were covered with "fer-blanc" for fire protection. Good examples are the old houses facing the halls of Justice in Montreal and the St. Peter Street house in Quebec (sheet No. 2). Every Manor had its own mill and these, built with thick stone walls, were scattered over the country. They were usually round, terminating in a conical shingled roof, mediaeval in its picturesqueness. Superimposed dormers were used on the St. Peter Street house at Quebec as well as on the Chateau Bellevue, built in 1778, which is a sophisticated Renaissance design. Quite frequently the dormers were shuttered. In Ontario the New England type of roof and gabled dormer prevailed as in the Bâby House (sheet No. 2), at Sandwich.

CORNICES

The verge or barge boards on the 17th century gables, were often quite elaborately moulded as shown in some of the old Salem prints. Occasionally they were placed away from the main wall, a free space being left when the verge board was footed on the projecting plate. We find on the Capen House carved finials or "pyramids" in the gable centres. The roof rafters, of various sizes, rested on the plates and were tied together by a collar beam, at about a six foot height from the attic floor. The $3'' \times 4\frac{1}{2}''$ purlins, parallel to the side girt, and generally about half way between the plate and the unimportant ridge, were pinned into the "principal rafters", which extended various distances beyond the plate, forming the cornices or overhangs. The earlier cornices were simple overhangs as at the Ward House where the plain verge board is cased with a simple boarding called the "jet", and finished along the gable edge with a cover mould. Later in the 17th century,

the coved plaster cornice was formed over curved projections pinned below the plate and extending to the second floor window heads, as in the Bray House at West Gloucester. In the Goodhue House, at Danvers, Mass. (1690); the windows penetrated the plaster cornice cove. The Capen House had double beaded verge boards returned on themselves.

An earlier type is the 1663 Batiscan Presbytery in Quebec which shows the cased bell-cast and simple tight cornice moulds. Bacon's Castle (1660) has a later type, guttered cornice returning on itself against the brickwork end. The tight return and simple effects of the Terhuen House at Hackensack, New Jersey, are achieved by delicate mouldings. As we arrive in the 18th century the "ordered" cornice, with its various adaptations became more prevalent. It is increasingly hard to "date" the 18th century house by its cornice, as the variations were so numerous. The simpler house, as the Thompson or Count Rumford House at Woburn, Mass., built in 1714, has a plain classical cornice similar to the Marrett house of 1765 at Cambridge and the Hancock House at Lexington, built in 1789. The house on St. Peter Street, Quebec, of much later date (1781-4) is even simpler. It has the metal gutter butting the corbelled stone parapet.

The originality of the carpenters' art, in New England particularly, was expended on cornices, whether for roof, porch, window or door decoration. Usually the main cornice was a simple classical outline, as in the Burbank House, Suffield (c1736), the Vernon House, Newport (1758), and Lee House (1768). The carpenter-builder tried to follow the decorative ideas shown in his book of plates. He showed a great originality in decoration, produced only by the very primitive tools. By 1798, in a house at Chatham Centre, N.Y., we find an increase in detail. Here a dentil-course, produced by long and short rounded "reeds" and guttae, is enhanced by a regular design of bored holes in the under surface of the modillions. Reeding alternating with square or diagonal blocks was a favorite frieze ornament. Rope ornament on bed moulds was produced by the use of chisels as on the Caesar House at Duxbury, Mass. (1794). Triglyphs are suggested by sets of incisions spaced regularly, while metopes were frequently spaced reeded blocks. Another Chatham Centre House (1789) shows the same pendant dentils but with a carved ornamental "sunburst and lamp" frieze. Swags, drops, frets and dentils were used in varying ways as cornice enrichment and in friezes, as on the richly ornamented door of the Arnold House at Weymouth, Mass. (1790). As we go southward, the Read House at Newcastle, Del. (1791) shows a colonial adaptation of the familiar bead and reel motif. Colross House, Alexandria, Va. (1799) shows a beautifully designed cornice with enriched bed mould and a frieze of coved drops. The ornament on cornices was quite frequently,

particularly in New England, gained by grooving and auger holes. This method is used on "Woodlawn", Virginia (1799), and on the "King" Caesar House at Duxbury, Mass. (1794). The Moravian settlements at Salem, N.C., eschewed enrichment on their buildings while the Charleston houses of Nathaniel Russel and William Blacklock permitted restrained ornament. In the Miles Brewton House ornament is unsparingly used in the main cornice and the secondary entablature of the portico. The cornice of the doorway of the Izard House (E, sheet No. 5), is a fine example of decorative restraint.

Plaster coving is very unusual in this later period, although we have a fine example in the Amstel House at New Castle, Delaware. Ornamentation reached its zenith in the South where such charming examples as the Brice House at Annapolis, Maryland, with its arcaded frieze, and the Colross House at Alexandria, Va., show a wealth of originality although almost 60 years apart in point of time. The Dutch houses of New York were slightly archaic in their detail and heavy in proportion. On Long Island, on the charming East Marion House, built about 1799, the simple detail can hardly be duplicated for proportion. Here the cornice is received on shaped verge boards projecting from the gable end.

WINDOWS

The windows during the 17th and 18th centuries show a marked development. Reminiscent of, and contemporary with, Jacobean and Tudor England, are the early casement windows placed high from the floor, usually in sets of three. In some cases, as in the Capen and Scotch-Boardman houses, they were originally single. The Ward House at Salem, compromises by having them double with a mullion between. The casements opened out and were fitted with small, square, diamond, or rectangular shaped, glass quarries. The sash was thin, frequently ¾″ and had the wide lead muntins wired to the horizontal or vertical wood or metal "calmes". The casements of the Buffem House at Essex (see sheet No. 4), as also those of the Fairbanks House at Dedham are ¾″ thick. An old casement preserved in the Essex Institute at Salem shows the unusual thickness of 7/8″. Occasionally, as in the Paul Revere House in North Square, Boston, and Bacon's Castle, two casements were used as a transom (sheet No. 4). Later the casements were replaced by double hung sash, of unequal size (see the Hancock House of 1698), (sheet No. 2). Each sash was divided into square lights or panes of small size, sometimes to the number of twenty-four. Usually there were twelve panes separated by narrow muntin bars. This continued the leaded glass tradition with the convenience of the more

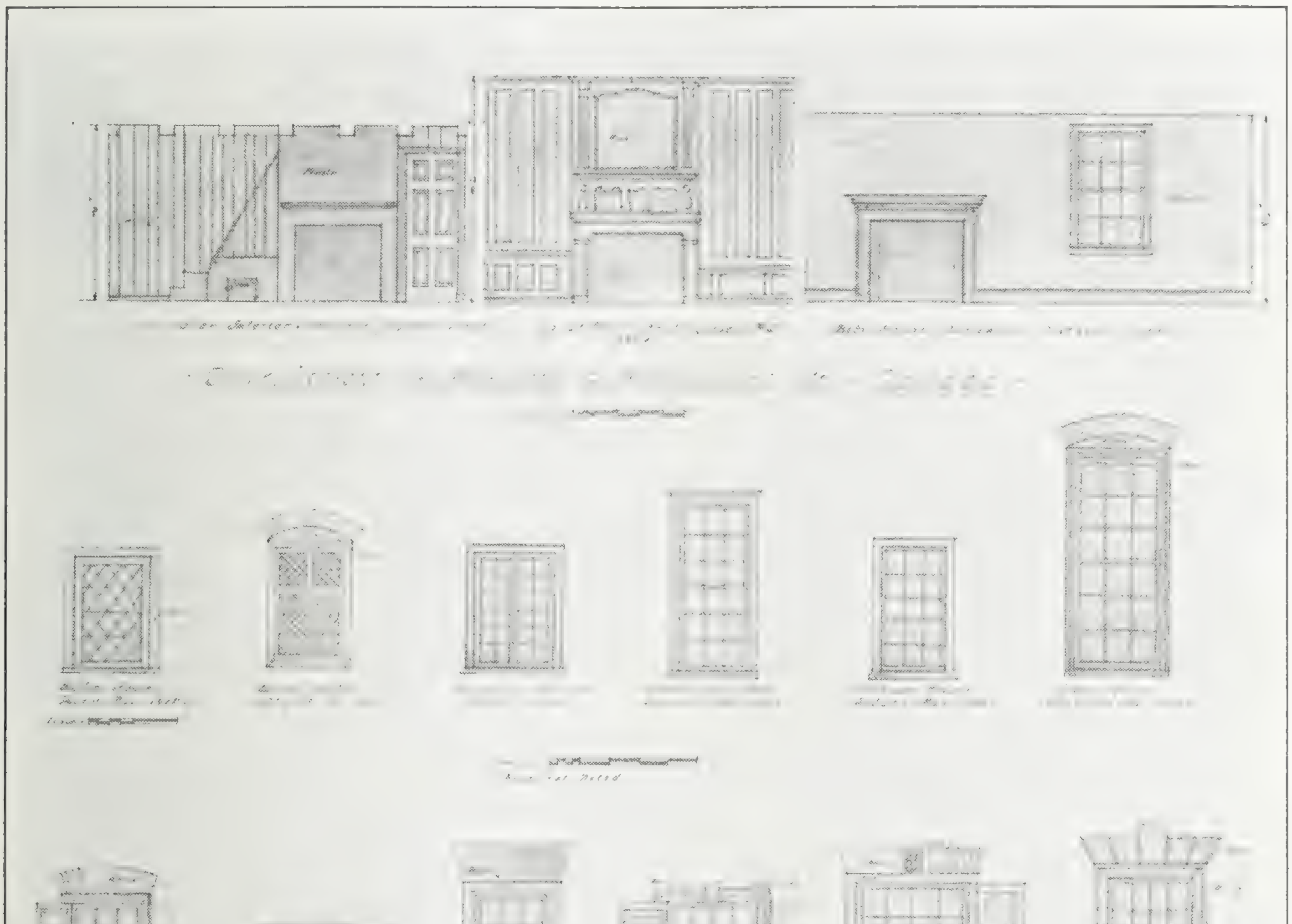
wind-proof "guillotine" type. A large number of the early houses which now show double hung windows, had them inserted to replace the earlier casements. The north windows of the Ward House are proof of this. In turn they were displaced by the ugly larger size panes. In the Acadian House at Guildford, Conn. (1670) the simple Connecticut type is in evidence. Here the window sashes are not equal in size but have the larger twelve light sash below and the smaller eight light sash above. This did not allow the lower sash to be fully opened. (Notice the different placing of the sashes in the Holabird and the Thompson Houses (sheet No. 4). The Baldwin House at Branford, Conn. (1645) shows a fifteen light sash, while the Walker House at Stratford of later date (1670), shows the larger sash. Solid shutters were occasionally made to close over the windows. Shutters which slide into the space between the studs were occasionally used. In cold weather these almost hermetically sealed the room and may have been in use before glass or oiled paper became prevalent for window glazing.

At first no particular heed was paid to fenestration in the North or Middle colonies. Although fenestration was more studied and symmetrical in the South, Kimball claims that all the windows opening horizontally had transomed casement sash, due to their large size, but later double-hung sashes with heavy muntins, were inserted. Windows and doors were dependent on the plan, being placed wherever they would most conveniently light the room behind. This resulted in a symmetrical arrangement as in the Capen and Ward houses in New England. In the Hancock House (sheet No. 2), we find two windows placed close beside the entrance. Further south in the Shenks-Crook House, Staten Island, the single windows each side of the door are far from being symmetrical. In some of the later Dutch farmhouses, casement windows were placed in the gables to light the attic, but as the chimneys cut across them on the inside, they do not seem to have adequately fulfilled their mission. Shutters were architectural and practical features common from Canada to South Carolina, although all houses did not have them. In Quebec, we find the Gagnox house at Louisville, as early as 1660, and the Tremblay House (1669) having the double casement windows closed with louvred shutters. The Vassal-Craigie House at Cambridge (sheet No. 2) and the Van Cortland House at New York, as well as the Brice House at Annapolis, have shutters inside and out. They were invariably of the panelled type on the interior reveals and of the immovable louvred typed, with or without beaded rails and stiles, on the exterior. We find the Dutch and German influence paramount in the Central region where the shutters were of the solid panelled type. The shutters on the Van Cortland house at New York, the Morris House at Philadelphia (1786), and "Cliveden", Pa. (1763) were of the solid type,

although at "Stenton" (1728) and Graeme Park, they were absent. The heavy panelled shutters as on the Bergen homestead, the Philipse Manor House and the Penn (Letitia) house, seem to be preferred. Frequently the second storey had shutters as well, as in the Lake Tyson House at Nieuw Dorp, Staten Island. This custom may have been a precaution against the attack of marauding Indians. The Reed House, at New Castle, has the shutters panelled on the first floor and louvred on the second.

In Quebec, we find the small paned, mullioned double casement type universally used, and set in a wide wood frame slightly back of the wall surface. The sills, usually of wood, project and return on themselves (sheet No. 4). The tendency, as in Bacon's Castle (1660) was to keep the frame as close to the edge of the masonry as possible, leaving no reveal. Later a four inch masonry reveal was used almost exclusively. The early type of wood frames in New England was simple, but by the beginning of the 18th century, it had become moulded, and with a projecting drip board as in the Hancock-Clarke House. This later developed into the moulded hood on the first floor, while on the second floor the bed mould of the cornice was mitred around the frame. The window heads were flat as in the Holabird, Bâby and Townshend frame houses, but arched in masonry. Where stone was used we generally find brick arches over the openings, as at Graeme Park. Keyblocks, in wood in the Old Manse, Suffield, Conn., and in stone (carved in the Van Cortland House and plain in "Mt. Pleasant" sheet No. 4), were prevalent. In "Mt. Pleasant" the flat-arch stones were rusticated, a pleasant contrast to the brick walling.

The latter part of the 18th century saw greater elaboration on the window frames. The second and third floor windows, more particularly in the South, were not as high as those on the main floor. This was accomplished by having unequal sashes with the smaller on the bottom. The moulded frame was pulled forward to project slightly beyond the brick line, as in the John Ridout House at Annapolis, Maryland, "Mt. Airy", Gunston Hall, and "Whitehall". In Charleston the prevalent usage was to place the narrow architrave at least four inches back of the wall face, as the builders have done on the Izard House (c1757). Of course, in the wooden houses of the Northern regions, the shingles or clapboards butted the projecting moulded wooden architrave. The Taylor House at Roxbury, built about 1790 (sheet No. 4), shows an unusual three sash window from floor to ceiling, with broken cornice and pilastered frame. Palladian windows are used as decorative adjuncts above the entrances or on the rear to light a staircase or principal room. "Rosewell", Va., and the McPhedris House in Portsmouth, N.H. use long narrow windows for this purpose. On sheet No. 4, we have three


Types of Colonial Windows
COLONIAL DOMESTIC ARCHITECTURE OF 17th & 18th CENTURIES

distinct types of the Palladian window. The Brice House window, profusely carved and arcaded, is in contrast to the New England Bowdoin House, with its true Palladian motif. Here the fixed side sash with panels below, is separated from the round-headed central three-sash window by fluted pilasters. In the Fairfax House at Alexandria, Virginia, the thin muntins have almost a leaded glass effect. In looking over the various types of windows on sheet No. 4, we notice, as the century closes, the increase in the size of the panes and the decrease in the muntin widths.

FRAMES, SASH AND TRIM

The Buffum house had ¾″ simple leaded sash, with a flat moulded exterior frame and simple flat interior trim. This type was used almost exclusively in the very earliest buildings but a few years later we find, as at Bacon's Castle, Virginia (1660), the original casements replaced by the "guillotine" or double hung type with thin sash (1½″) and simple moulds and wide, 1¼″ muntins. The earlier types of double-hung windows persisted almost to the 19th century and we find an absence of the separator stile between the sashes. They slid directly in contact with each other. The Van Cortland (1748), Imlay (1790), Brewton-Sawter (c1795), and the Smallwood Jones (c1800) houses show this stile.

At Bacon's Castle the solid frame without weights, is set just inside the brick reveal. The depth of the reveal is increased at Graeme Park and the Van Cortland House, and is very deep in the Hammond House, later by 22 years. The tendency was to provide space for a heavier ornamental exterior cover mould, and to throw the pulley box and weights, which were now in vogue, deeper into the reveal. This gave the windows greater depth and consequently the shadowed openings counted more in the general façade. The earlier solid, moulded, wooden, exterior frame, projecting greatly on the wooden Townsend-Sweetser house of 1720, was superseded by a flatter and more highly moulded frame as on the Brewton-Sawter porch of c1795, and on the Smallwood-Jones house. In the interior reveals which were usually splayed, were placed the panelled wooden shutters. These were prevalent in all the later houses due to the thickening of the exterior walls.

The interior trim or architectrave developed from the simple round-edged type on the Buffam house to the beaded architrave of the Ward House. The Buckman Tavern at Lexington had a narrow moulded trim and wooden splay, which was somewhat similar to the Townsend-Sweetser House. After the opening years of the 18th century, we find a greater refinement in the mouldings and also an increase in the width and projection of the trim. The beautifully proportioned moulds of

the Brewton-Sawter (c1795), Imlay (c1790) and Read (1791) houses are fine examples of this.

The sash width remains approximately the same, varying from the $\frac{3}{4}''$ width on the casements of the Buffum house, through the 7/8″ narrow sash of the Snowden-Long house, to $1\frac{1}{2}''$ width in the Smallwood-Jones house (c1800). The commonest width was $1\frac{3}{8}''$. The width of the exposed sash varied with the individual taste of the designer and was not set for any one type but varied from 1″ to $2\frac{1}{4}''$.

The muntin profiles on the double-hung windows remained very similar throughout the century and a half, being mostly the fillet and quarter-round. The main difference was the width which varied from 7/8″ in the Snowden-Long house to $1\frac{1}{2}''$ in the Van Cortland and Potts houses. The Central States persisted in clinging to the wide heavy muntin, typical of the sturdy inhabitants' characters. The mouldings of the German and Dutch colonists were heavier than those of the contemporary English. The cavetto and cyma were used sparingly in muntin profiles. A charmingly naive and delicate profile is that shown in the Snowden-Long muntins.

DOORS

Early exterior doors in New England were of the simple low batten type, vertically sheeted outside and horizontally lined within, or with heavy wooden battens. Built of unequal, wide moulded boards, they were diagonally studded with hand wrought iron nails as shown on the Ward House (sheets No. 2 and No. 5); also on the Capen House, as well as on Bacon's Castle in Virginia. They were framed by a simple round edged board, similar to the window frames. In Quebec and Connecticut the doors were more ornate, being panelled, as in the Philo-Bishop House (c1665), with nine panels, long at the top and with a small square one in the centre. The Hyland-Wildman House (1668), at Guildford, Conn., has attenuated moulded panels above and square panels below. Double "Dutch" doors were used in the Peirce-Little House at Newbury, Mass.; on the Richards House at Litchfield, Connecticut, 1730 (sheet No. 5) and on the Halsey House, Long Island, in 1690.

The doors themselves were fairly consistent, in that panels in various combinations were used. We rarely find an early sheathed door, with an "ordered" enframement. These architectural embellishments allow us to separate doors into various groups with similar detail.

I have given representative types of each group on sheet No. 5. They are not segregated to any one locality, with the possible exception of the "Connecticut" type doorways, which seem to have developed as a most interesting type of their own in the early 18th century. Two

charmingly different examples of this type are the Porter House at Hadley, Mass., in which the heaviness of the mouldings is apparent, and the delicate, almost modernistic, doorway of the Richards House at Litchfield, Conn., with its arcaded glass transom in the door itself, not as is usual, above.

Below we have the various early types differentiated:

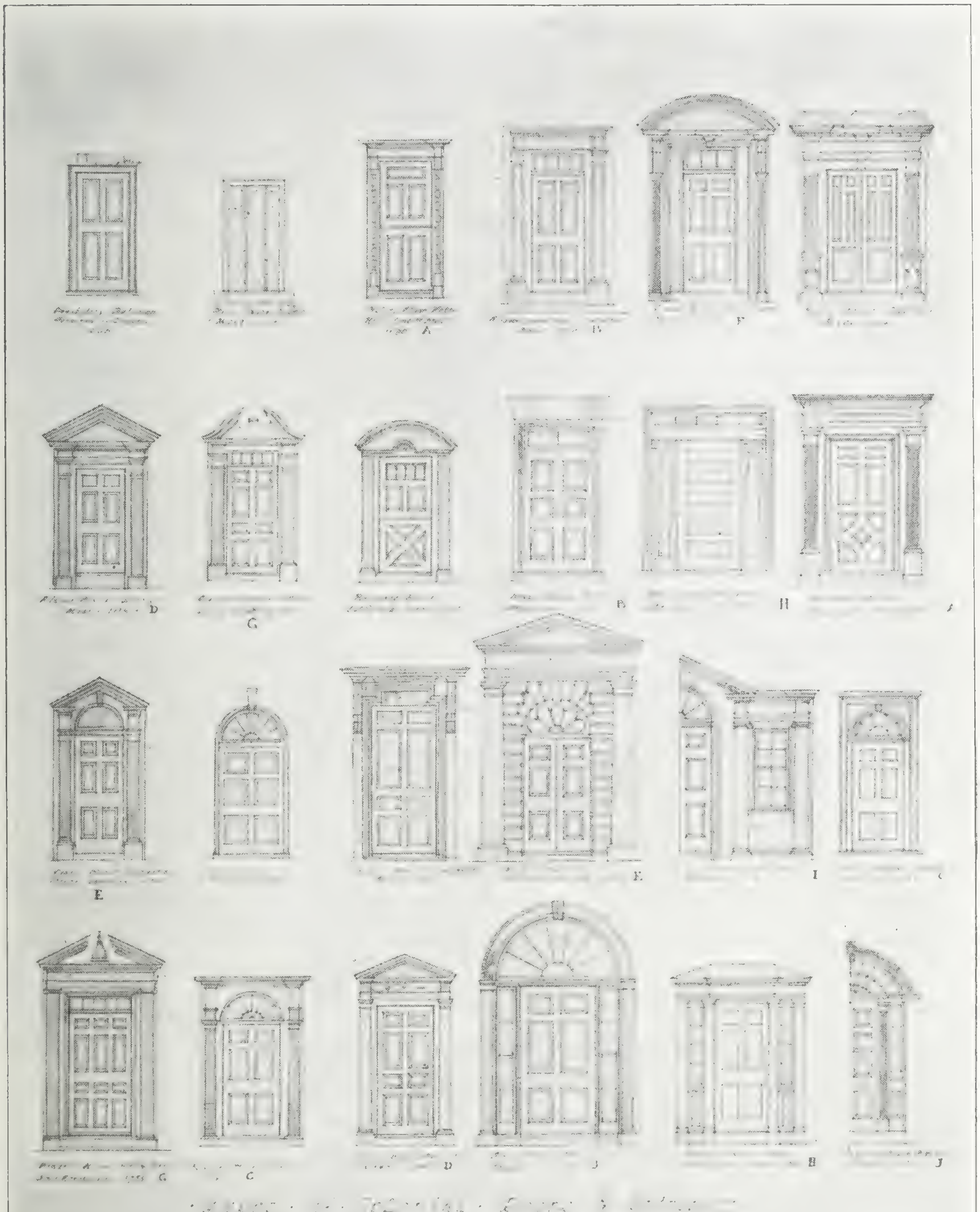
TYPE A—Represented by the Halsey House on Long Island. This early type is SQUARE HEADED, WITHOUT TRANSOM OR SIDELIGHTS and has a flat cornice. A heavier and more complete order is used in the door shown on the Marratt House at Cambridge in which the interesting lower panelling is worthy of note. The Vassal-Craigie door is an example of this type where glass has been inserted in the upper panels as a transom. Other later examples are: the Thos. Archer House in Suffield, Conn. (1795) and the Colross House, Alexandria, Va. (1799).

TYPE B—is SQUARE HEADED WITH A GLASS TRANSOM which lighted the entry hall. The same type without an order, but with a carved architrave frame, is used on the Brice House; the Stebbins House, Deerfield (1772); the Morris-Jumel House (1765); Lincoln House, Hackensack, N.J. (1773); the Brothers' House at Salem, N.C. (1768), and the Webb House, Long Island (1702).

TYPE C—Transoms were introduced to light the entries and the combination of a SQUARE HEADED DOORWAY WITH A CIRCULAR LEADED GLASS TRANSOM over the door gives us this third type, as on the farmhouse at Milton, Mass. The typical method of "carpenter" ornamentation was used in the varied vertical and horizontal fluting and reeding. The plainer doorway of this type on the Bâby House at Sandwich, Ontario, is well proportioned. Other well known examples are on the Phelps House, Suffield, Conn. (1795); Gunston Hall, Va. (1758); and Maginault House at Charleston, S.C.

TYPE D—This type is perhaps the most prevalent of any. It is the SQUARE HEADED DOOR WITH A PEDIMENTED GABLE in a complete Doric order, as on the Adams House at Quincy, Mass. Further South on the Kensey-Johns House at Newcastle it has lost its New England huskiness for more delicate mouldings and a geometrical frieze band. This type is found on the Crawford House (1740), and Ridge House (1700), both at Providence, R.I.; Kensey-Johns House, New Castle, Del.; Griswold House, Guildford, Conn. (1780); and the Blake houses at Charleston, S.C. (1760-72).

TYPE E—is the combination of a CIRCULAR HEADED GLASS TRANSOM IN A PEDIMENTED DOORWAY. This type is also very common, as the doorway could be kept low, yet still have room for the transom sash. In the Izard House in Charleston, S.C., the doorway is



Colonial Domestic Architecture of 17th & 18th Centuries

profusely carved and delicate in detail, although well proportioned. Other examples are on the Amstel House, New Castle, Del. (1730); Van Dyck House, New Castle (1799), and the Jones House, West Maryland (1795). In the entrance at "Mount Pleasant Mansion" near Philadelphia, we have the same idea, although the transomed doorway is set behind the wooden Doric porch and not in the tympanum of the order. This type is used on the St. Peter Street House at Quebec and Poirè House at Beaumont (sheet No. 2).

TYPE F—Here we have a SQUARE HEADED OPENING IN CONJUNCTION WITH THE CIRCULAR PEDIMENT. In the Doak House, which is shown, we have interesting breaks in the entablature and the slender fluted pilasters ending in attenuated consoles. The door itself is the usual six-panelled type. The old Manse at Deerfield, Mass. (1768); the Richards House, Litchfield, Conn. (1730); and Benson House, Providence, R.I. (1786) are also interesting examples of this type.

TYPE G—A variation of Type F, gives the BROKEN PEDIMENT IN COMBINATION WITH THE SIMPLE FLAT TRANSOM and the eight panelled doorway as on the Townsend-Sweetser House at Lynnfield, Mass. and on the Gay Manse at Suffield, Conn. The entablature is usually broken also. This is frequently elaborated into the charming scroll pediment ending in a carved rosette, and with a carved pineapple or a central vase as on "Westover", Va.; the Pickman House, Salem; and the Wentworth-Gardiner House at Portsmouth, N.H. The vase in conjunction with the straight broken pediment is used on the Wendell House at Portsmouth. Note here the unusual wide twelve panelled door and the Ionic pilasters.

TYPE H—Here we have a common expedient of procuring a greater amount of light in the halls. It can be used only where the hall is broad and we find it most frequently in the "central hall" type of house. IT IS A COMBINATION OF TRANSOM AND SIDELIGHTS WITH A FLAT HEADED OPENING. The Van Cortland house has the transom only, above the panelled "Dutch" door. The sides are wood panelled instead of the glass and wood panels, as that shown on the Bowdoin House (1798); the Commodore Loring House at Old Roxbury, Mass. (1775); and the "Old" Wetherald and Root houses at Deerfield, Mass. In Deerfield also is the Wetherald House (1752) with its H type door combined with the pilastered Connecticut doorway, similar to the Porter House. This is also seen on the Lefferts House, Brooklyn, N.Y. (1780).

TYPE I—is a COMBINATION OF THE CIRCULAR HEADED TRANSOM DOOR IN A TRIANGULAR PEDIMENT WITH SIDELIGHTS SEPARATED BY PILASTERS—an elaboration over the preceding type yet different in

massing. The Chase House at Annapolis, Maryland (1769) is a simple yet dignified interpretation of this. Here the Ionic pilasters support the well-proportioned order with its modillions and carved bed mould. The Colonel Joseph Nightingale House at Providence, R.I. (1792); the Gay Mansion at Suffield, Conn. (1795); the Admiral Cowles House at Farmington, Conn.; the Smith House at Deerfield, Mass.; and the Wye House, Talbot Co., Maryland (1782) are others of this type scattered throughout the country.

TYPE J—This is perhaps the most delicate of the various types. It shows the refinement and studied proportions prevalent towards the latter part of the century. The COMBINATION OF THE SIDELIGHTED DOOR AND THE ROUND, ELLIPTICAL OR SEGMENTED TRANSOM HEAD, is set in a deep panelled recess as on the Lord Fairfax House at Alexandria, Va. (c1750) or on the wall surface as at the Colross House in the same town (1799), (see sheet No. 5), and in the Gore House at Waltham, Mass., where it is indeed a worthy entrance to any building. The sidelights, simple fixed sash as in the Colross House, or double-hung as in the Reed House, were frequently shuttered. In a house on Prince Street, in Alexandria, the sidelights have a charming geometric design in leaded glass. The transoms frequently owe their charm to ornate leaded glasswork, with stamped lead roundels at the intersections or on the narrow muntins as in the Gore House. The Blacklock House at Charleston, S.C. (c1799) shows the elliptical-headed doorway with stone keyblock, delicate engaged-columned order and charmingly narrow muntins in the transom and the sidelights. The deeply recessed doorways with panelled reveals seem to be a local custom in Alexandria, where the Lord Fairfax, Floyd and Harry Lee houses all have them.

STAIRS

Perhaps the most interesting interior details are shown in the staircases. The early Canadian, similar in practically all the Quebec farmhouse types, were most primitively simple. Steep ladder-like steps, very seldom with a handrail, were placed in the corner of the kitchen. Frequently, as in the Dutch Farm houses of New York, they were boxed in and had a door at the bottom to conserve the heat in the downstairs room. The earliest known New England type of staircase was a simple wooden rail between posts, while a rope has been known to serve as a handrail. In New England, they were usually placed in the entry against the brick chimney stack where they are models of ingenuity. They had winders at either end, with short runs between, and were invariably quite steep, sometimes with eight inch treads and eight or

nine inch risers. Heavy, square newel posts, often carved similarly to the drops on the exterior of the house, received the turned balusters and moulded handrails in the more pretentious examples as that of the Capen House. In the Benaiah Titcomb House at Newburyport, built about 1680, the staircase, with its Jacobean panelling and low turned balusters, is perhaps the finest example of this period extant.

The Dutch stairs, in the farmhouses of the Hudson and Delaware Valleys, are most simple, being steep boxed, "ladders" to the sleeping quarters above. The stair at Bacon's Castle (1660), in Virginia is placed in an extruding stair hall addition and was of the simplest kind.

The 18th century developed the stairs as an artistic end in themselves, paying no attention to whether they were approaches to important or unimportant rooms. The 17th century staircases had closed strings, while now open strings were adopted with brackets at the ends. The earlier type is seen at Graeme Park (1721). The Van Cortland House still retained the low closed string. The ends of the treads were treated with plain docks, as in "Stenton", although panelled blocks occur any time up to the end of the century. Floral carved scroll ends are seen early at "Tuckahoe" (c1730) and plain modillion ends at "Westover" (1737). The Hancock House in Boston used a scroll block sawn in an unusual design. This became increasingly common from Boston to Charleston. Quite frequently the profile of the stair ends was carried across the soffit as at the Hancock House and the Chase House (1769).

The balusters at first were wooden slats but were later sawn to shape from flat boards. This method appears as late as 1749 in the attic stairs of the Van Cortland House. The "stumpiness" of the early turned balusters denote their antiquity, according to Messrs Isham and Brown. Balusters of the 18th century became increasingly ornate. They were turned and carved and usually were placed three on each tread, although at "Tuckahoe" two were used. As in the Hancock House at Boston, a favourite custom was to have the three balusters turned differently, adding to the rich effect of the stairs. Square newels were common before the middle of the century, but increasingly rare after. "Stenton" and Graeme Park have square newels, a survival of the 17th century custom. The circular newels at "Rosewell" and "Tuckahoe" have "swash"[1] turnings. A single spiral newel, almost an enlarged baluster, is frequently seen as at "Carter's Grove". In the Jeremiah Lee House of 1768, the moulded handrail easement forms a square cap for the heavy spiral newel. The double spirals, one within the other, are an ingenious piece of carving for the newels at the Hancock House. Here the inner spiral twists in the opposite direction to the outer one. This

[1]"Swash" is the term used where the heavy spiral is turned in the direction opposite to the baluster turnings.

became a common type in many New England houses. Often the newel itself was left plain, while the balisters formed a surrounding "wind-up" on the lower projecting tread. The handrail frequently terminated as a console scroll against the square newel, when it was elaborately carved and panelled. After 1750, a common practice was to round the turns of the landing balustrade. In the Brewton House at Charleston the railing between the landing newel makes a semicircle and in the Chase House it curves around without the intermediate newels.

The designers of the latter part of the 18th century delighted in curving the stair runs, instead of using the former straight style. This necessitated the use of winders in the curved portions. At "Woodlands", Philadelphia (1788) straight runs with curved ends were used. This is used very successfully by Bulfinch in the Barrell House at Charlestown, Mass. A semicircular staircase is used in the Hersey-Derby House (1799) and in the Gore House at Waltham.

Exterior Ornamentation

Exterior architectural adjuncts were in the 17th century confined almost wholly to constructional necessities as the "overhang". On the State House (ante 1700); Penn House (1682); both at Philadelphia and the Province House, Boston (1676) we find stone string courses, while brick is used for the string course on the Tufts (Cradock) House at Medford, Mass. (1677-80). In the early years of the 18th century, we find decorated doorways, cornices, windows, stone keyblocks and string courses. Quoins are used on the Hancock House, Boston (1737-40) as well as further south on "Mt. Airy", Richmond Co., Va. (1758). On John Bartram's house at Philadelphia (1731), are found engaged stone columns and carved stone window frames with ears and consoles. Stone pilasters with festooned caps, flanking the doorway and on the corners, were used in 1681 on the Hutchinson House at Boston. Here also, string courses were placed over the first and second storey windows. The Marston House at Salem (1707) had stone Corinthian capitals. We find stucco used between the quoins on "Mount Pleasant" (1761).

The wall surface itself was grooved or enriched by "rustication" and was not confined to masonry as at "Mt. Airy" (1758) but was used in wood in the Royall House (c1750), the Pickman House, Salem (1751), and the Jeremiah Lee House at Marblehead (1768). The more pretentious houses were treated with pavilions, pilasters and porticoes, as noted formerly. "Rosewell" (1730) was the first Colonial house to have a projecting pedimented pavilion at each end. After 1750, shallow pavilions are noted on the Pickman, Apthorp and Vassal (sheet No. 2),

houses in Massachusetts; "Mount Pleasant" and "Cliveden", Philadelphia; the Chase House at Annapolis, and "Mount Airy" and the Tryon Palace in the South.

Walls and pavilions were adorned with an "order". In the Tryon Palace and in Dudley House near Newport, the order rises through two storeys over a lower storey used as an architectural basement. The common practice was for the colossal "order" to rise from the ground or from the pedestal to the cornice. The Pinckney House at Charlestown, Mass. (1746) and "Shirley Place", Roxbury (c1746) are examples of this. In the west façade of the Royall House (c1733); in the Vassal-Craigie House at Cambridge (1759) (sheet No. 2); as well as in the Apthorp House (1758); we find the order comprising the pilasters, fluted or plain, with individual pedestals, cap, architrave and frieze tied together by the cornice. Occasionally four pilasters will be used to accent the pedimented pavilions. On "Shirley Place" the pilasters are coupled on the end bay, where they turn the corners. Occasionally engaged columns, as on the Brown House in Beverly, were used to flank the more important central motif.

One wishes for more time to delve deeper into the exhaustive study of the 17th century work of the early Colonists, both in Canada and the United States. A much greater amount of the 18th century domestic work is still extant and can be studied *in situ*. Canadian work of the 18th century is very scarce in Ontario where the New England influence was paramount. I have not touched on the domestic work of the 19th century in the United States. The same period of Canadiana, I leave for later research.

As in many cases the exact date of erection of the various houses is not fully proven by documentary evidence, the approximate date will be given in the following chronological table. At the end will be placed a list of reference books where more detailed information concerning the foregoing data may be garnered.

CHRONOLOGICAL TABLE

CANADA

Quebec Houses—17th Century

Jesuit House	Sillery, Que	1637-8
Vallee	Rue St. Anne, Quebec City	c1640
Denechaud Manor	Berthier-en-Bas	c1650
Marcoux	Beauport	1655
Gagnox	Louis	c1660
Maizerts Chateau	Canardiere	c1665
Kent House	St. Louis St. Quebec City	c1660-5
Boucher Manor	Boucherville	1668
Gouin-Bureau	St. Anne de la Perade	1669
Tremblay	St. Anne de la Perade	c1669
Gamache	Cap St. Ignace	1672-89
Repentigny Manor	St. Henri de Mascouche	1672-1702
Dupuis Manor	Montmagny	1677
Vezina	Boischatel	c1677
Montcalm	St. Louis St., Quebec City	1677
Georges Larue	St. Jean, Isle d'Orleans	1678-80
Villeneuve	Charlesbourg	c1684
Houses opposite Palais de Justice	Montreal	c1685
Marsil	St. Lambert de Chambly	c1690
Girardin	Beauport	c1690
Forget	St. Francis de Sales	1694
Turgeon	Beaumont	c1695
Chevalier	Cape-Sante	1696
Desmarchais	Notre dame de Neiges, Montreal	1698

Quebec Houses—18th Century

Ursuline Monastery	Three Rivers	c1700
Pérade Manor	St. Anne de la Perade	c1707
St. Dizier	Verdun	c1710
Presbytery	Caughnawaga	1716-21
Dorion	St. Anne de la Perade	1720
Mackenna	St. Famille	1720
Barbibeau	St. Anne de la Perade	1723
Denis	Neuville	c1725
Bouliane	La Malbaie	c1725
Mauvide Manor	St. Jean, Isle d'Orleans	c1734
Cantin	St. Romvald D'Etchemin	c1740
Recollect Convent	Three Rivers	1742
Blais	St. Foy near Quebec	c1747
Paquet	St. Nicolas	c1750
Breton	Beaumont	c1750
Dêchene	St. Roche des Aulnaise	c1750

Langlois	Kamouraska	c1750
Gannes	Three Rivers	1754
Octave de Lisle	Deschambault	c1755
Olivier DeLisle	Deschambault	c1755
Manoir Boucher de Niverville	Three Rivers	c1756
Goulet	St. Joachim	c1759
Gaudiose Pouliot	St. Jean, Isle d'Orleans	c1759
Samuel Pouliot	St. Laurent	c1759
Blouin	St. Jean	c1759
Têtu	Montmagny	c1763
Bourdages	St. Denis sur Richelieu	c1765
Gagnon		c1765
Ferland	St. Pierre	c1770
Tourangeau	Quebec City	1770-5
Basilica Presbytery	Quebec City	1773-5
Auger	Neuville	1775
Bellevue Chateau	St. Joachim	1778
Cartier	St. Antoine sur Richelieu	1779-82
Picotte	St. Paul L'Ermite	1780
92 St. Peter Street	Quebec City	1781-4
"Boulangerie"	Terrebone	c1784
Longtin	La Prairie	1790
Lemay	Nicolet	1794
Noblet	Contrecoeur	1794
Cherrier	Repentigny	c1798

ONTARIO HOUSES—18TH CENTURY

Old Mission House	Sandwich	1747
Moy House	Windsor	(post) 1776
William Hand Homestead	Sandwich	1780
Bâby House	Sandwich	1780-90
Poplar Hall	Prescott	1795
Munro House	Long Point	1796
Count de Puisaye's House	Niagara	1799

NOVA SCOTIA HOUSES—18TH CENTURY

Officers' Quarters	Fort Anne, Annapolis Royal	c1721

UNITED STATES

NEW ENGLAND HOUSES—17TH CENTURY

Fairbanks House	Dedham, Mass.	1636-8
Doten House	Plymouth, Mass.	1640
Baldwin House	Branford, Conn.	c1645
"Scotch-Boardman" House	Saugus, Mass.	1651
East Part of Pickering House	Salem, Mass.	1651-60
Thomas Lee House	East Lynne, Conn	1660
Older Cowles House	Farmington, Conn	1661

Narbonne House	Salem, Mass	1661-71
Philo Bishop House	Guilford, Conn	c1665
Starr House	Guildford, Conn	c1665
Governor Bradstreet House	North Andover, Mass	1667
Hyland-Wildman House	Guildford, Conn	1668
Whipple House	Ipswich, Mass	1669
Acadian House	Guilford, Conn	c1670
Henry Bridgeham House	Boston, Mass	1670
Walker House	Stratford, Conn	1670
West Part of Pickering House	Salem, Mass	1671
Deliverance Parkman House	Salem, Mass	1673-82
Jonathan Corwin House	Salem, Mass	1675
Hollister House	South Glastonbury, Conn	c1675
Sergeant (Province) House	Boston, Mass	1676-9
Paul Revere House	Boston, Mass	1676
Burnham House	Ipswich, Mass	1670
Old Ship Tavern	Essex, Conn	1675
Tufts (Cradock) House	Medford, Mass	1677-80
Daniel Epes House	Salem, Mass	1679
Turner House	Salem, Mass	c1680
Old Feather Store	Boston, Mass	1680
Foster (Hutchinson) House	Boston, Mass	1681-91
Hooper-Hathaway House	Salem, Mass	1682-93
Capen House	Topsfield, Mass	1683
Philip English House	Salem, Mass	1683-92
John Ward House	Salem, Mass	1684
Abbott Farm House	Andover, Mass	1685
South Wing of Turner House	Salem, Mass	1692
Benaiah Titcomb House	Newburyport, Mass	1695
Usher (Royall) House	Medford, Mass	1697
Hunt House	Salem, Mass	1698
Goldsmith House	Guilford, Conn	c1700

NEW YORK AND NEW JERSEY HOUSES—17TH CENTURY

Bergen House	Flatlands, Brooklyn, N.Y	1655
Shenks-Crook House	Bergen Beach, Flatlands, N.Y	1656
Lake Tyson House	Nieuw Dorp, Staten Island, N.Y	c1656
Terhuen House	Hackensack, N.J	1670
Lefferts House	Brooklyn, N.Y	1675
Senate House	Kingston-on-Hudson, N.Y	1676
Philipse Manor House	New York, N.Y	1682
Old Houses	Hurley, N.Y	c1690

PENNSYLVANIA AND DELAWARE HOUSES—17TH CENTURY

Old Dutch House	New Castle, Del	1665
William Penn (Letitia) House	Philadelphia, Pa	1682
Tile House	New Castle, Del	1687
"Wynnestay"	Philadelphia, Pa	1689
"Wyck"	Germantown, Pa	1690
Meeting House	Merion, Pa	1695
State House	Philadelphia, Pa	c1697

VIRGINIA AND SOUTHERN HOUSES—17TH CENTURY

Adam Thoroughgood House	Princess-Anne County, Va	c1640
Warren House	Smith's Fort, Va	1651-2
"Bacon's Castle"	Surry County, Va	1660
Original Wye House	Talbot Co., Maryland	1662
"Country House and Ludwell House"	Jamestown, Va. (ruins)	1662-66
"Fairfield", Carter's Creek	Cloucester Co., Va	1692

NEW ENGLAND HOUSES—18TH CENTURY

Doak House	Marblehead, Mass	1705
Marston House	Salem, Mass	1707
Miller House	Byfield, Mass	1710
Porter House	South Hadley, Mass	1713
Thompson or Count Rumford House	Woburn, Mass	1714
Shortt House	Newbury, Mass	1717
Townsend-Sweetser House	Lynnfield, Mass	1720-30
McPhedris-Warner House	Portsmouth, N.H	c1728
Richards House	Litchfield, Conn	1730
Royall House	Medford, Mass	1733
Holabird House	Falls Village, Conn	1735
Challoner House	Newport, R.I	1735
Hancock House	Boston, Mass	1737-40
Ayrault House	Providence, R.I	1739
William Brown House	Beverley, Mass	1744-50
Pinckney House	Boston, Mass	1745-6
"Shirley Place"	Roxbury, Mass	1746
Pickman House	Salem, Mass	1750
Treadwell House	Portsmouth, N.H	1750
Webb House	Wetherfield, Conn	1753
Colton House	Longmeadow, Mass	1753-5
John Williams House	Deerfield, Mass	1756
Ebenezer Grant House	East Windsor, Conn	1757-8
Vassal-Craigie House	Cambridge, Mass	1759
Timothy Orne House	Salem, Mass	1761
Apthorp House	Cambridge, Mass	1761-4
Moffat-Todd House	Portsmouth, N.H	1763
Pickman House	Salem, Mass	1764
Jeremiah Lee House	Marblehead, Mass	1768
Copley House	Boston, Mass	1771
"Lord" Timothy Dexter House	Newburyport, Mass	1772
Peirce-Nichols House	Salem, Mass	1780
Derby House	Salem, Mass	1780
Cutler-Bartlett House	Newburyport, Mass	1782
Boardman House	Salem, Mass	1782-9
Governor Langdon House	Portsmouth, N.H	1784
Capt. George Benson House	Providence, R.I	1786
Brown House	Providence, R.I	1789
Arnold House	Weymouth, Mass	1790
Barrell House	Charlestown, Mass	1792

Nathan Reed House	Salem, Mass	1793
Knox House	Thomaston, Maine	1793
Lyman House	Waltham, Mass	c1793
Swift House	Andover, Mass	1795
Joseph Hosmer House	Salem, Mass	1795
Harrison Gray Otis House	Boston, Mass	1795
Elias Derby House	Salem, Mass	1795-8
Morton (Taylor) House	Roxbury, Mass	1796
Bowdoin House	South Hadley, Mass	1798
Ezekiel Derby House	Salem, Mass	1799

New York and New Jersey Houses—18th Century

Miller House	Long Island, N.Y	1700
The "Willows"	Gloucester, N.J	1720
Ackerman-Brinckerhoff House	Hackensack, N.J	1740
Van Cortland House	Lower Yonkers, N.Y	1748
Schuyler House	Albany, N.Y	1761-2
Van Rensellaer Manor House	Albany, N.Y	1765-8
Morris-Jumel House	New York, N.Y	1765
Lincoln House	Hackensack, N.J	1773
Dyckman House	New York, N.Y	1783
Board-Zabriskie House	Hackensack, N.J	1790
Jan Ditmars House	Hackensack, N.J	1798

Pennsylvania and Delaware Houses—18th Century

Graeme Park	Horsham, Pa	1721-2
"Stenton"	Germantown, Pa	1728
Amstel House	New Castle, Del	1730
John Bartram House	Philadelphia, Pa	1730-1
Pastorius House	Germantown, Pa	1748
Whitby Hall	Philadelphia, Pa	1754
"Woodford"	Philadelphia, Pa	1756
Manor House	Nazareth, Pa	1759
"Mt. Pleasant"	Philadelphia, Pa	1761
"Mill Grove"	Lower Providence, Pa	1762
"Cliveden"	Germantown, Pa	1763
Johnston House	Germantown, Pa	1768
Corbit House	Odessa, Del	1772-74
"Lansdowne"	Philadelphia, Pa	1773-7
"Solitude"	Philadelphia, Pa	1784
Nazareth Hall	Nazareth, Pa	1785
Reynolds-Morris House	Philadelphia, Pa	1786-7
Kensey-Johns House	New Castle, Del	1787-90
Bingham House	Philadelphia, Pa	1788
George Reed House	New Castle, Del	1791
Robert Morris House	Philadelphia, Pa	1793
"Upsala"	Germantown, Pa	1798

SOUTHERN COLONIES' HOUSES—18TH CENTURY

Governor's Palace	Williamsburg, Va	1705-6
The "Mulberry"	Goose Creek, S.C	1708-25
"Stratford"	Westmoreland Co., Va	1725-30
"Westover"	Charles City Co., Va	1726
"Tuckahoe"	Goochland Co., Va	1730
"Rosewell"	Gloucester Co., Va	1730
"Ampthill"	Chesterfield Co., Va	1732
Robert Brewton House	Charleston, S.C	c1733
Brice House	Annapolis, Maryland	1740
Eveleigh House	Charleston, S.C	1743-53
Snowden-Long House	Laurel, Md	c1745
Lord Fairfax House	Alexandria, Va	c1750
"Carter's Grove"	James City Co., Va	1751
Izard House	Charleston, S.C	1757
Drayton Hall	Ashley River, S.C	c1758
Gunston Hall	Fairfax Co., Va	1758
"Mt. Airy"	Richmond Co., Va	1758
"Elsing Green"	King William Co., Va	1758
Brewton House	Charleston, S.C	1759
Ridout House	Annapolis, Md	1760
"Whitehall"	Anne-Arundel Co., Maryland	1763
Miles Brewton House	Charleston, S.C	1765-9
Tryon's Palace	New Bern, N.C	1767-70
Moravian Brothers' House	Salem, N.C	1768
Chase House	Annapolis, Md	1769-71
Hammond House	Annapolis, Md	1770
Maginault House	Charleston, S.C	c1770
"Monticello"	Albermarle Co., Va	1771
John Stuart House	Charleston, S.C	1772
"Beverley"	Pocomoke River, Md	1774
Christ Church	Lancaster, Va	1782
New Wye House	Talbot, Co., Md	1782
John Marshall House	Richmond, Va	1789
"Acton"	Anne-Arundel Co., Va	1790
"Montpelier"	Orange Co., Va	1793
Middleton (Pinckney) House	Charleston, S.C	1796
"The Octagon"	Washington, D.C	1798-1800
Colross House	Alexandria, Va	1799
Blacklock House	Charleston, S.C	c1799
Smallwood-Jones House	New Bern, N.C	c1800

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THE WATER TURBINE TESTING FLUME OF THE UNIVERSITY OF TORONTO

By Robert W. Angus[1]

The hydraulic laboratory of the University of Toronto has been, for many years, well equipped with apparatus for proper instruction of students in hydraulics. The close proximity to Niagara Falls and many other sources of water power, coupled with the fact that fuels are not available in Canada within short distances, has given the Province of Ontario a very vital interest in water power development and other kindred problems. The University has endeavoured to meet the demand created by this condition by having water turbines of various types, turbine and other pumps, orifices, weirs, meters, etc., available for student instruction.

While this equipment is available for research work also, the limitations imposed upon it by instruction prevent any very extensive research being done, and therefore a good deal of the latter work can only be carried out on special equipment. Some few years ago the turbine testing flume described in this article was installed for research work, and has been extensively used since for investigations on turbines and for other problems requiring a large flow of water at low head.

It should be stated at the outset that this flume was erected in an improvised place in a building that had been built some years before. There were, therefore, very decided limitations with regard to space and disposition of equipment, which prevented the design being carried out in as effective a way as was desired.

For turbine study it has usually been found advisable to use a vertical setting, for various reasons, and the design had this in view, although the flume may also be used for horizontal turbines. The details of the flume are shown on Figs. 1, 2 and 3, which show a plan in Fig. 1, longitudinal section in Fig. 2 and front elevation in Fig. 3. The flume consists of a concrete tank 8 ft. square at the top and 7 ft. 8 in. square at the bottom, the inside depth being 9 ft. 3 in. There is a circular opening 3 ft. dia. in the front of this tank with the centre line of the opening 2 ft. above the bottom of the tank, this circular opening being provided for setting up horizontal turbines or for other studies where the apparatus needs to be horizontal. This opening was used in the case of the calibration of the reducer and the determination of the losses in a 14 in. tee, as reported in a paper by the author in an earlier Bulletin.

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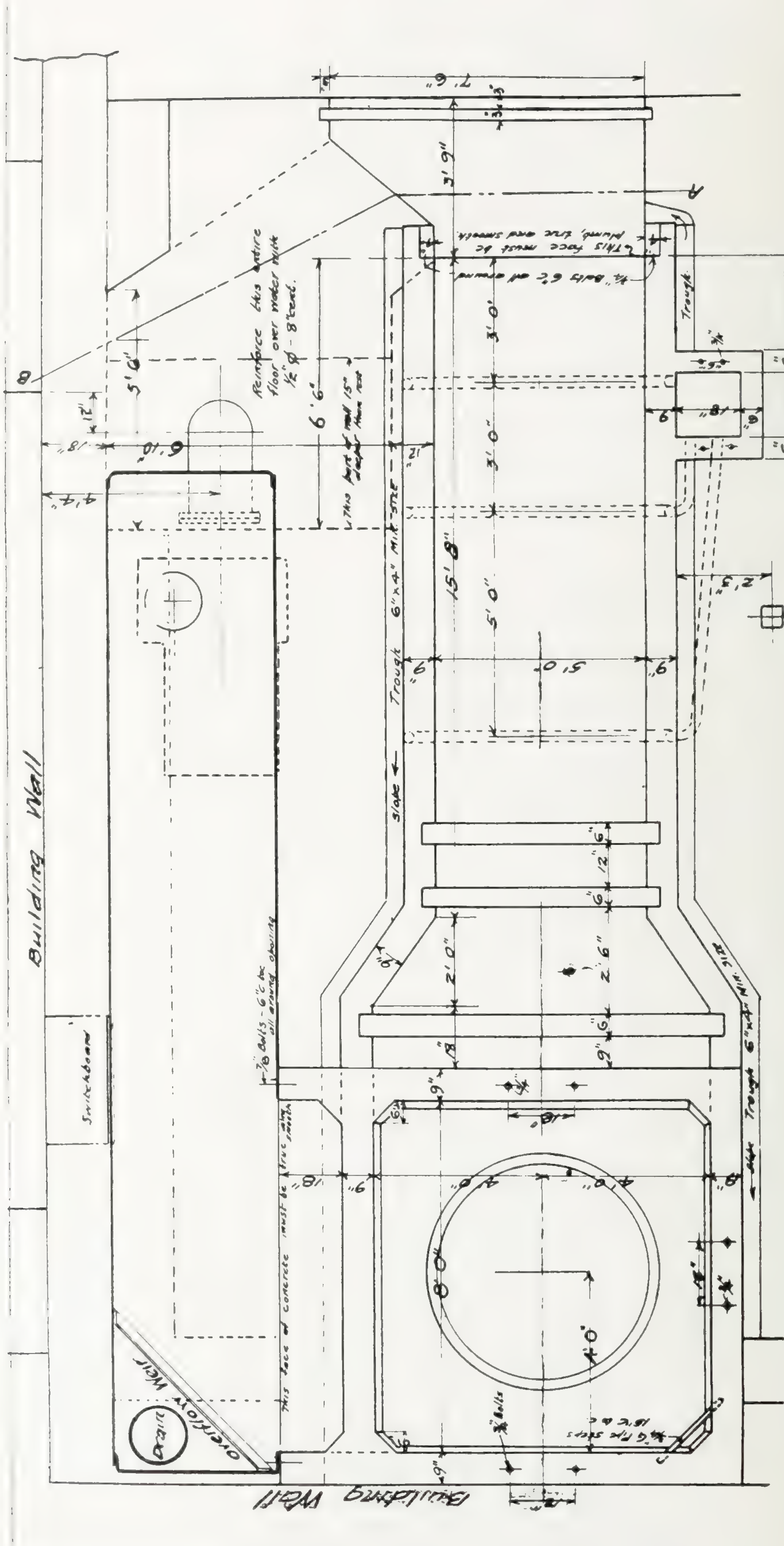

Fig. 1. Plan of Testing Flume.

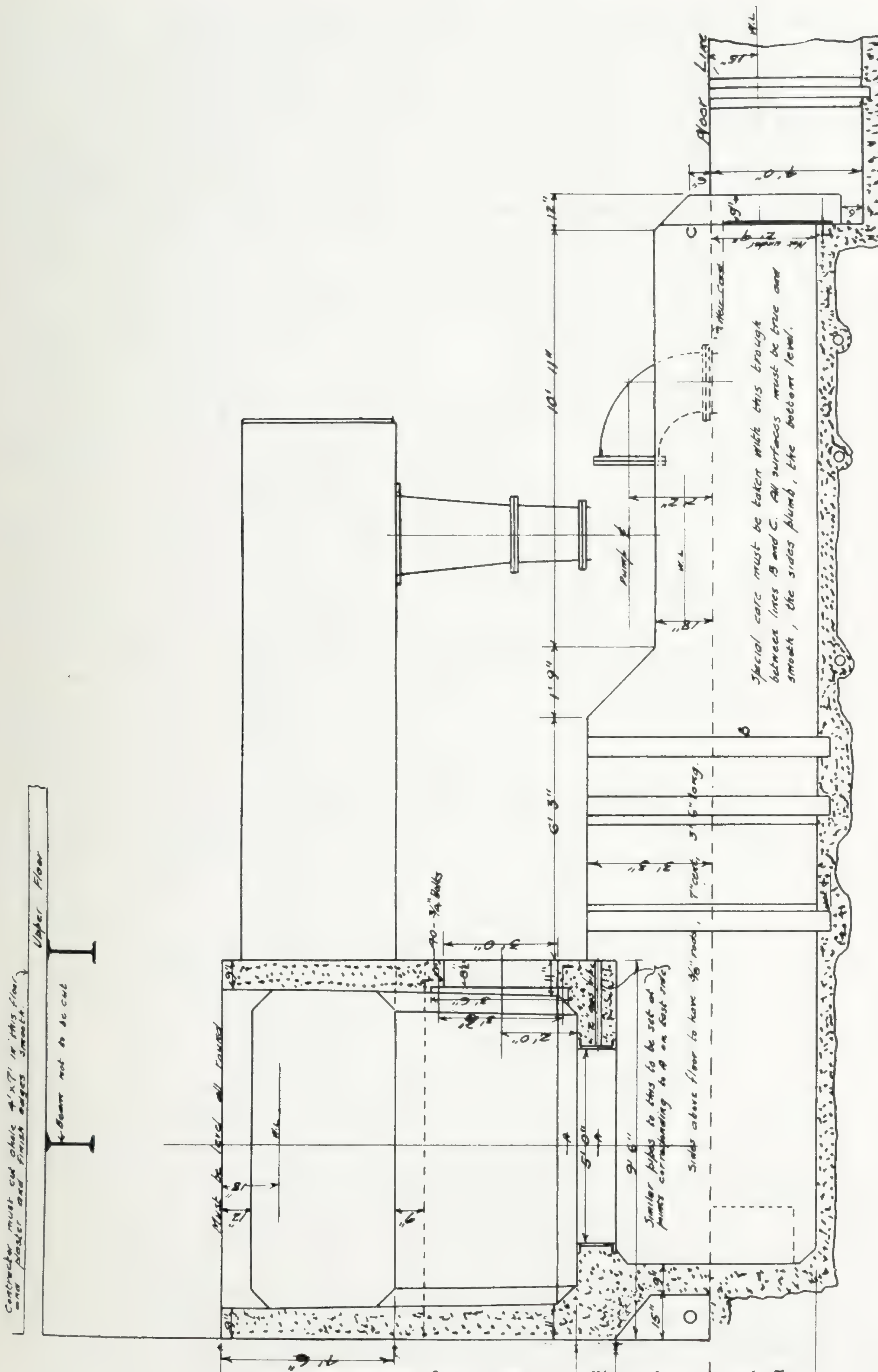

Fig 2. Longitudinal Section of Testing Flume.

The main opening is in the bottom of the tank where a circular cast iron ring, 5 ft. inside diameter, faced on top and bottom, is accurately set level and provides a base for any vertical turbine or similar apparatus. This ring has been designed so that a turbine may be set on the top side of it, or fastened to the lower side, and by the use of additional annular rings the turbine may be set at any height desired, but on the discharge side the lowest tail water level is 3 ft. 9 in. below the inside of the bottom of the tank.

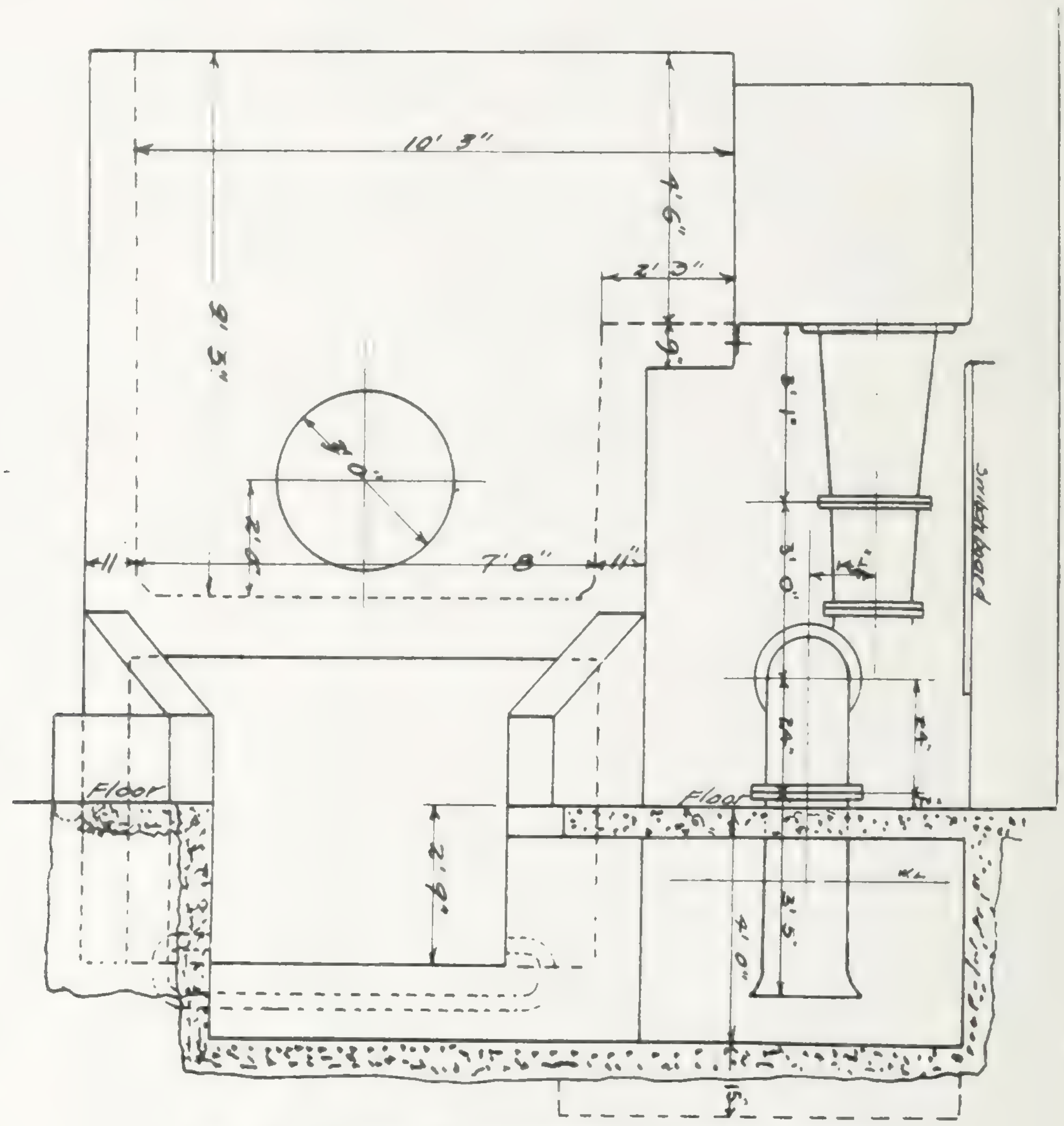

Fig. 3. Front Elevation of Testing Flume.

Below the bottom of the turbine tank just described, is a concrete channel 8 ft. wide in the part near the tank, see Fig. 1, and with a level floor 6 ft. 3 in. below the inside of the bottom of the latter, see Fig. 2. Looking at the plan of this trough, as shown on Fig. 1, it is seen that the concrete has recesses at three points, 9 in., 3 ft. 9 in. and 5 ft. 3 in. away from the front of the tank, these recesses being arranged for setting in baffles or stop logs, as desired. In the ordinary method of testing, a dam is inserted in the grooves nearest the tank, for the purpose of maintaining the tail water at a desired level, and the design permits it to be brought up against the bottom of the tank. The other recesses

hold the baffles for smoothing out the flow and destroying the eddies as the water enters the part of the channel that is 5 ft. wide.

The bottom of the part that is 5 ft. wide is level with that of the other part, and this section is 15 ft. 8 in. long, with a carefully made suppressed weir at the end of it, the head on this weir being taken on a hook gauge set in a stilling box 18 in. square in the position shown on Fig. 1. After passing over the weir the water falls into an open channel, whence it runs back into the pump well. The levels of the head and tail water are taken by floats.

Water for the experiments is supplied by a spiral pump to be described later. This pump draws water from the well mentioned above, and discharges it into a steel tank 4 ft. wide, 4 ft. deep and 23 ft. long, this tank being connected to the top of the concrete tank, as shown on the drawings. This tank has been constructed on quite liberal lines to enable the disturbances from the pump discharge to be damped out, and for this purpose it also contains a series of baffles so arranged that the water is quite calm when it reaches the concrete flume. Since the opening from the steel to the concrete tank is 8 ft. wide and 4 ft. deep, the mean velocity at this point is always under 6 in. per sec.

Running diagonally across the corner of the steel tank is an overflow weir with a 4 ft. 6 in. width of crest, the latter being of variable height, and in this way the level of the head water may be maintained constant during the different tests. The damping out of cross currents in the testing tank has given much trouble, and naturally it would have been much more easily done had it not been for the restrictions imposed by the existing building.

TURBINE TESTING EQUIPMENT

Up to the present time the turbine testing has been entirely done on vertical units, and one such turbine is shown in place in the tank on Fig. 4. This shows the general set-up of the unit with its dynamometer and a partial front view of the equipment is shown on the right, which indicates the position of the scales, etc. The vertical wheels have been employed principally because the mechanical friction in them may be reduced to an extremely small amount by the elimination of shaft packing boxes, but this setting also prevents the entrance of any air to the draft tube, a difficulty often met with in a horizontal unit where the shaft projects through the tube. This drawing shows the turbine in the lowest setting possible without the use of additional rings.

The turbine shaft terminates in a half coupling, which is connected to the half coupling on the permanent upper mechanism of the testing plant. The permanent upper bearings and brake are mounted on a

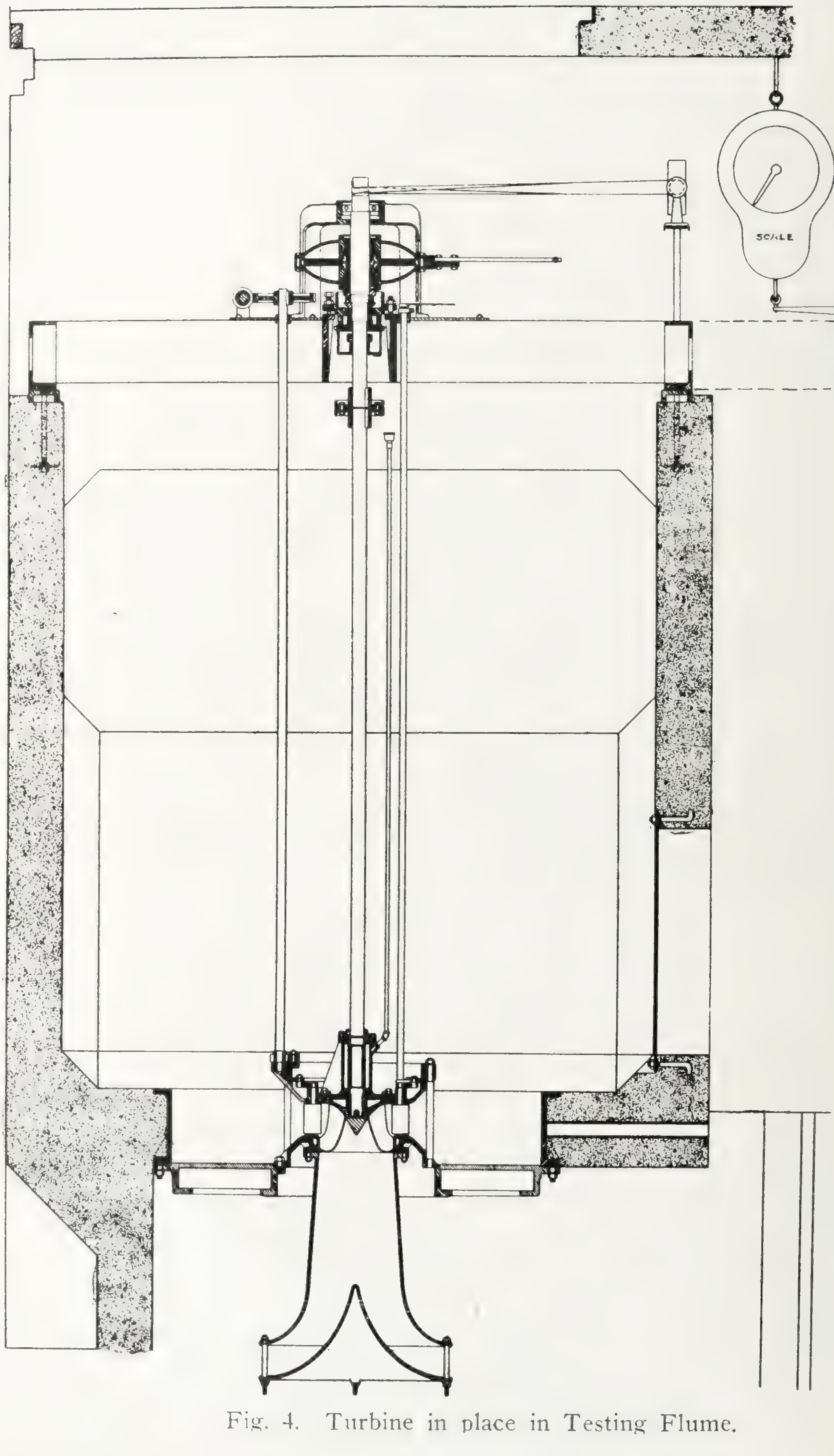

Fig. 4. Turbine in place in Testing Flume.

bridge across the tank, this bridge being composed of two 10 in. channels bolted to a casting at each end, these end castings being set true with one another and each having a planed lower surface. Bolted to the concrete wall of the tank at each side is a cast iron sole plate, and each plate is faced on top so that both are true, these forming permanent base plates for the bridge already described. Dowel pins give this bridge a fixed setting on the sole plates.

In the centre of the bridge, directly in the vertical axis of the tank, another casting is bolted between the two channels, this casting holding the shaft bearing, and the steel plates shown are also bolted to the tops of the channels on each side of the central casting. When turbines are being set in for testing, the entire bridge and all of the machinery above the half coupling is just lifted away from the planed sole plates, but the use of the dowel pins in each end enables it to be put back in place again with great exactness and without loss of time. The central casting carries the ball thrust and steady bearings for the upper part of the shaft and dynamometer, Fig. 4, these bearings being carried in a second casting mounted in the stationary central casting, and the second casting is adjustable for height by set screws, so that the coupling may be connected to the shaft of the turbine, and the exact height of the entire shaft may be adjusted to give free running to the turbine.

A detail of the upper bearings, etc., is illustrated on the drawing, Fig. 5, which shows the three ball bearings, the lower one carrying the entire weight of all the moving parts, including the runner, the central one being a steady bearing and the upper one carrying the dynamometer of itself. It was also found necessary to put a steady bearing above the dynamometer as indicated. The castings are all designed to prevent oil getting down into the water. Near the top of the shaft is an Alden dynamometer, shown in section in Fig. 4, and consisting of a central cast iron disk keyed to the shaft and revolving in oil between two copper diaphragms attached to the dynamometer casing and which are forced toward the iron disk by water pressure. Regulation of the water pressure varies the load applied by the device. The dynamometer requires a good deal of skill for successful operation because it takes some time for the oil to reach its final temperature, and, further, variation in city water pressure is enough to cause much annoyance unless a separate supply at constant pressure has been provided. With proper care and experience, however, the dynamometer is an excellent one, and loads may be maintained constant throughout long periods, and the magnitude of the load is easily ascertained from the scales.

The load taken by the dynamometer is transferred from the dynamometer arm through a right angled bell crank lever to the scales as shown on the right hand drawing in Fig. 4. This bell crank is mounted on ball bearings and the pressure from the arm is transmitted to it

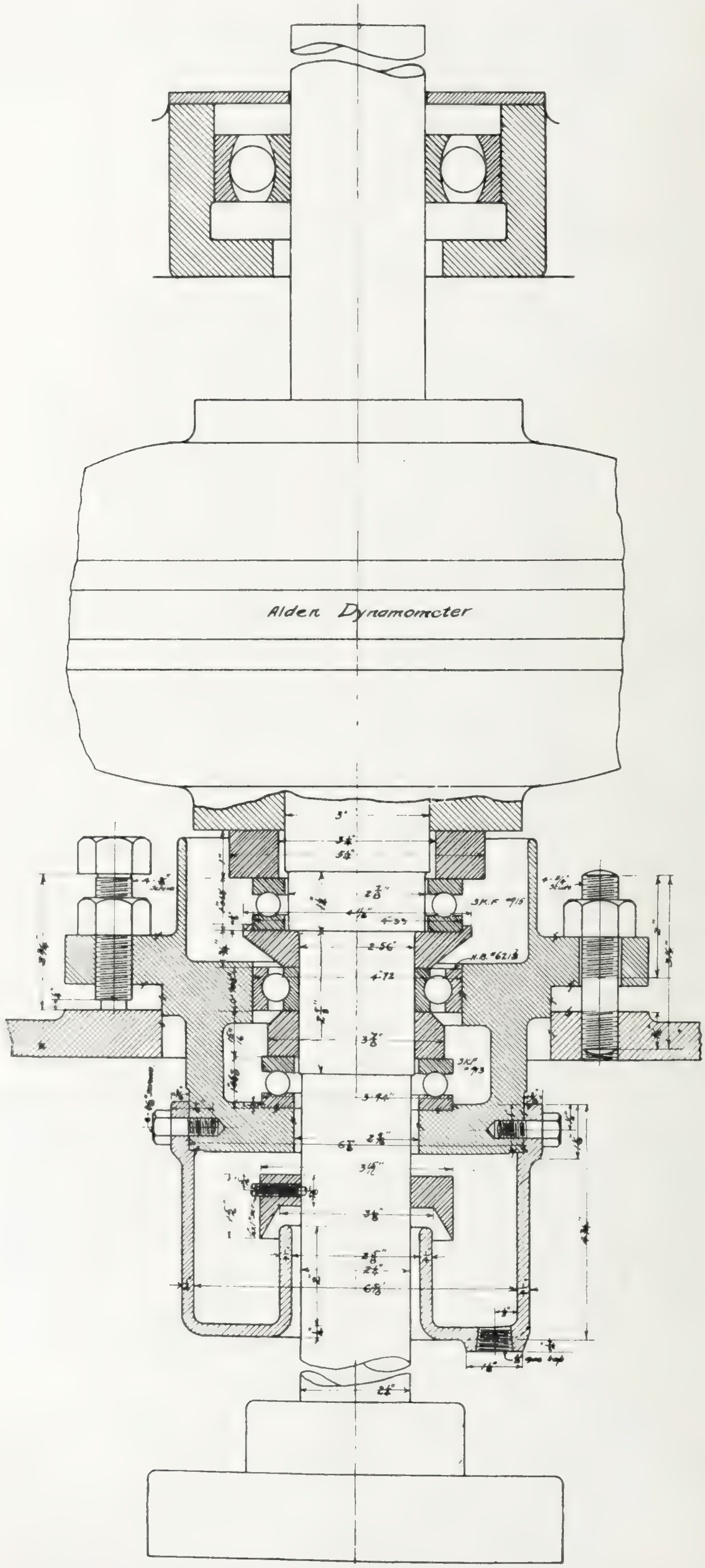

Fig. 5. Top Bearing and Dynamometer on Testing Flume.

through a strut with pointed ends so that the lever arms have perfectly constant and definite length. In addition, the force is transmitted to the scales through a pivot point which avoids friction or uncertainty in the leverage during transmission of the brake effort to the scales.

The load is weighed on Toledo springless scales which are carefully adjusted. The relation between the torque produced on the brake and the scale reading has been determined by very careful experiments, and a curve has been constructed connecting the two. This calibration was done by attaching a fine wire to the brake arm and running it over a bicycle wheel, mounted on ball bearings, to a scale pan on which standardized weights were placed; the torque applied to the brake arm was, therefore, known with accuracy and the corresponding reading on the scales was taken at the same time. A curve was then plotted showing the relation between the torque and the scale reading, and points on this curve were repeatedly checked before the curve was finally established.

In determining the speed, two methods are used. A tachometer is driven by belt from the top of the shaft, as indicated, and this is frequently checked up by direct counting, but this instrument is only used for setting the turbine close to the desired speed and holding it there. The actual speeds are taken by revolution counter and split seconds stop watch, the average over a two minute interval being used.

As already mentioned, water for the tests is supplied by a pump direct coupled to a Westinghouse variable speed, direct current motor for 220 volts, 20 h.p. to 40 h.p. at speeds between 1050 r.p.m. and 1500 r.p.m. The pump is of the high speed spiral type with 16 in. suction and $14\frac{1}{2}$ in. discharge branches, the latter, however, has an enlarger increasing the size gradually to 16 in., and a tapering pipe from this to the large tank has a diameter of 22 in., where it enters the latter. There is only one valve on all the pump piping and it is a flat check valve where the water discharges into the tank, but this valve is pulled out of the way of the stream after the pump starts. The pump has a capacity exceeding 12 c.f.s. and in order to allow for high speed of rotation at the low head at which it works the flow through the impeller is conical instead of radial, as is shown on the section shown on Fig. 6.

A larger section through the impeller and shaft is shown on Fig. 7, from which it is seen that the shaft is mounted on two ball bearings, while a third ball bearing provides for thrust in either direction. The pump has very little mechanical friction and hydraulic losses are avoided by providing fixed guides in the suction side of the impeller and also by discharging the water into a snail shell type of casing where the velocity head is partly regained.

The maximum head available in the plant is somewhat under 11 ft. under full load tests.

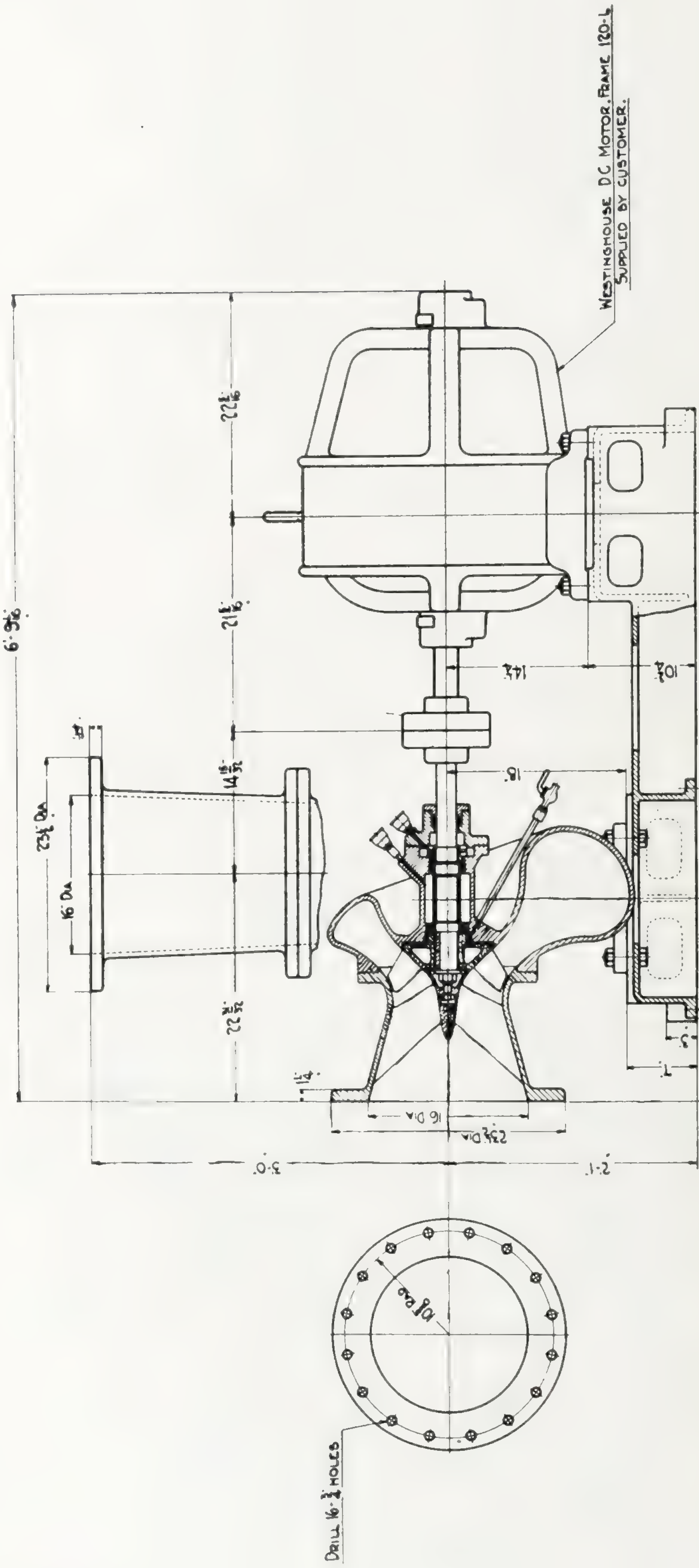

Fig. 6. High Speed Pump for Supplying water for tests.

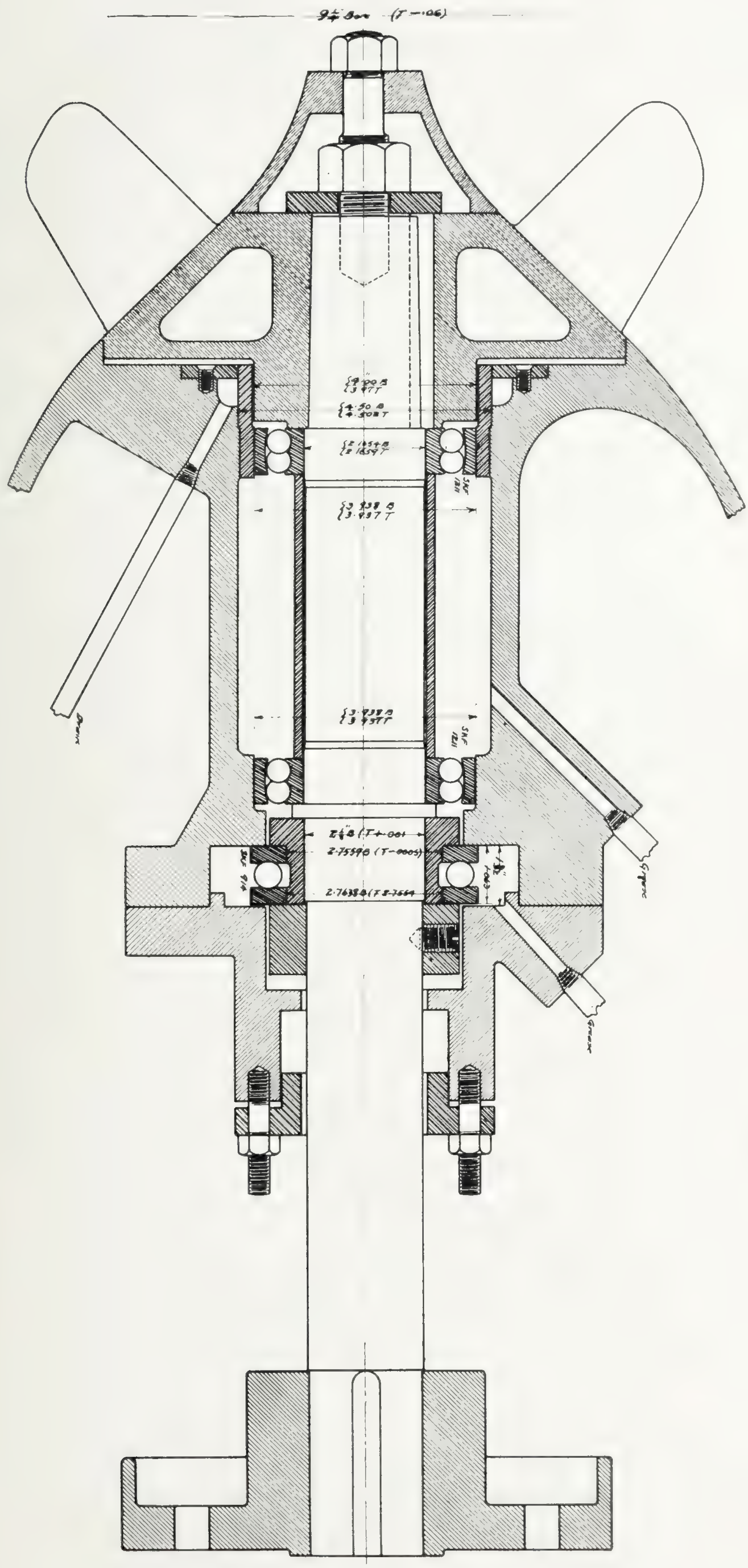

Fig. 7. Section through shaft and impeller of High Speed Pump.

TESTS ON A KAPLAN HYDRAULIC TURBINE

By Robert W. Angus,[1] R. Taylor[2] and C. G. Heard[3]

Part I

DESCRIPTION OF THE KAPLAN TURBINE TESTED

By Robert W. Angus

A vertical section of the turbine referred to in this paper is shown on Fig. 1 and a sectional plan on Fig. 2. The turbine has ten distributor vanes operated by a ring which is free to rotate through a small angle, and a link is connected from each vane to the ring, as shown on Figs. 1 and 2. These links offer some slight resistance to free flow of the water through the distributor, although the interference has been very much reduced by bevelling off the edges of the links and making them as small as possible.

The runner has four blades mounted with bearings on the central hub, and each blade has a crank pin on the inner end to allow its angular position to be adjusted. Since this is an experimental turbine the blades are not automatically controlled, but are set by hand to each running position, for which purpose the turbine has to be stopped and the central screw turned till the blades reach the desired position. In order to avoid friction, packing has not been used on the shaft, but excessive leakage has been avoided by the close running fits shown on the figures, and detailed discussion of this feature is unnecessary. The drawing shows that there is no thrust bearing on the turbine itself, the entire weight being supported by the bearing at the top of the shaft, as explained in the author's paper in this Bulletin, describing the testing flume. There is, however, a ball guide bearing on the turbine and the arrangement to keep water away from it is shown.

Friction of the shaft in the water has been entirely eliminated by surrounding the shaft by a water tight casing which extends above the head water surface, and the lower end of which is shown on the drawing, Fig. 1. The turbine shaft was very carefully trued up and a half coupling on the upper end of it was turned and faced true after putting it on the shaft. The turbine was then set and carefully lined with the upper part of the shaft carrying the dynamometer, the entire shaft and dynamometer running so true that the friction in the moving parts is too small to be accurately measured.

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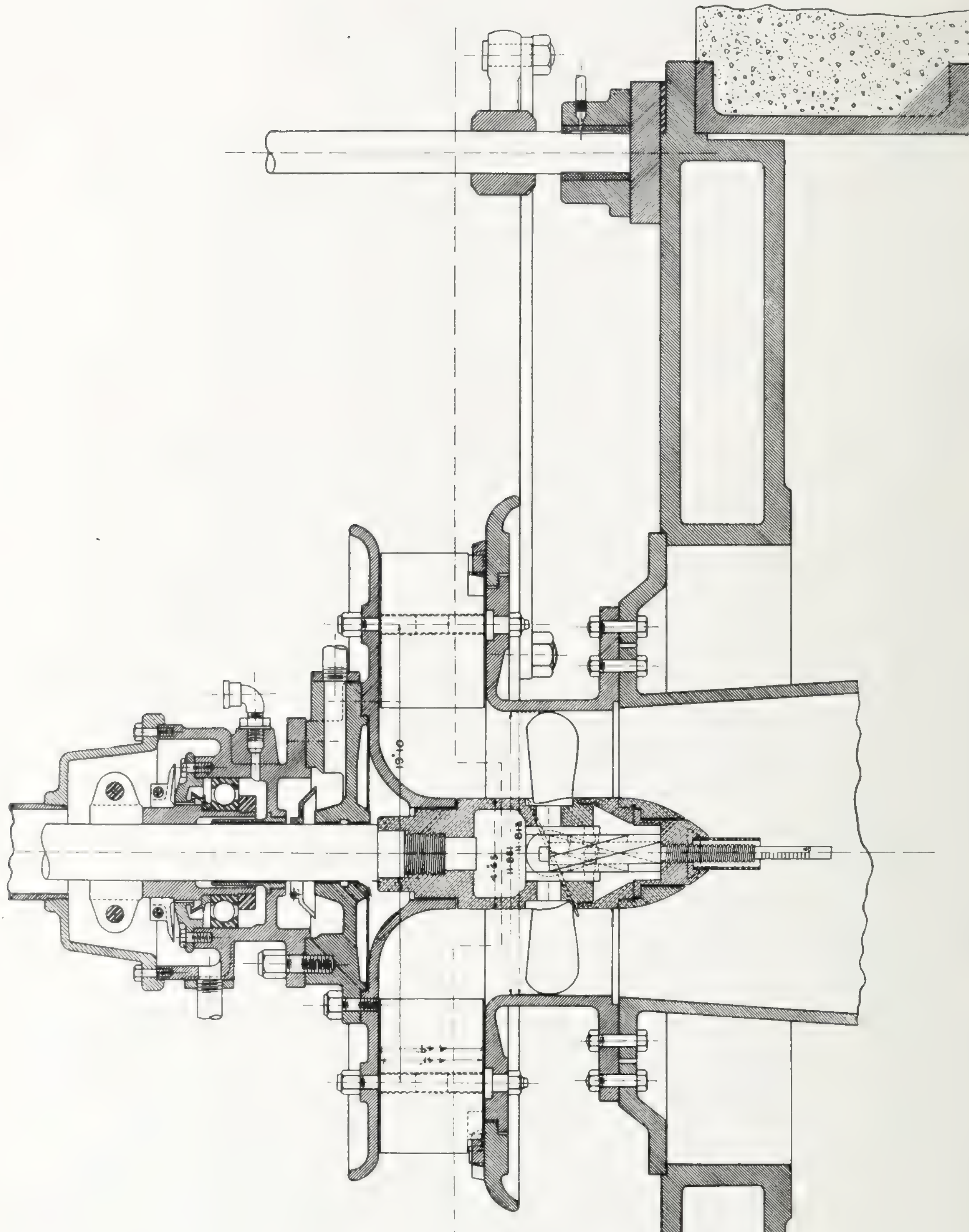


Fig. 1. Vertical Section of Kaplan Turbine Tested.

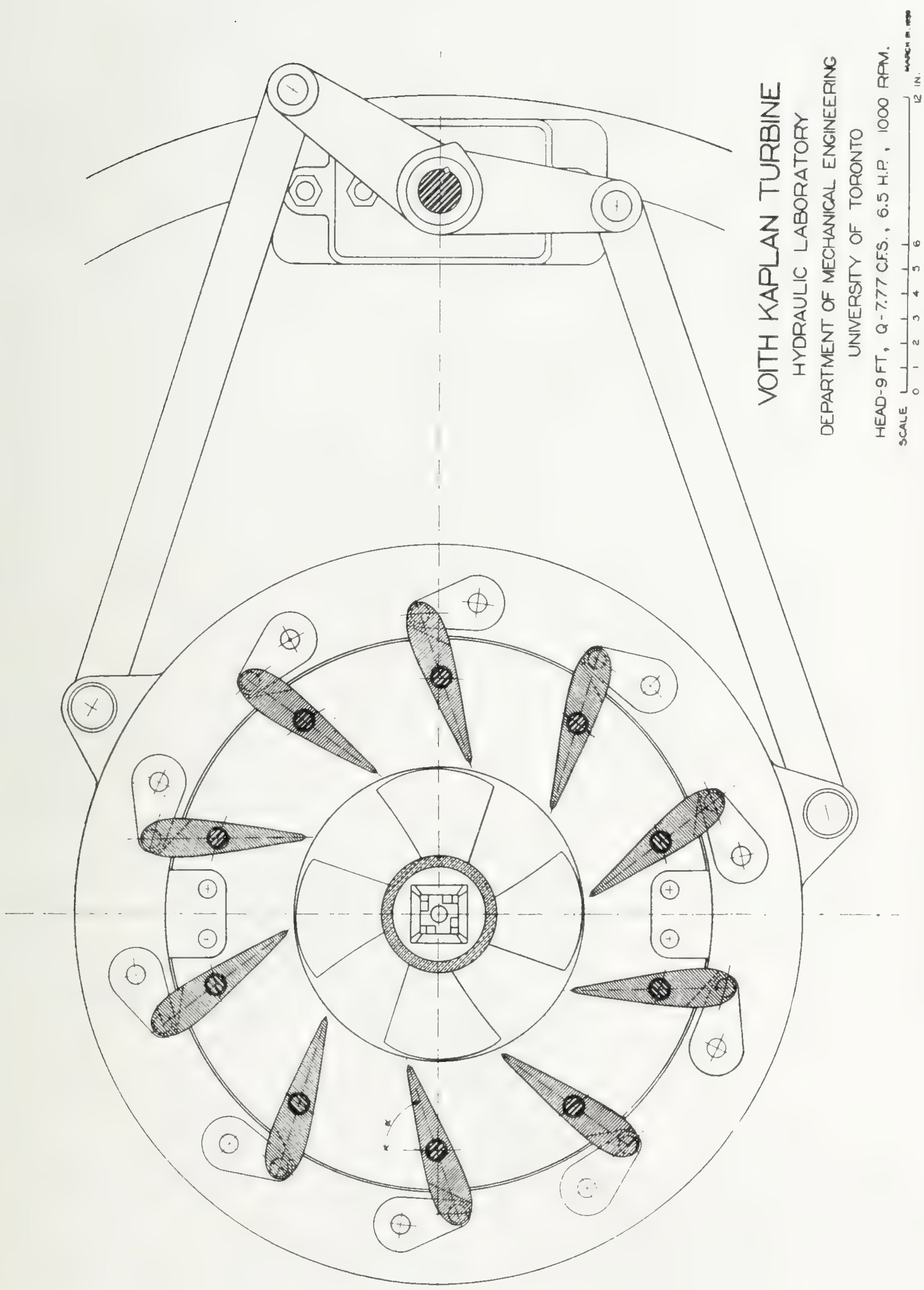


Fig. 2. Horizontal Section of Kaplan Turbine Tested.

A few of the leading dimensions of the turbine are marked on the drawing, as follows: Outside diameter of runner 11.812 in., runner hub diameter 4.65 in., throat diameter 11.851 in., shaft diameter 2.36 in., height of distributor vanes 4.41 in., height of distributor passages 4.49 in., diameter of top of draft tube 12 in., axial length of outer part of draft tube 31.1 in., extreme diameter of tube at exit 31.5 in.

In order that the details of the turbine may be fully available, drawings of the hydraulic elements are presented. Four sections of the runner blades are shown on Fig. 3, these sections having been determined with great accuracy. Before beginning the measurements the blades were set in the intermediate position designated as No. 12, in which position it was found that the entire tip of the blade exactly touched a cylinder 11-13/16 in. (11.812 in.) diameter. The runner was then set up in a lathe by putting a mandrel through it and holding this in the lathe centres, and while in this position four circles A, B, C, D were laid off on the blades, these circles having diameters 11.812 in. (outside diameter of runner), 10 in., 8.188 in. (8-3/16 in.) and 6.375 in. (6-3/8 in.), respectively. By the use of the headstock gears the runner was then gradually revolved and for each small angle of revolution a point on the contour was determined by the aid of a pointer attached to the tool holder.

By the means described, cylindrical sections of two of the blades were made on each of the four cylinders A, B, C and D, and the sections of the blades were plotted. While the drawing is not to a large enough scale to show the several points, it was found that the two vanes measured were almost identical and an average curve was drawn for them at each cylinder; these are shown on Fig. 3, the axis of rotation of the blade being marked by 0 in each section. Four tangents were then drawn to each section of the blade, two at the entry edge and two at the exit edge, and from these the blade angles β'_1 and β''_1 at entry and β'_2 and β''_2 at exit were measured for the position 12.

TABLE I

BLADE ANGLES MEASURED AS SHOWN ON FIG. 3, FOR SETTING NO. 12

Angle of tangent Fig. 3	Entry Angles		Exit Angles	
	β'_1	β''_1	β'_2	β''_2
Section A	8° 13′	18° 53′	11° 33′	6° 38′
Section B	16° 38′	29° 53′	13° 48′	7° 48′
Section C	19° 43′	41° 43′	16° 58′	8° 28′
Section D	24° 33′	47° 23′	19° 13′	6° 33′

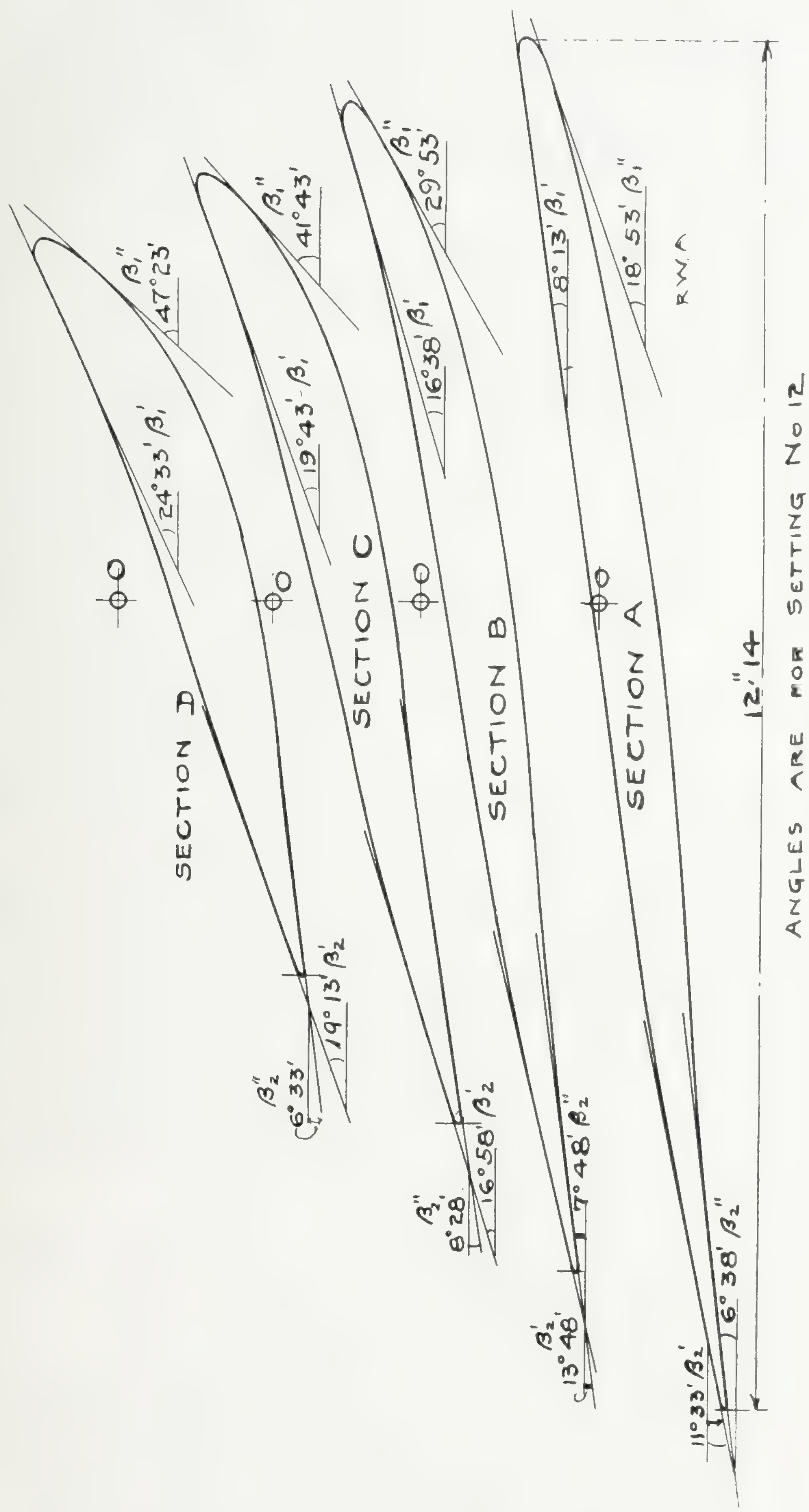


Fig. 3. Blade Section for Runner. Kaplan Turbine.

It was next found that 13 turns of the central screw were required to move the blades from one extreme position to the other and, hence, it was decided to select 14 settings, one for each revolution of the screw (including the extreme positions) and these have been designated as settings 0, 2, 4 . . . 26, respectively. Starting with the position 26 (wide open) the angle through which the blade turned to reach each setting is shown in Table II; thus the blade turns 10° 37′ in going from setting No. 26 to setting No. 12.

TABLE II

ANGULAR MOVEMENT OF THE RUNNER BLADES FOR EACH SETTING, STARTING IN EACH CASE FROM SETTING No. 26, WHICH IS THE MAXIMUM OPEN POSITION

Setting No.	26	24	22	20	18	16	14
Angular movement of blade from setting No. 26.......	0	1° 29′	3° 7′	4° 43′	6° 3′	7° 28′	8° 59′
Setting No.	12	10	8	6	4	2	0
Angular movement of blade from setting No. 26.......	10° 37′	12° 17′	13° 43′	15° 17′	16° 56′	18° 48′	20° 41′

A comparison of Tables I and II enables the blade angles for any setting to be computed; for example, in the full open position, setting No. 26, all of the angles are 10° 37′ greater than those in Table I, or for Section A they are 18° 50′, 29° 30′, 22° 10′ and 17° 15′, respectively, while for the most nearly closed position, setting No. 0, the angles are 20° 41′ − 10° 37′ = 18° 4′ less than in Table I, or for Section A they are −1° 51′, 8° 49′, 1° 29′ and −3° 26′, respectively.

In addition to those made on the runner, careful measurements were also made on the distributor vanes. There are ten vanes, each 4.41 in. high (although the opening is 4.49 in. high) and each mounted on a pin for turning, these pins being equally spaced around a circle 19.10 in. diameter. A horizontal cross section through a vane is shown on Fig. 4, this being made from the average of the measurements on the whole ten. To locate the vanes, Table III gives the angle α, Fig. 2, for the different distributor settings, and this was made up by setting the adjusting ring, Figs. 1 and 2, in the two extreme positions for gates full open and tight closed and measuring the angle through which the ring turned from one position to the other.

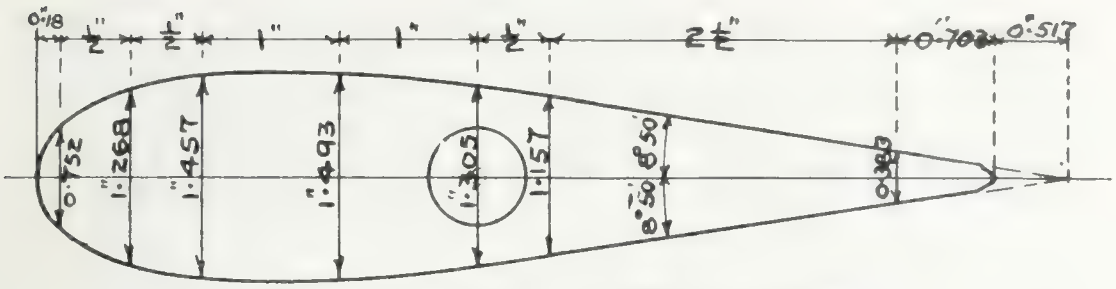


Fig. 4. Detail of Distributor Vane.

TABLE III

ANGLE BETWEEN THE CENTRE LINE OF THE DISTRIBUTOR GATE AND THE TANGENT TO THE CIRCLE DRAWN THROUGH THE CENTRE OF THE GATE PIN

Gate position	A	B	C	D	E	F
Angle α Fig. 2..............	(open) 70° 42′	63° 8′	55° 47′	48° 26′	41° 9′	34° 38′
Gate position	G	H	I	J	K	
Angle α Fig. 2..............	28° 25′	23° 20′	18° 23′	14° 34′	(shut) 11° 24′	

Dividing this angle into ten equal parts gives the eleven gate settings, *viz.*, A (full open), B, C, D, E, F, G, H, I, J and K (closed). The angles corresponding to these settings are marked in this table and the same designation is used in connection with the curves and tables presented herewith from the tests.

All measurements on the blades and angles were made by Mr. C. G. Heard.

PART II

RESULTS OF TESTS ON KAPLAN TURBINE

By R. W. ANGUS, R. TAYLOR and C. G. HEARD

The results of the tests, as measured by R. Taylor and C. G. Heard, are given on the accompanying tables and curve sheets, these tests being made in the testing flume in the hydraulic laboratory of the University of Toronto. These are reported in four series, one for each of the four runner blade settings that was used on the tests, and at each runner blade position a group of tests was made for each of six distributor vane settings. To give a definite illustration, the case of runner blade setting No. 26, which is the full open position, may be taken; then under this condition six distributor settings, designated as A (full open), B, C, D, E and F (partly closed), were used. For each of these distributor gate settings tests were run at various speeds, these being so selected that the corresponding speed at unit head ranged from 160 revs. per min. to 300 revs. per min., on the larger gate openings eight or nine speeds being used, while on the smaller openings there were six speeds over the range.

As might be expected, it was not possible to cover the same range of speeds with the small gates as with the large ones, because the turbine became unstable, and the load could not be properly measured, but the work was carried out to as great a range as possible. The curves have been drawn on different sheets so as to make the results more clear, and the first four sheets, Figs. 5 to 8, cover the complete series, each sheet containing three sets of curves showing efficiency, brake horse power under one foot head, and discharge under one foot head, the base line in all cases being the speed in revolutions per minute under one foot head. Each of these curves has been run through the several test points and is remarkably smooth.

From Figs. 5 to 8 on the first four sheets computations were made for two other sheets, Figs. 9 and 10, each containing two series of curves, all of which are plotted on a base of unit speeds in revolutions per minute. Each series shows the specific speed for one runner blade setting and six distributor vane settings. The seventh curve sheet, Fig. 11, also contains four series of curves deduced from the tests as plotted on the first four sheets; this sheet shows the efficiency, plotted on a specific speed base, for each of the four runner blade settings and for the six gate settings.

On the eighth curve sheet, Fig. 12, the curves were plotted from those already described, and the lower sets of curves have a base showing

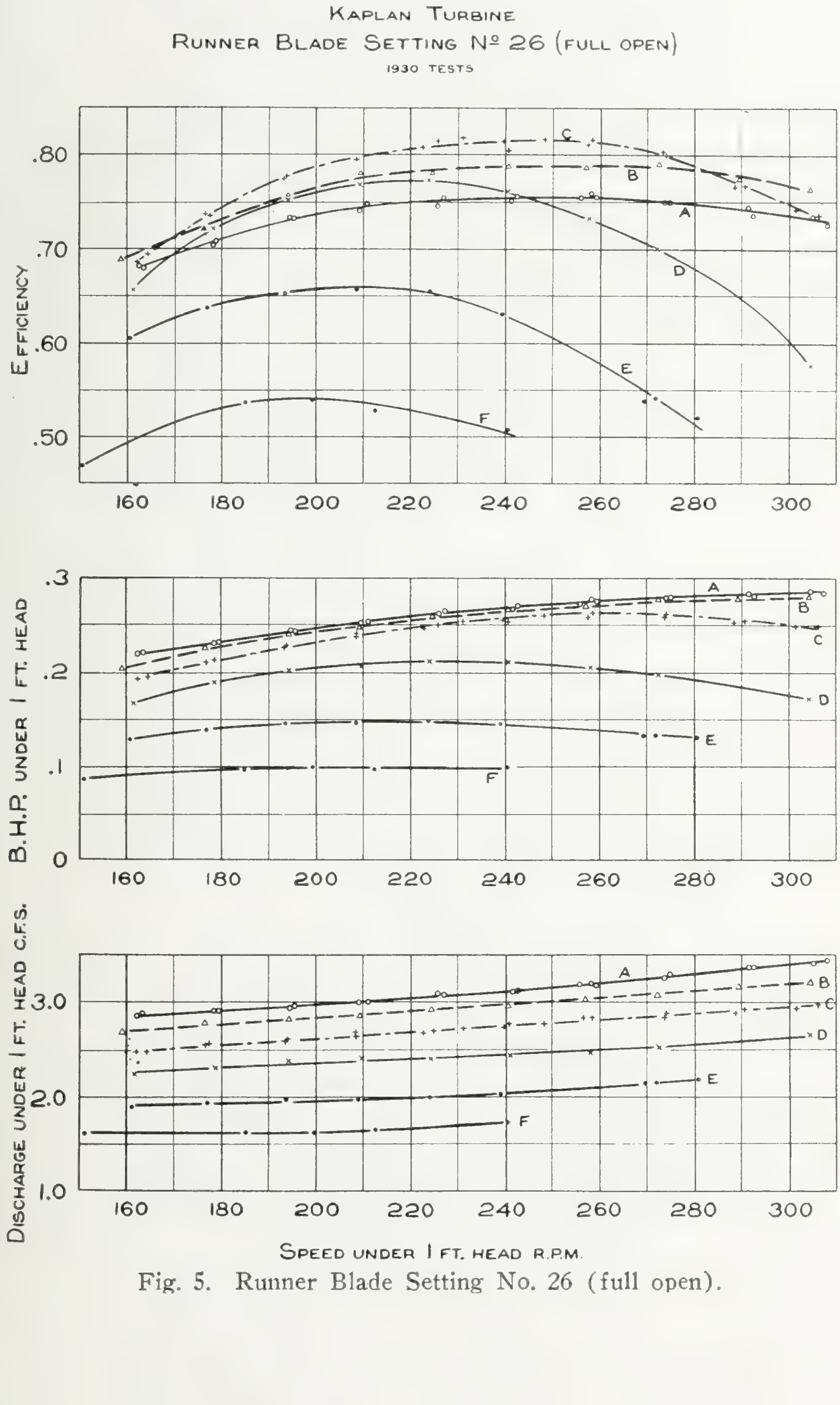


Fig. 5. Runner Blade Setting No. 26 (full open).

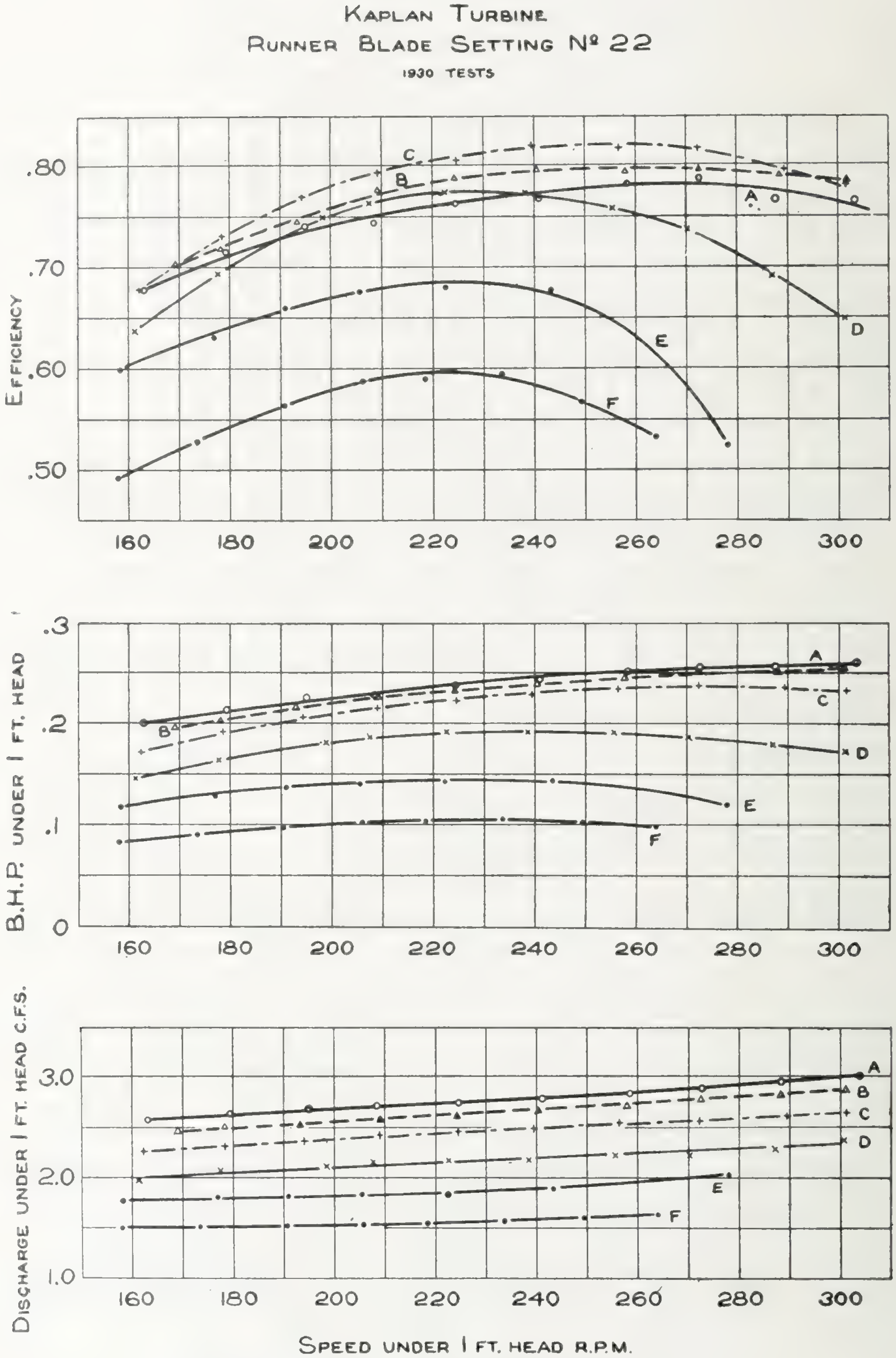


Fig. 6. Runner Blade Setting No. 22.

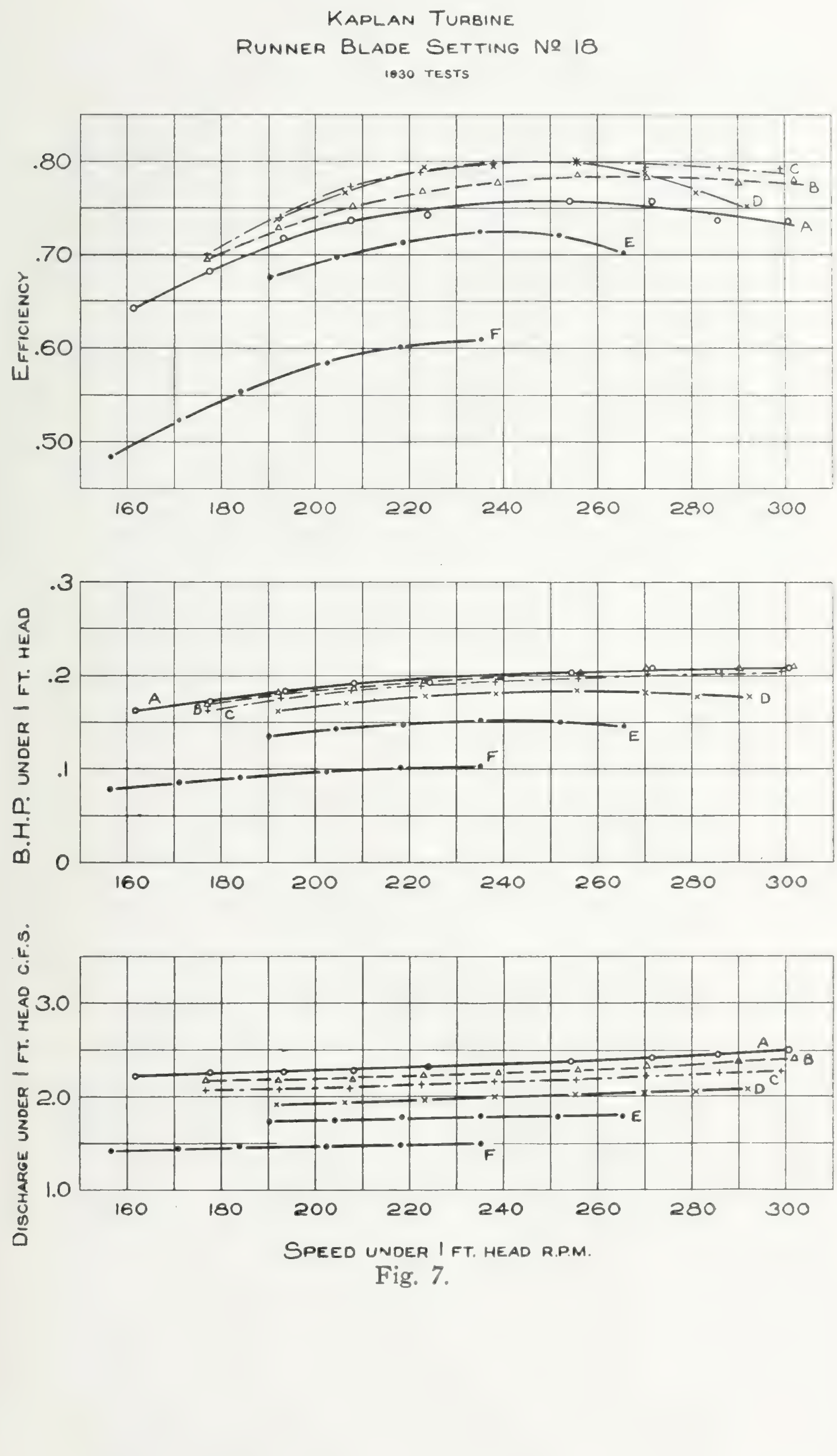

KAPLAN TURBINE
RUNNER BLADE SETTING Nº 18
1930 TESTS
EFFICIENCY
.80
.70
.60
.50
A
B
C
D
E
F
160
180
200
220
240
260
280
300
B.H.P. UNDER 1 FT. HEAD
.3
.2
.1
0
DISCHARGE UNDER 1 FT. HEAD C.F.S.
3.0
2.0
1.0
SPEED UNDER 1 FT. HEAD R.P.M.

Fig. 7.

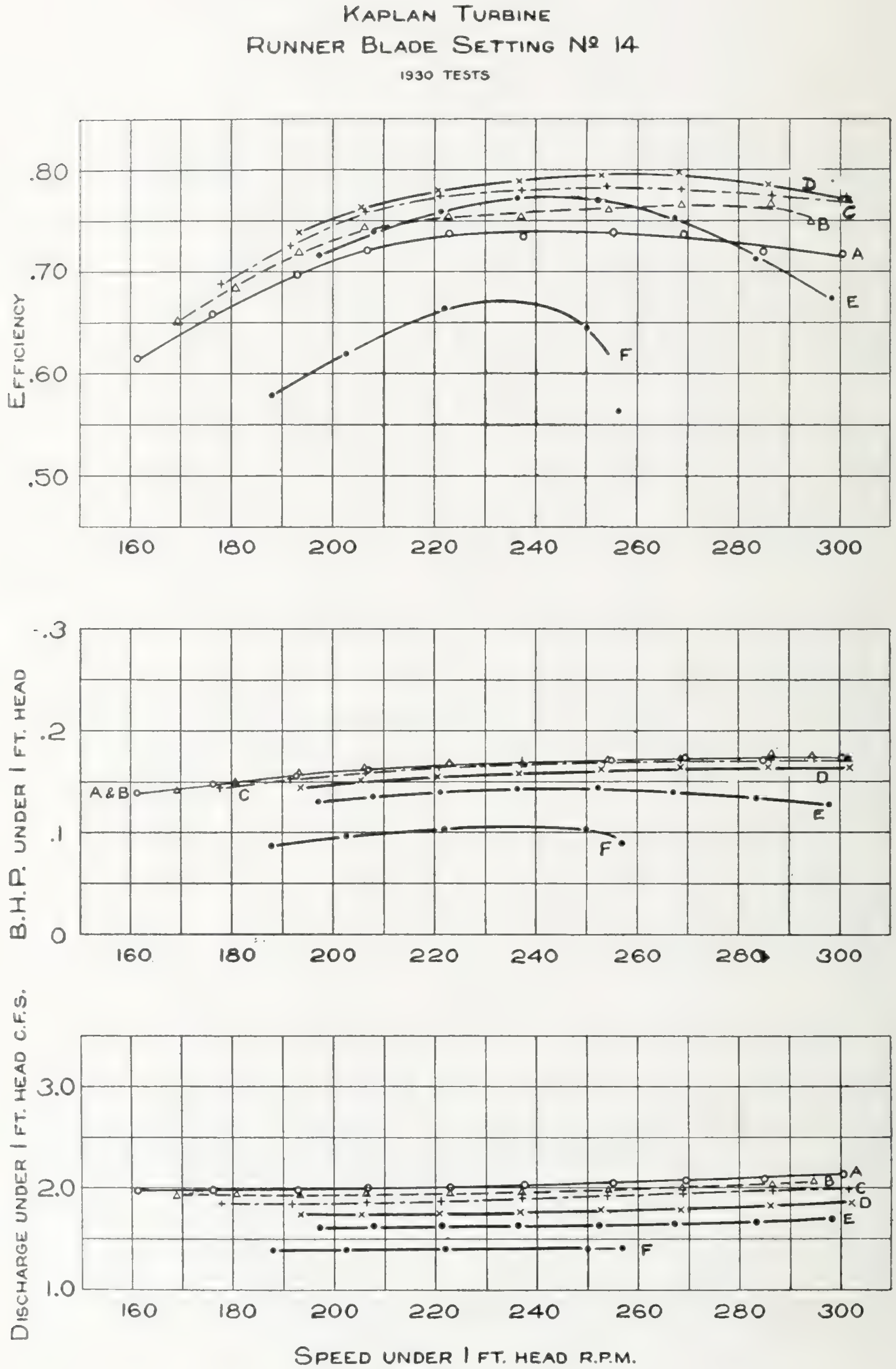

KAPLAN TURBINE
RUNNER BLADE SETTING № 14
1930 TESTS
EFFICIENCY
.80
.70
.60
.50
A
B
C
D
E
F
160
180
200
220
240
260
280
300
B.H.P. UNDER 1 FT. HEAD
.3
.2
.1
0
A & B
DISCHARGE UNDER 1 FT. HEAD C.F.S.
3.0
2.0
1.0
SPEED UNDER 1 FT. HEAD R.P.M.

Fig. 8.

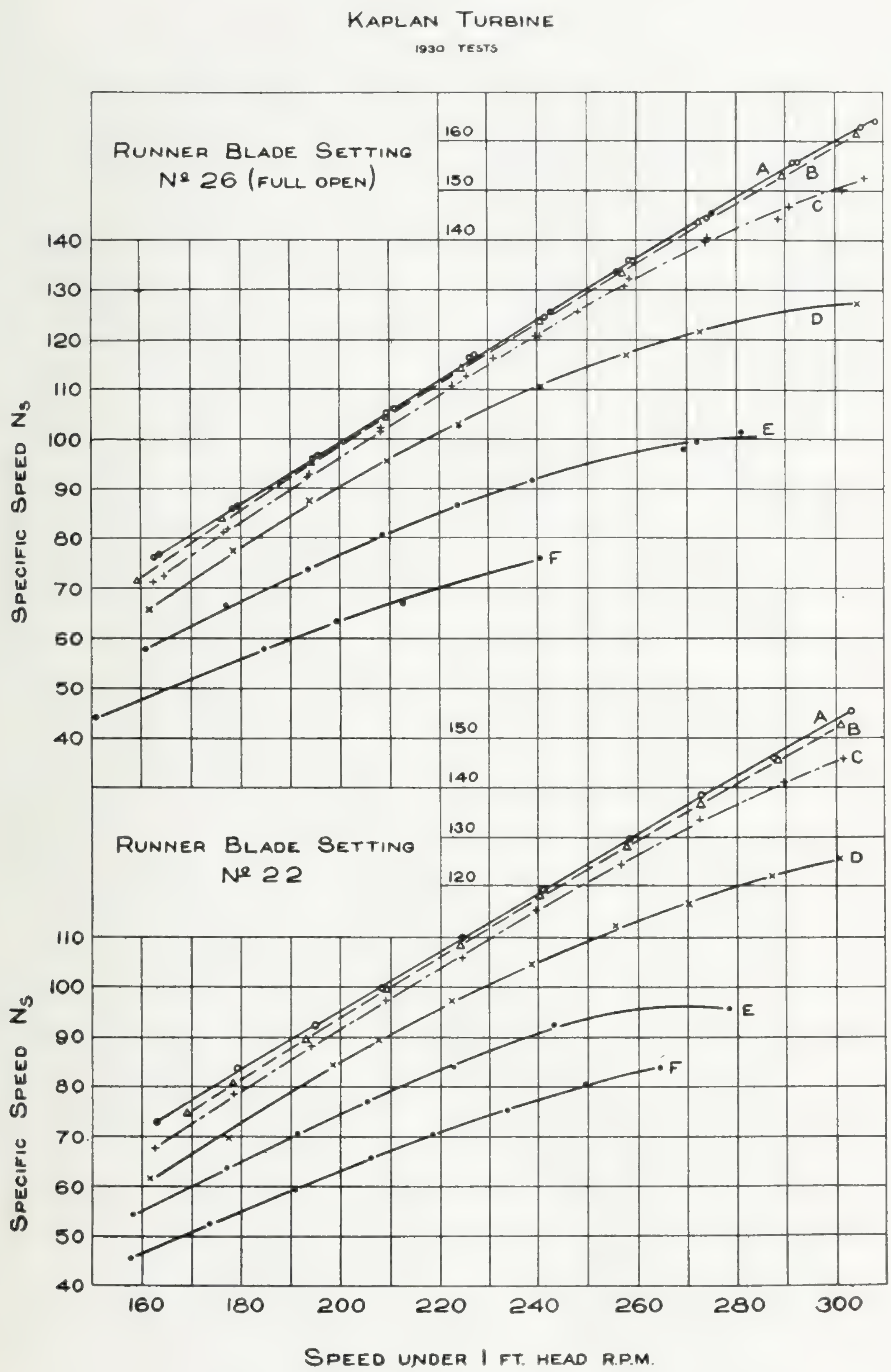

KAPLAN TURBINE
1930 TESTS
RUNNER BLADE SETTING
Nº 26 (FULL OPEN)
RUNNER BLADE SETTING
Nº 22
SPECIFIC SPEED N_S
SPEED UNDER 1 FT. HEAD R.P.M.
A
B
C
D
E
F

Fig. 9.

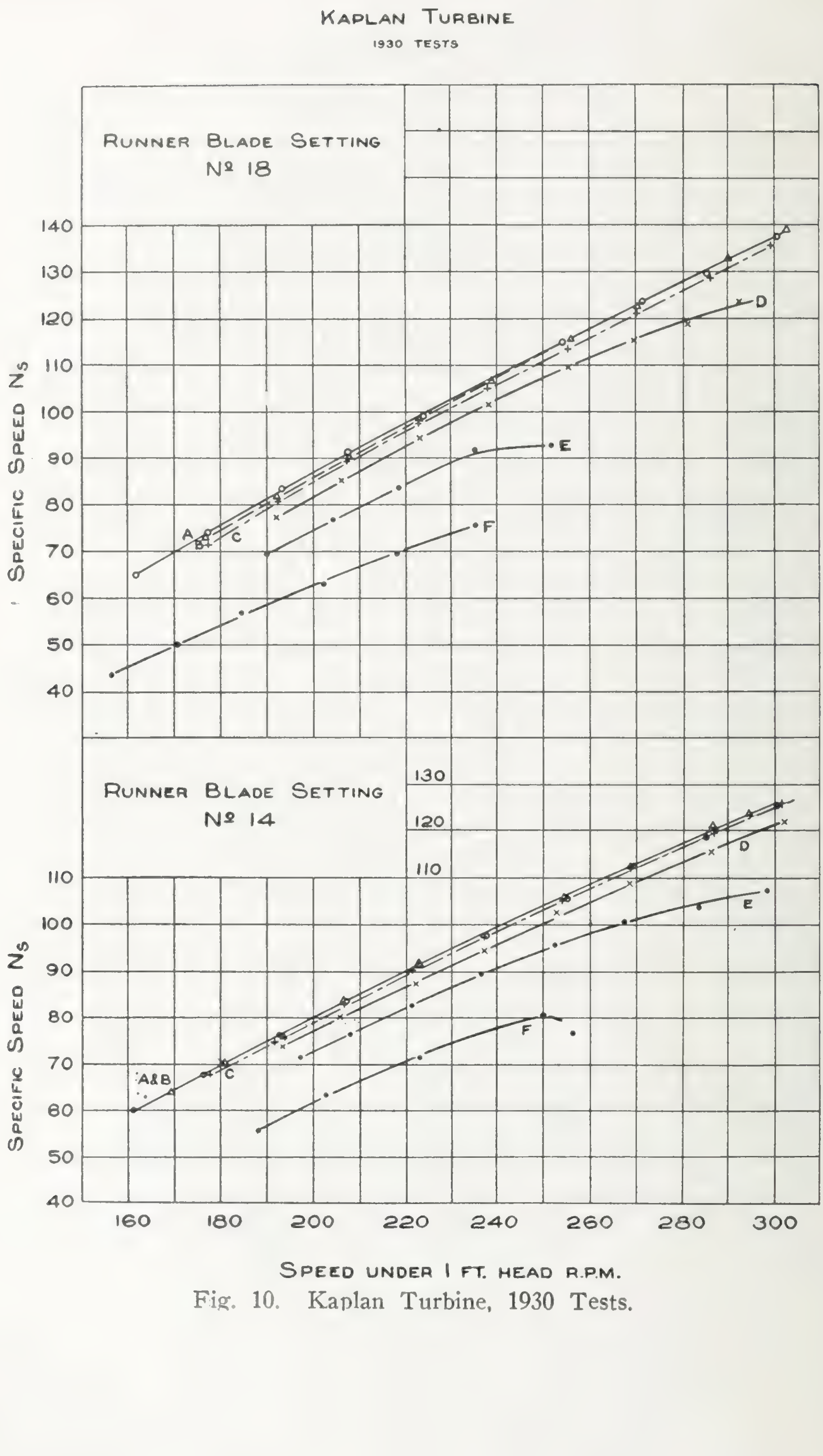


Fig. 10. Kaplan Turbine, 1930 Tests.

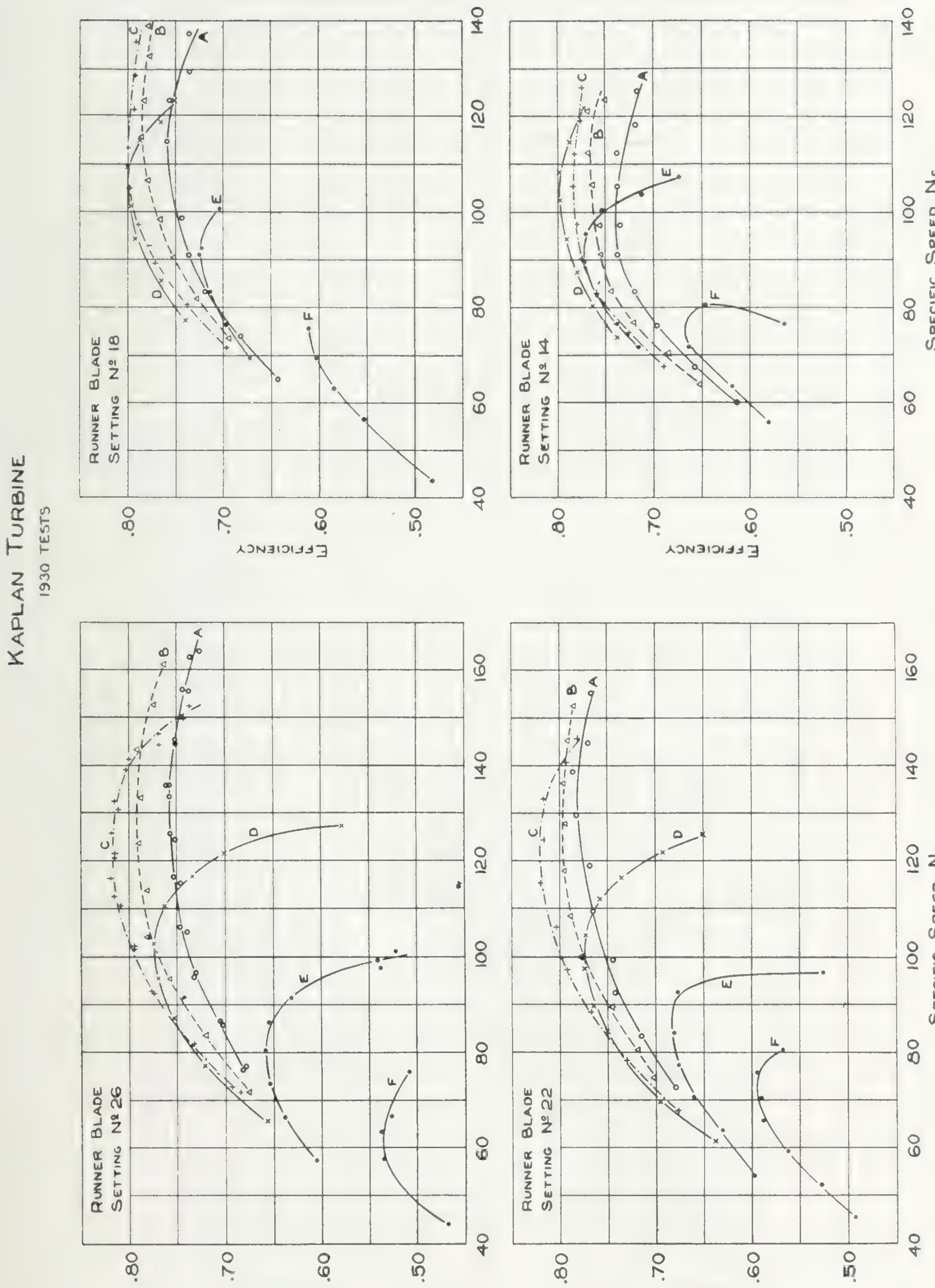

KAPLAN TURBINE
1930 TESTS
RUNNER BLADE SETTING № 18
RUNNER BLADE SETTING № 14
RUNNER BLADE SETTING № 26
RUNNER BLADE SETTING № 22
EFFICIENCY
SPECIFIC SPEED N_S

Fig. 11.

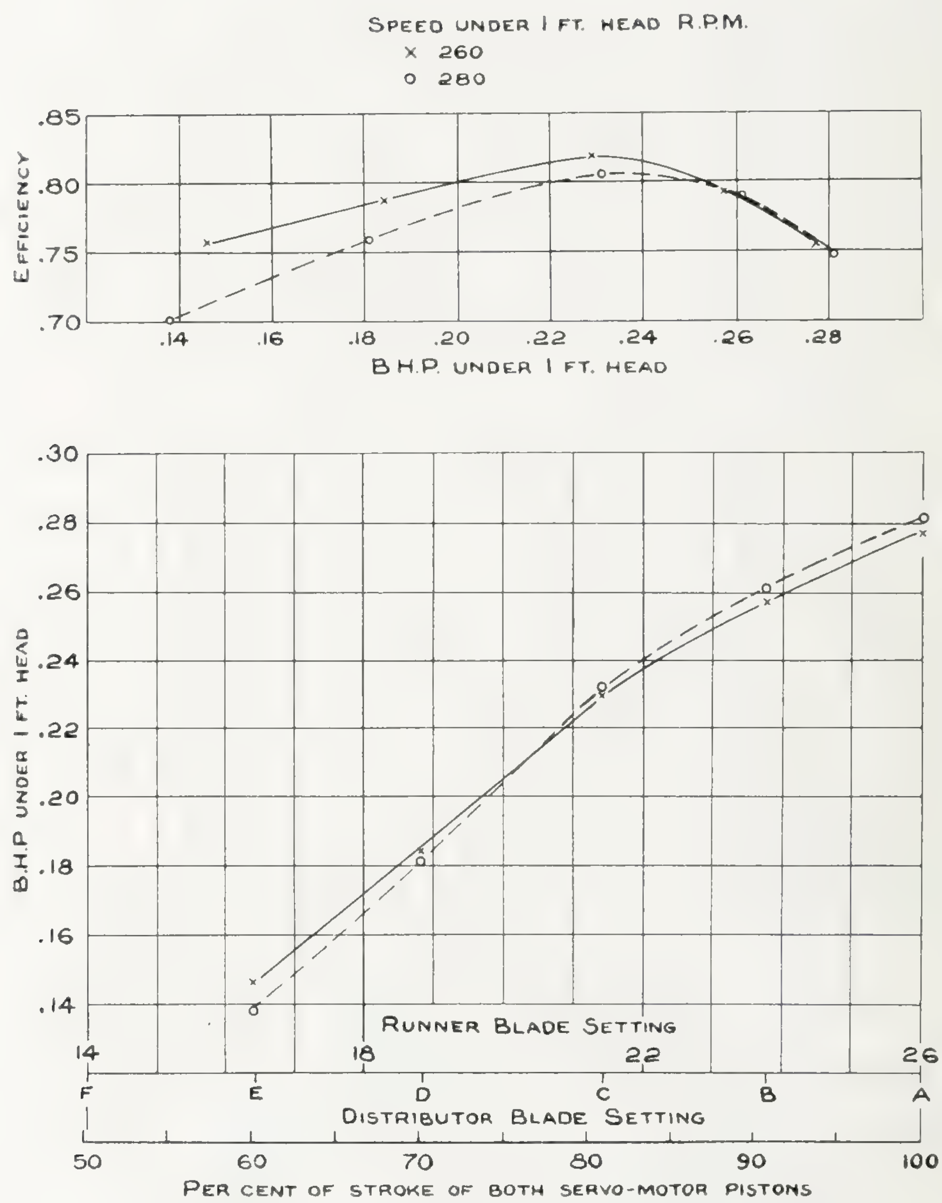


Fig. 12. Kaplan Turbine 1930 Tests.

the per cent. of stroke of the servo-motor pistons. That is, it is assumed that there are separate servo-motors on the blades and distributor guides, and the base line shows percentage of motion of both pistons, taking the zero at the closed distributor gate position and the flattest position of the runner blades, respectively, and 100 per cent. as the maximum opening in each case. On the same base line the various runner blade and distributor gate settings used on the tests are shown, and the vertical axis represents horse power under one foot head at various speeds. The upper pair of curves show efficiencies plotted against unit power output at two speeds; they are the curves that are ordinarily obtained on commercial tests, and are plotted for the unit speeds of 260 and 280 revs. per min., because these are the speeds corresponding most nearly to best efficiency at the larger power outputs.

The curves of Fig. 12 show that at full open positions the turbine gives 0.278 and 0.282 brake horse power under one foot head at unit speeds of 260 and 280 revs. per min., respectively, in each of which cases the efficiency is 75 per cent. When both servo-motor pistons have moved 20 per cent. of their stroke toward the closed positon, or are set at 80 per cent. of the full open position, corresponding to a distributor vane setting slightly below C and a runner blade setting somewhat below No. 22, the unit power output is approximately 0.222 h.p. in each case. At this output the efficiency will be 82 per cent. at unit speed of 260 revs. per min. and 80 per cent. at unit speed of 280 revs. per min. The highest efficiency of 82.3 per cent. occurs at 260 revs. per min. unit speed, with an output of 0.23 h.p. at unit head, which is 82.1 per cent. of the maximum output. Under these conditions the specific speed is only 126, which would appear to be rather low.

The highest specific speed obtained with the turbine was 164 at maximum setting of both distributor vanes and runner blades and at a unit speed of 309, but the efficiency under this condition was only 73 per cent., which is relatively low. However, at maximum runner setting No. 26 and at distributor setting B, a specific speed of 161 was obtained with 76 per cent. efficiency, which is much more favourable.

A further investigation of the turbine is now being made to determine the directions of flow and velocities in it and the results of this study will be presented later.

The draft tube used in these tests is shown on Fig. 13.

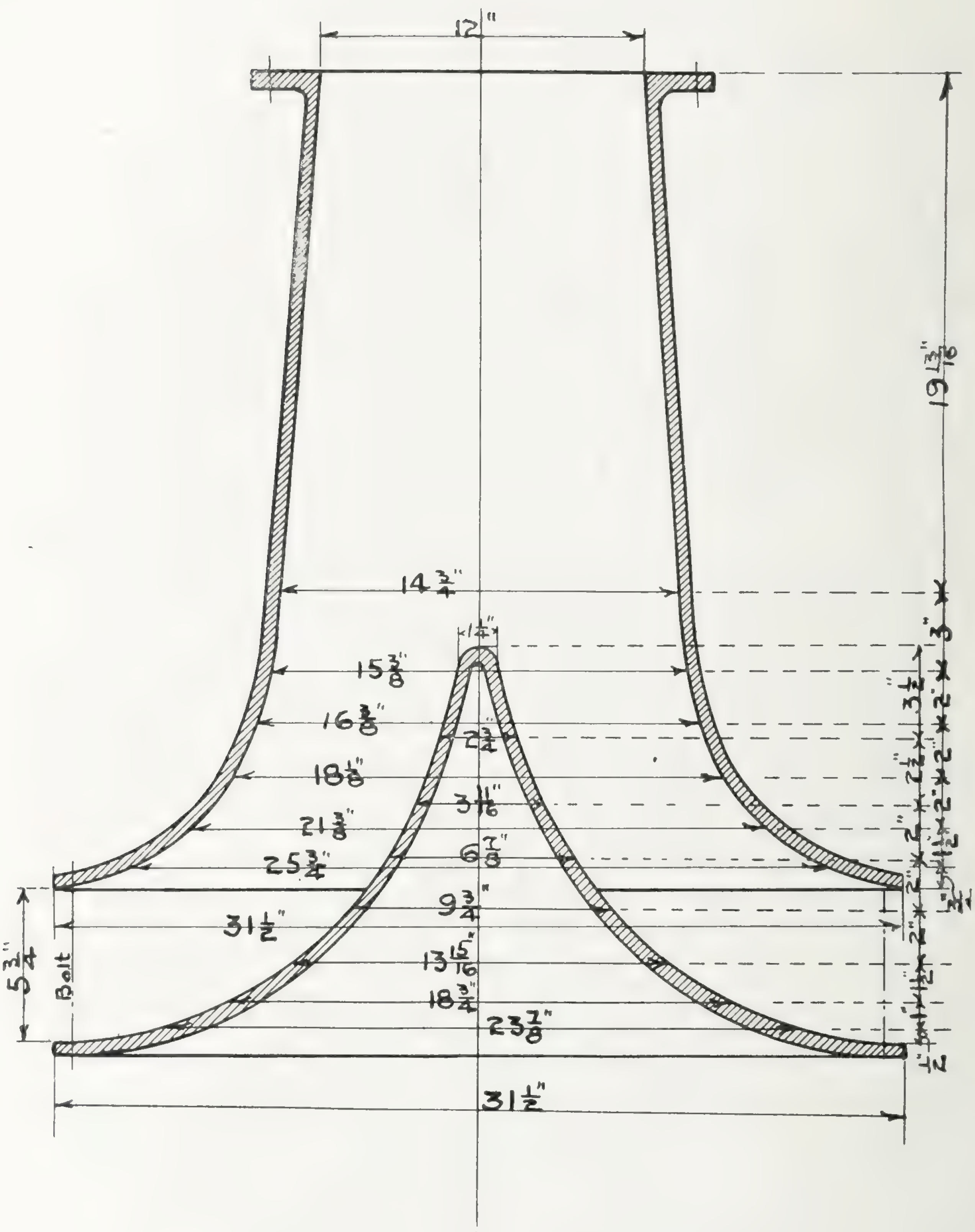


Fig. 13. Draft Tube used in Tests.

ELECTRICAL REFRACTORY PORCELAIN BODIES CONTAINING MAGNESIUM OXIDE

By J. M. Higgins[1] and Robert J. Montgomery[2]

The series of electrical refractory porcelain bodies studied was based upon known commercial bodies used in Canada. It was assumed that an increase of the alumina content at the expense of silica and an increase in the magnesium oxide content would improve the body. Eight bodies were selected as shown on the $MgO-Al_2O_3-SiO_2$ triaxial diagram given in Fig. 1 (1).

Compositions 1 and 2 approximate present commercial bodies. Nos. 1-2-8 contain clay, talc and potters flint. Nos. 3 and 7 contain only clay and talc. Nos. 4 and 6 contain clay and fused MgO. No. 5 contains clay, talc and calcined Al_2O_3. Talc was favoured as the source of MgO in the bodies which would allow its use as it is cheap and may be obtained fairly pure. The silica it contains is combined and the ignition loss is not large as is the case with magnesium carbonate. MgO is used in bodies 4 and 6 as the silica content of talc would not allow it to be introduced without the use of Al_2O_3.

Literature contains many articles on the use of MgO in ceramic bodies but the interest has been largely the substitution of MgO for some other constituent of certain bodies. Most of these bodies contain alkali and vitrification is usually a requirement. An electrical refractory porcelain must have a high resistance to thermal shock and low coefficient of thermal expansion, high dielectric strength at high temperature, high mechanical strength, controlled drying and burning shrinkage and lowest possible absorption. The body need not be vitrified. Alkali may be omitted from the composition with advantage. However, specifications for such bodies have not been standardized and little or no testing of the refractory is done by manufacturers of electric stoves and appliances. While the Ontario Hydro Electric Commission recommend a body having an absorption of less than 1% (2), this point is not tested and commercial bodies are quite porous.

Important points on the subject of composition obtained from published information may be listed as follows:

---

[1]Research Assistant in Ceramics, University of Toronto.
[2]Assistant Professor of Ceramics, University of Toronto.

1. MgO in small amounts lowers the maturing point of bodies containing clay, flint and feldspar but does not shorten its vitrification range as does CaO (3).

2. In such triaxial bodies not fired to vitrification the first small additions of MgO lower the porosity more than further additions (4).

3. Porous bodies containing MgO tend to be resistant to thermal shock (5).

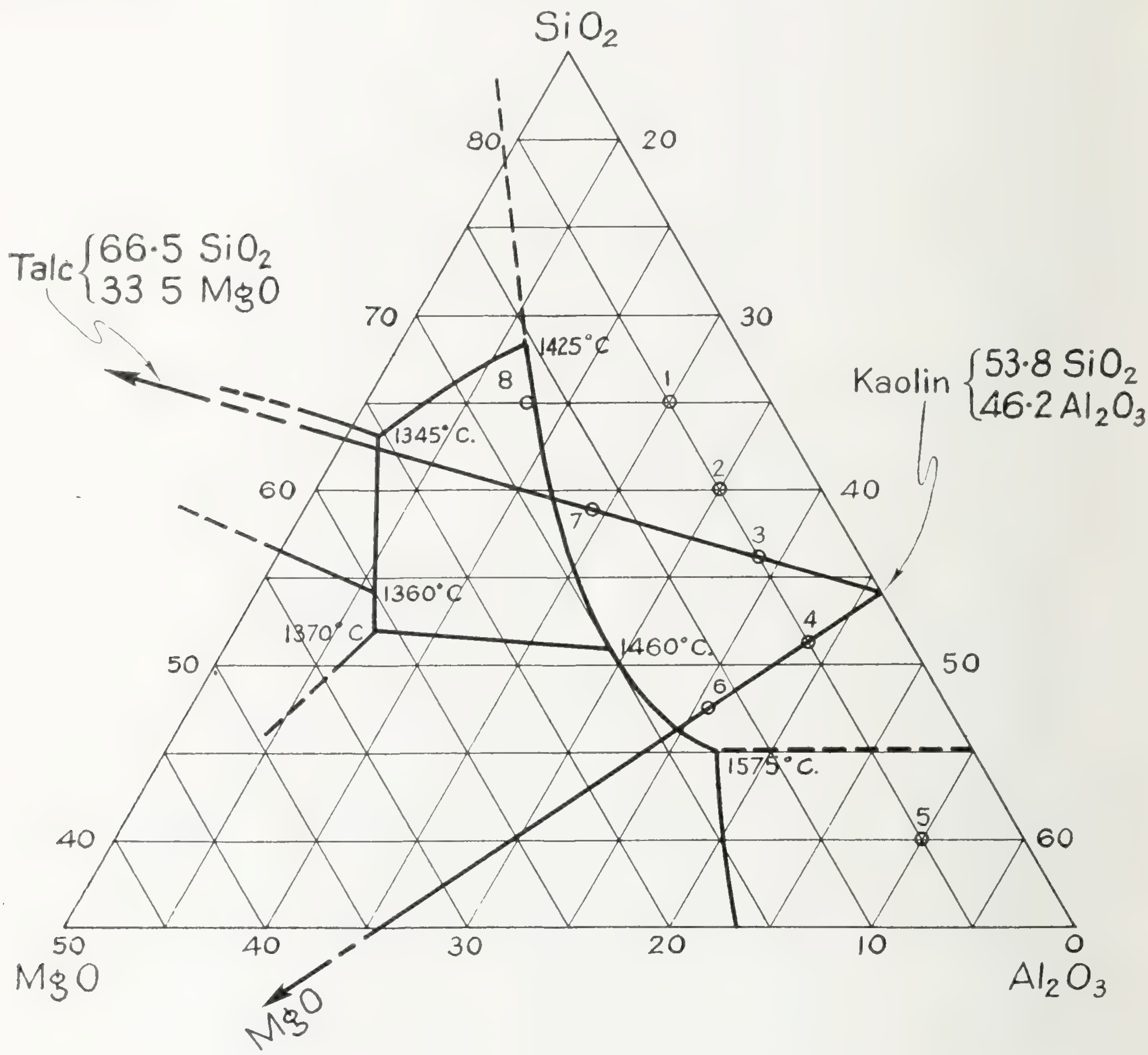


FIG. 1

4. Vitreous bodies containing MgO also tend to be resistant to thermal shock (6).

5. MgO greatly accelerates the formation of sillimanite (7).

6. The hot dielectric strength of a vitrified body composed only of MgO, Al_2O_3 and SiO_2 is much higher than that of a feldspar porcelain (7).

7. The partial or total substitution of MgO for K_2O in vitreous electrical refractories gave improved properties (7).

8. Gas mantle rings have been successfully made using $MgCO_3$ as a flux and up to 15% have given good rings but with 25% or above

the rings had little mechanical strength (8). A gas mantle ring must have practically the same requirements as an electrical refractory with the exception of high dielectric strength at high temperatures.

9. The use of 20% talc with clay gave a porous body which was quite resistant to thermal shock and had a good electrical insulating property at high temperatures (9).

10. Gas mantle rings have also been made using 15 to 20% talc with a wholly clay body giving a porous body very resistant to thermal shock (10).

11. The elimination of quartz and the substitution of substances not subject to crystal inversions or other volume changes gave improved bodies. These may be calcined kaolin, alumina, zirconia or sillimanite (11).

Many other references could be given but most of them suggest ideas without having any direct connection with the subject at hand. The ones listed above aided in the selection of the compositions studied.

MATERIALS AND BODIES

The talc used in this series came from New York State (11) and was selected in preference to Ontario talc because of purity. The Ontario talc (13) as stated by the producers, is high in lime and this is particularly objectionable in the type of body desired. The fusion point (P.C.E.) of the Ontario talc is cone 14 (1400° C.) as compared to cone 16 (1465° C.) for the New York material. According to the work of M. Kramer and S. J. McDowell (14) a commercial talc containing 7% CaO melts at 1500° C. Pure talc ($3\ MgO-4\ SiO_2-H_2O$) has a melting point of approximately 1543° C. (1). The lowest temperature eutectic of the system $MgO-Al_2O_3-SiO_2$ forms at 1345° C., therefore there is only about 200° C. between the temperature at which at fluid phase will start to form and that of complete melting, if pure talc were obtainable commercially. Using the New York talc the temperature difference is probably less as the melting point is only 1465° C. The vitrification range at best will be very short.

A comparison of the two talcs as shown by the analysis submitted by the producers is given.

Composition	*New York Talc*	*Ontario Talc*	*Pure Talc*
SiO_2	56.54	47.50	63.5
$Fe_2O_3+Al_2O_3$	1.04	3.65	....
MgO	30.74	29.00	31.7
CaO	6.25	8.00	....
Loss on ignition	4.60	10.85	4.8
P.C.E.	1465° C.	1400° C.	1543° C.

The low silica content and high ignition loss of the Ontario talc indicates that in all probability part of the MgO is present as the carbonate. Then part of the MgO as well as the CaO is present not as a silicate. The presence of any considerable amount of lime gives us a 4 component system and the lowest temperature eutectic of such a system is probably below 1345° C.

Published analyses (15) indicate that pure talc in commercial grades is difficult to obtain and that the New York talc selected is among the better grades.

From the foregoing it will be seen that to fire bodies to vitrification, with commercial talc as an important constituent, is a doubtful possibility except with perfect temperature control. In this investigation no attempt was made to obtain vitrified bodies although this property might be desired by the users of electrical refractory porcelain. Kramer and McDowell (14) state that the nearer the composition approaches that of fostirite ($Mg_3 SiO_4$), having a melting point of 1890° C. (1), the greater the improvement in the firing behaviour. Hence the addition of MgO to talc bodies will lengthen the firing range. The per cent. of low temperature eutectic in the body is reduced. In the series studied the composition was restricted to bodies containing as little MgO as possible as the amount of ground which could be covered was limited.

Fused MgO (16) was used as the only source of this oxide in bodies 4 and 6. An analysis as furnished by the producer is:

Composition of Fused MgO

MgO	95.68
Al_2O_3	.75
SiO_2	2.00
Fe_2O_3	.07
CaO	1.50
	100.00

The clays were selected to give a good dry-press body containing 50% raw clay on a dehydrated clay basis, as follows:

15% Tennessee Ball, No. 3
20% English China clay
15% Georgia kaolin

The remainder of the clay content was made up of calcined Delaware kaolin (17). The calcination was carried to cone 12 and the material was ground so that approximately 50% would pass a 200 mesh while the remainder was finer than 28 mesh. Additional grinding was done in the preparation of the batch which gave a residue of less than 0.5% on a 48 mesh screen.

The potters flint and oxide of alumina necessary to complete the batches were of normal commercial grade.

The compositions of the eight bodies as taken from Fig. 1 are:

CHEMICAL COMPOSITION—OXIDE BASIS

Body	MgO	Al_2O_3	SiO_2
1	5%	30	65
2	5	35	60
3	5	39.1	55.9
4	5	43.6	51.4
5	5	55.0	40.0
6	12	40.5	47.5
7	12	29.4	58.6
8	12	23.0	65.0

PERCENTAGE BATCH WEIGHT[1]

Body	Talc	Flint	Tenna. Ball	Eng. China Clay	Georgia Kaolin	Calcined Delaware Kaolin	Fused (14) MgO	Al_2O_3
1	14.56	18.04	15.40	20.50	15.40	16.10	....	....
2	14.56	7.98	15.62	20.82	15.62	25.40	....	....
3	14.56	.....	15.80	21.07	15.80	32.77	....	....
4	.....	.....	16.02	21.34	16.02	42.00	4.62	....
5	14.56	.....	15.17	20.32	15.17	7.58	....	27.20
6	.....	.....	15.87	21.17	15.87	35.97	11.12	....
7	34.49	.....	15.34	20.42	15.35	14.50	....	....
8	34.59	12.65	15.08	20.01	15.08	2.59	....	....

[1]Allowance is made in the raw clay content for the ignition loss of both clay and talc.

PREPARATION AND FORMING OF BODIES

2000 gr. batches were ground in ball mills, screened to 48 mesh and filter pressed. The filter press cakes were aged in a damp box for 48 hours, dried sufficiently to allow pulverizing. After going through the pulverizer several times the bodies were screened to 12 mesh and dry pressed into test pieces. For transverse strength, resistance to thermal shock, shrinkage and porosity, test bars $\frac{3}{4} \times 1\frac{1}{4} \times 4$ inches in size were made. For electrical resistivity pieces $\frac{3}{4}'' \times 1\frac{1}{4}'' \times 1\frac{1}{4}''$ were cut from the test bars when partly dry and smoothed upon emery paper. For the coefficient of expansion test, bars $\frac{3}{8} \times \frac{3}{8} \times 4$ inches were made by pressing the piece $\frac{3}{8}''$ thick then cutting away and smoothing the 4″ piece to obtain the other dimension while the bar was partly dry.

FIRING

Preliminary firing tests indicated that burning to cones 8-10 and 12 should give well matured bodies without over-fire. The burns were made in a gas fired downdraft laboratory kiln taking 15 hours to reach the finishing temperature. The time temperature curve is given in Fig. 2. Four hours were allowed for the finishing period to equalize the temperature. The temperature difference between the outside and inside of the saggers was less than one cone instead of 2 cones as ex-

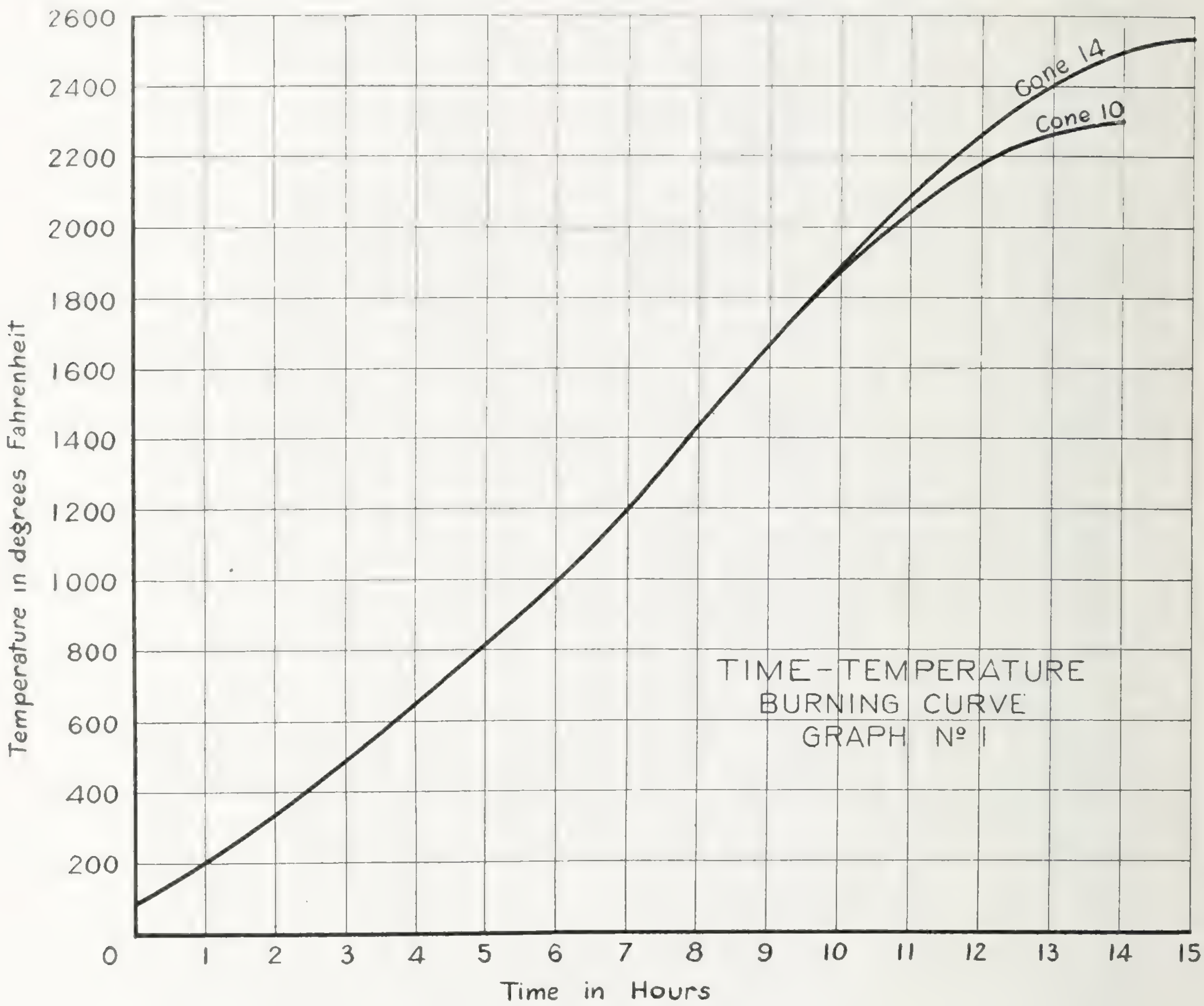


FIG. 2

pected, so the actual temperature obtained for three burns where cones 10-11 and 13½ instead of 8-10 and 12. Tests were therefore made only on the samples from burns at cones 10 and 13½.

METHODS OF TESTING

The following physical tests were applied to the burned test pieces and the average results tabulated. Except where otherwise specified the averages given were calculated from the tests of five samples.

1. *Linear Shrinkage.* The drying shrinkage of the dry pressed bars was too small to measure and record separately from the burning shrinkage. The total shrinkage is given in % of the length as pressed.

2. *Apparent Porosity.* Samples of approximately 12 grams weight were used for this test applying the 1 hour boiling method to obtain saturation. The method is considered to be satisfactory for porous bodies.

3. Absorption was calculated from the weighings obtained for the porosity test.

4. *Transverse or Flexual Strength.* The usual method was used and agreed with the A.S.T.M. standards except for size of piece (18). The distance between supports was 3 inches.

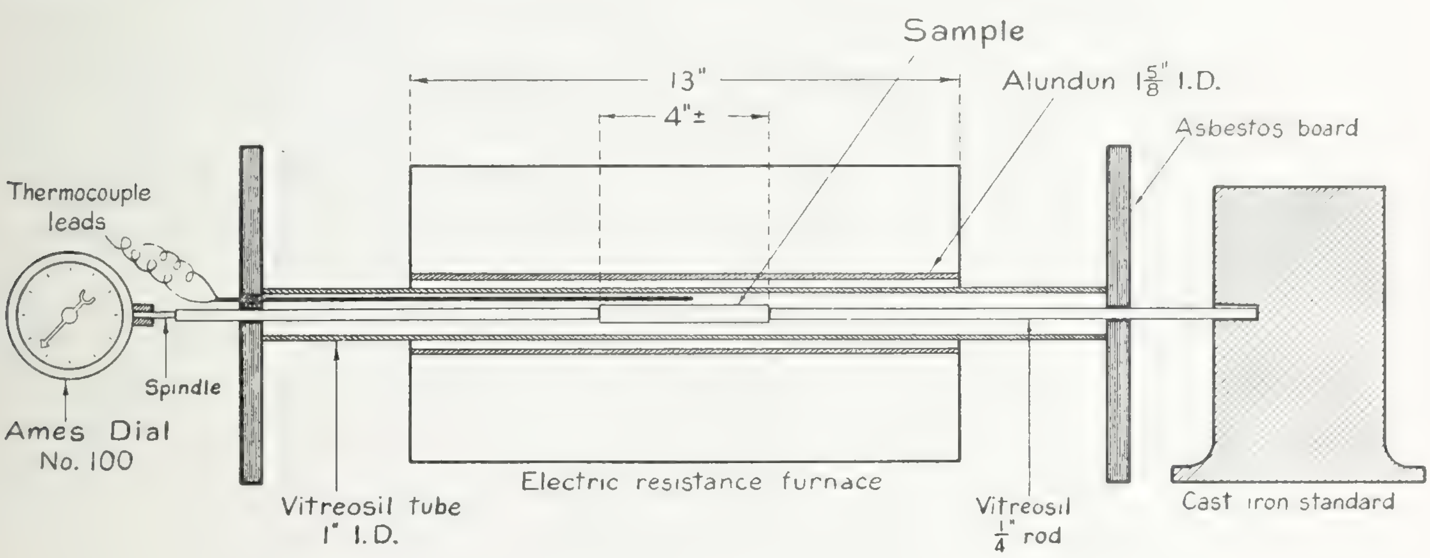


FIG. 3

5. *Resistance to Thermal Shock.* The samples were heated in an electric furnace to 900° C. (1652° F.) (19) in three hours. The furnace was held at this temperature and the samples removed, quenched in a bath of water at 14° C. (20). The samples remained in the water 3 minutes (21), were dried at 100° C. on a gas hot plate, returned to the furnace for 45 minutes to reach temperature equilibrium and then quenched again. All samples were subjected to 25 quenchings (22) whether cracked or not, except where complete failure (separation into 2 or more pieces) had taken place before 25 quenchings were reached. All specimens still intact were then subjected to the transverse strength test and the percentage loss of strength was calculated (19).

6. *Thermal Expansion.* The apparatus used for this test is very similar to that recommended by the American Ceramic Society (23), with the exception that sample was in a horizontal position. An electric furnace with a base metal resistor was used. The spring of the gauge was sufficient to overcome friction and return the dial reading to 0 after the test. An Ames No. 100 (24) dial reading to 0.0001 inch and having a total movement of 0.1 inches gave the expansion reading direct. A

sketch of the set-up is seen in Fig. 3. The hot junction of a platinum to platinum-rhodium thermocouple was placed directly above the centre of the sample but was left free to move either way to check the temperature variation within the working zone.

The apparatus was calibrated by heating a vitreosil blank taking the coefficient of linear expansion for the vitreosil (25) as 540×10^{-9}. A sample was then substituted for the vitreosil blank (all specimens were approximately 4 inches long) and the thermal expansion is calculated. The furnace as wound could only be heated to 900° C. so this was the maximum temperature for all tests as 900° C. is well above the temperature such bodies attain in use.

Satisfactory readings were not obtained with the set-up for 50° intervals of temperature rise and the temperature curve as used by Geller and Heindl (27). It was found necessary to record only the total expansion over the entire range as given in Table III. The time required to reach equilibrium at 900° C. and allow the checking of results was about 3½ hours. Further work on smaller temperature intervals is planned.

7. *Electrical Resistivity.* Of the many methods used for the determination of the electrical resistance of ceramic bodies at elevated temperatures, the one used in this investigation was chosen more to meet local conditions and apparatus available rather than for its advantages. A review of the literature brought out the following important points.

(1) The U.S. Bureau of Standards method (28), using alternating current and molten metal terminals gave good results but the molten metal could not be used in this investigation due to the very porous body being tested.

(2) P. H. Brace (29) used direct current and nickel wire grid terminals but the results were rather peculiar, most likely due to polarization caused by the direct current.

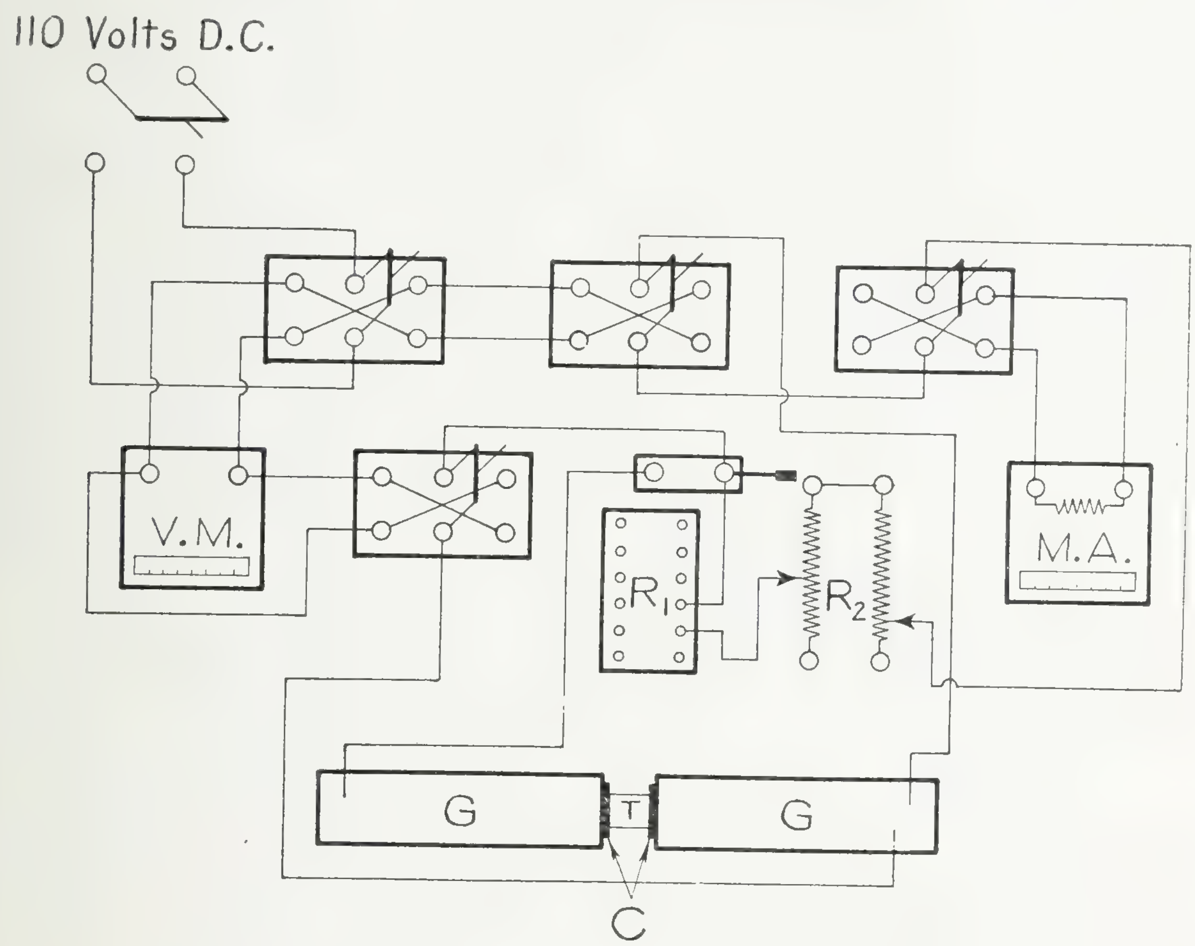
(3) Dr. E. F. Northup (30) used alternating current and graphite terminals and good results were obtained on alundum cements.

(4) A. V. Henry (31) used alternating current, graphite terminals and used a Wheatstone bridge to measure the resistance of the test pieces.

(5) R. M. King (32) used alternating current and nichrome terminals and measured the voltage drop across the test pieces to determine the resistance.

The methods differ in detail but the outstanding problems are (1) sufficiently sensitive circuit to allow accurate measure of small currents, (2) good contact between test piece and terminals, (3) terminals suitable for the kind of material to be tested.

As alternating current was not available in the laboratory 110 volt direct current was used. The current was measured with a micro-ammeter, and at the same time measuring the voltage drop across the terminals. A Hoskins carbon resistance furnace was used to heat the test piece to the desired temperature and the measuring circuit was set up as given in Fig. 4. In the first set-up the voltmeter drew a current



SET-UP FOR MEASURING THE ELECTRICAL RESISTIVITY

V.M. – Weston D.C. voltmeter.
R_1 – High resistance box having a maximum resistance of 19,150 ohms.
R_2 – Sliding-contact rheostat having a maximum resistance of 1,224 ohms.
M.A. – Western microammeter.
S – 22·18 ohm shunt, F.S. deflection 15·2 milliamperes.
G – Graphite terminals.
T – Test piece.
C – Silver contacts.

FIG. 4

of 6.75 milliamps and this was too great a current to be read on the microammeter so that it was necessary to construct a shunt to be attached across the terminals of the microammeter in order that the meter would read in milliamps. This shunt was calibrated so that a full scale deflection of the micro-ammeter was equal to 15.2 milliamps. This worked very well although the smallest possible reading was 0.05 milliamps or 50 microamps, which is not as sensitive as might be desired.

Using direct current and graphite terminals gave irregular readings

on reversal of the current direction. After reversal there was a sudden and decided increase in current value which then slowly decreased. The decrease was never as great as the increase probably due to the increasing temperature of the furnace. This polarization (29) has already been referred to and has been observed by Ferguson, Mulligan and Rebbeck on silicate glasses (33) and by the U.S. Bureau of Standards (28) on spark plug porcelain. It was necessary to overcome this polarization in order to obtain accurate results. Prof. J. B. Ferguson (34) explained that this polarization was caused by a migration of basic ions in the direction of the current thereby lowering the percentage of ions on one side of the test piece. Hence a poor contact would result and the resistance would increase with time. A reversal of the current would increase the current flow until gradually decreased as the ions migrate back to the other side. He suggested that silver foil contact between the graphite terminal and the test piece would prevent polarization as the ions migrating in the test piece would be replaced by those given up by the silver. An equilibrium would be established.

Accordingly two pieces of sterling silver (35) (92.5% Ag+7.5% Cu) having a thickness of .008 inches, were inserted at the terminals. As silver and copper form a eutectic (35) at 779° C. it was decided not to run the test above 700° C. The results of this test with the silver contacts were quite satisfactory and irregular current fluctuations disappeared. The current flowing throughout the circuit was recorded in milliamperes as well as the current required by the voltmeter and the voltage drop across the sample. The electrical resistivity was then calculated from this formula:

$$R=\frac{r\times a}{d}$$

where R = electrical resistivity in ohms per cm^3

r = resistance of sample in ohms calculated from Ohm's law ($E=IR$)

a = area of sample in cm^2

d = depth of sample in cm.

RESULTS

The results of both the cone 10 and cone 13½ burns were very satisfactory. All the samples had a uniform shrinkage and showed a minimum of warpage with the exception of bodies 7 and 8. Body No. 8 at cone 13½ was melted to a glassy mass. At this temperature most of the test pieces of body 7 were also melted but a few of the larger bars were in fair shape and were tested for transverse strength and resistance to thermal shock. The vitrification range of these bodies

is very short and Nos. 7 and 8 could not be burned commercially at cone 13½.

BURNING SHRINKAGE

Referring to Table No. I it may be seen that an increase in temperature from cone 10 to 13½ gives an increase in shrinkage in all eight bodies. At cone 10 the shrinkage increases as kaolin replaces flint in bodies 1, 2 and 3, and talc in bodies 7 and 3. Evidently under these conditions silica acts as a refractory, preventing shrinkage. Body No. 5 which contains a lower silica content than body No. 4, has a lower shrinkage because of the introduction of Al_2O_3, which also acts as a refractory. As the raw clay content of each body is the same, changes in properties must be due to the other constituents.

A comparison of body No. 3 with No. 7 and body No. 4 with No. 6 shows that an increase of MgO at the expense of clay decreases the shrinkage somewhat.

TABLE I

PHYSICAL PROPERTIES

Body No.	Firing Temp. Cone	Per cent. Total Shrinkage	Max. Fiber Stress lbs. per sq. inch	Per cent. Apparent Porosity	Per cent. Absorption
1	10	5.38	2218	29.9	16.2
1	13½	6.13	2019	29.8	16.7
2	10	7.19	2944	28.4	15.0
2	13½	8.96	4307	17.9	8.5
3	10	8.93	3954	23.6	11.7
3	13½	10.32	4868	14.3	6.9
4	10	10.19	3074	35.1	19.4
4	13½	11.00	3120	29.6	15.9
5	10	7.35	2847	36.9	20.8
5	13½	9.15	3768	34.1	18.8
6	10	8.34	2560	39.5	23.5
6	13½	8.44	3468	31.5	18.0
7	10	6.54	4223	23.5	12.6
7	13½	9.15	7130	11.1	5.3
8	10	4.66	2870	30.4	17.0
8	13½	....	....	....	....

Cone 10 is approx. 1260° C.
Cone 13½ is approx. 1370° C.

N.B.—As all bodies contained the same percentage of raw clays and the same percentage of non-plastic material the drying shrinkage was the same in all cases and was 0.65 per cent.

FLEXUAL OR TRANSVERSE STRENGTH

In all the bodies except No. 1 the increase in firing temperature gave an increase in strength. The decrease in free silica content in bodies 1, 2 and 3 increased the strength at both cone 10 and 13½. The introduction of fused MgO in bodies 4 and 6 tends to give lower values, while Al_2O_3 in body No. 5 has much the same effect. Bodies 4, 5 and 6

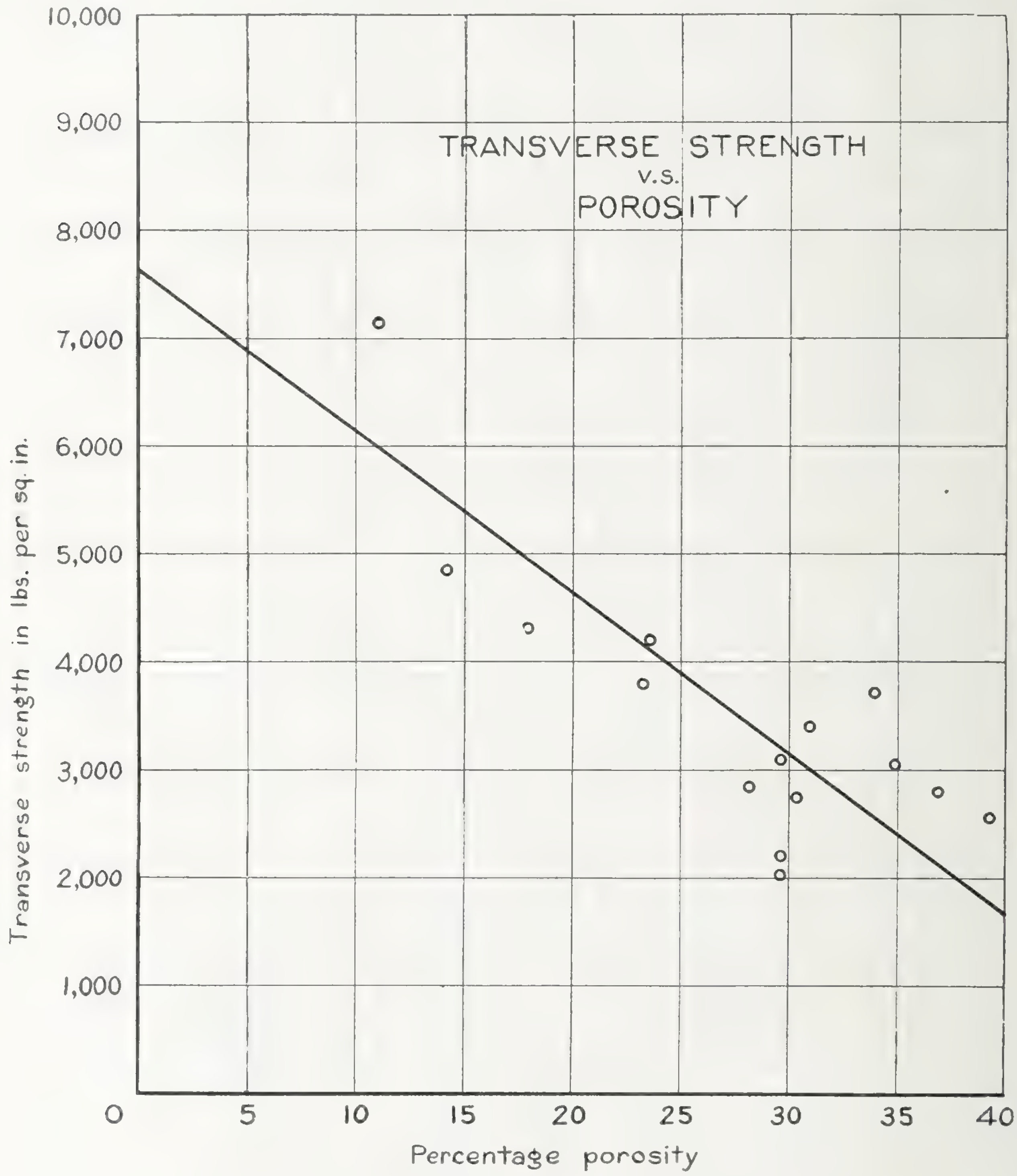


FIG. 5

would contain smaller amounts of the 1345° C. eutectic proportions and would require a higher burning temperature to cause them to approach vitrification. The porosity of these three bodies is 30% or above even at cone 13½. The strength in general depends upon porosity or vitrification (Fig. 5). The amount of vitrification conforms to a normal interpretation of the triaxial diagram as given in Fig. 1.

Apparent Porosity

Porosity determinations check the points already observed in connection with shrinkage and strength. Reduction in free silica (bodies 1, 2 and 3) from the amount usually found in commercial bodies now used in Canada, decreases the porosity obtained at a given temperature. Comparing bodies No. 3 and 7 shows that an increase in talc did not materially decrease the porosity as might have been expected, thus checking the shrinkage data. The same is true with bodies No. 4 and 6. The increase in fused MgO content gave increased porosity and lower shrinkage. Up to cone 13½ (1370° C.) MgO is not a very active flux in these bodies. Comparing bodies No. 1 and 8, having a constant silica content, we find the MgO is not active at cone 10 (1260° C.) but at cone 13½ (1370° C.) body No. 8 has melted. The porosity decreased from 30.4% to 0 with a temperature increase of about 100° C. From the triaxial diagram (Fig. 1) it is noted that body No. 8 is in a lower temperature field than the rest of the bodies and its failure at about the eutectic temperature of 1345° C. is not unexpected. Body No. 5 still shows a high porosity at cone 13½ and it must be considered as being immature. Its properties will probably improve with higher burning temperature, but commercial burning above cone 12 or 13 is avoided if possible.

Resistance to Thermal Shock

From the results given in Table II, only bodies No. 4 and 6 burned to cone 13½ appear to have retained a fair percentage of their original flexual strength. Of the two bodies, No. 6 gave the better results.

The test as carried out was too severe and differences from body to body are not as apparent as they should be. A modification of the test to quench only 10 times and then obtain the percentage loss in strength by the transverse test, would probably give a more satisfactory comparison. Otherwise the test seems to be satisfactory. However, body No. 6 has the lowest coefficient of expansion of the series and the results are therefore consistent. The decrease in silica in bodies 1, 2 and 3 appears to lessen the resistance to thermal shock. This is unexpected and does not agree with the coefficient of expansion data which shows a decrease in expansion for this series. The data here presented is not considered to be conclusive on this point. In all bodies, except Nos. 2 and 7, increase in burning temperature improved the resistance. Comparing bodies No. 3 and 7, talc additions increased the resistance at cone 10 but decreased it at cone 13½. It will be remembered that body No. 7 is quite dense at the higher temperature.

A comparison of bodies 3 and 7 with 4 and 6 indicates that at cone 10 talc gives better bodies but at cone 13½ fused MgO gives increased resistance. The coefficient of expansion data at cone 10 checks this conclusion but is not quite so consistent at cone 13½. Also the cone 13½ MgO bodies are better than the cone 10 talc bodies. Body No. 5 containing Al_2O_3 did not give good results. This body is immature and the increase in temperature from cone 10 to 13½ greatly improved its performance. Body No. 8 gave a fair body at cone 10 and is probably next to body 7 in resistance for this temperature.

TABLE II

THERMAL SHOCK

		Visible Cracking			Failure			
Body No.	Firing Temp. Cone	Minimum No Quenchings	Maximum No Quenchings	Average No Quenchings	Minimum No Quenchings	Maximum No Quenchings	Per cent. Loss of Strength	Ring (when struck with Steel)
1	10	7	24	15	No failure	No failure	100.0 (3)	None
1	13½	16	25 (1)	21	" "	" "	65.4	Fair
2	10	5	17	14	" "	" "	81.0	None
2	13½	5	13	7	10	" "	100.0	None
3	10	5	16	10	15	" "	82.0	None
3	13½	11	25	20	No failure	" "	79.8	Fair
4	10	4	5	5	7	11	100.0	None
4	13½	17	25	23	No failure	No failure	61.4	Fair
5	10	5	6	6	7	" "	100.0	None
5	13½	11	25	14	11	" "	100.0	Fair
6	10	5	6	6	12	21	100.0	None
6	13½	25	25	25	No failure	No failure	58.2	Good
7	10	25	25	25	" "	" "	87.8	Fair
7	13½	9	10	10	19	19 (2)	100.0	None
8	10	12	25	22	No failure	No failure	82.1	None
8	13½	..	..	..	..	..	..	..

Failure means that the bar cracked into two or more pieces during handling.

(1) No visible cracks after 25 quenchings.

(2) Failure probably due to pressure by hand in trying to locate cracks as they were very small and no doubt these specimens (only 2) would easily have stood 25 quenchings without failure under ordinary circumstances.

(3) Broke with pressure of machine therefore 100.0% loss.

COEFFICIENT OF LINEAR THERMAL EXPANSION

The results are given in Table III.

The decrease in free silica in bodies No. 1, 2 and 3 show a corresponding decrease in coefficient of expansion. The introduction of MgO

and Al_2O_3 in bodies No. 4 and 5 increased the values obtained above those for body 3. This is probably due as much to the degree of vitrification as to the composition. All of the bodies showed a definite lowering of the coefficient of expansion with increased temperature from cone 10 to 13½. Comparing bodies 4 and 6 we find the increase in MgO content definitely lowers the coefficient of expansion and that body No. 6 at cone 13½ gave the lowest value and best resistance to thermal shock.

TABLE III

COEFFICIENT OF LINEAR EXPANSION $\times 10^{-8}$
80 to 900° C.

BODY NO.

Firing Temp. Cone	1	2	3	4	5	6	7	8
10	416	406	361	439	454	362	299	387
13½	408	386	329	376	391	213	...	...

% POROSITY VS. RESISTIVITY

Body	% Porosity	Resistivity at 700° C. ($\times 10^3$)
6	39.5	1194
8	30.4	2315
7	23.5	3150

POROSITY AND RESISTIVITY OF BODY NO. 6

Cone Temperature	% Porosity	Resistivity at 700° C. ($\times 10^3$)
Cone 10	39.5	1194
Cone 13½	31.5	9510

Comparing bodies 3 and 7, talc additions also lower the coefficient of expansion but becomes too active as a flux in body 7 at cone 13½ to be of commercial value. Body 8 is intermediate in its value as it was in the test for thermal shock resistance.

The coefficient of expansion of bodies of this type is low because they do not contain alkali which has a factor of 8.5 to 10 (depending on whether the alkali is potassium or sodium) in calculations of the coefficient of expansion of fused silicates as compared to 0.8 for SiO_2, 5.0 for Al_2O_3, and 0.1 for MgO (36). As the values obtained for the expansion coefficient decrease with increased temperature and decreased porosity, we may assume that there is a relation between the amount of fluid phase developed as compared to the unmelted or undissolved

part of the body and the expansion. A calculation of the coefficient of expansion of body No. 6 (the best of this series) using the factors of Winkelman and Shott (36) would give a value of 59×10^{-7}. This might be compared to a calculated value of 110×10^{-7} for pyrex glass based upon the composition as given in this same reference. The actual coefficient of expansion for pyrex glass is given as 32×10^{-7} as

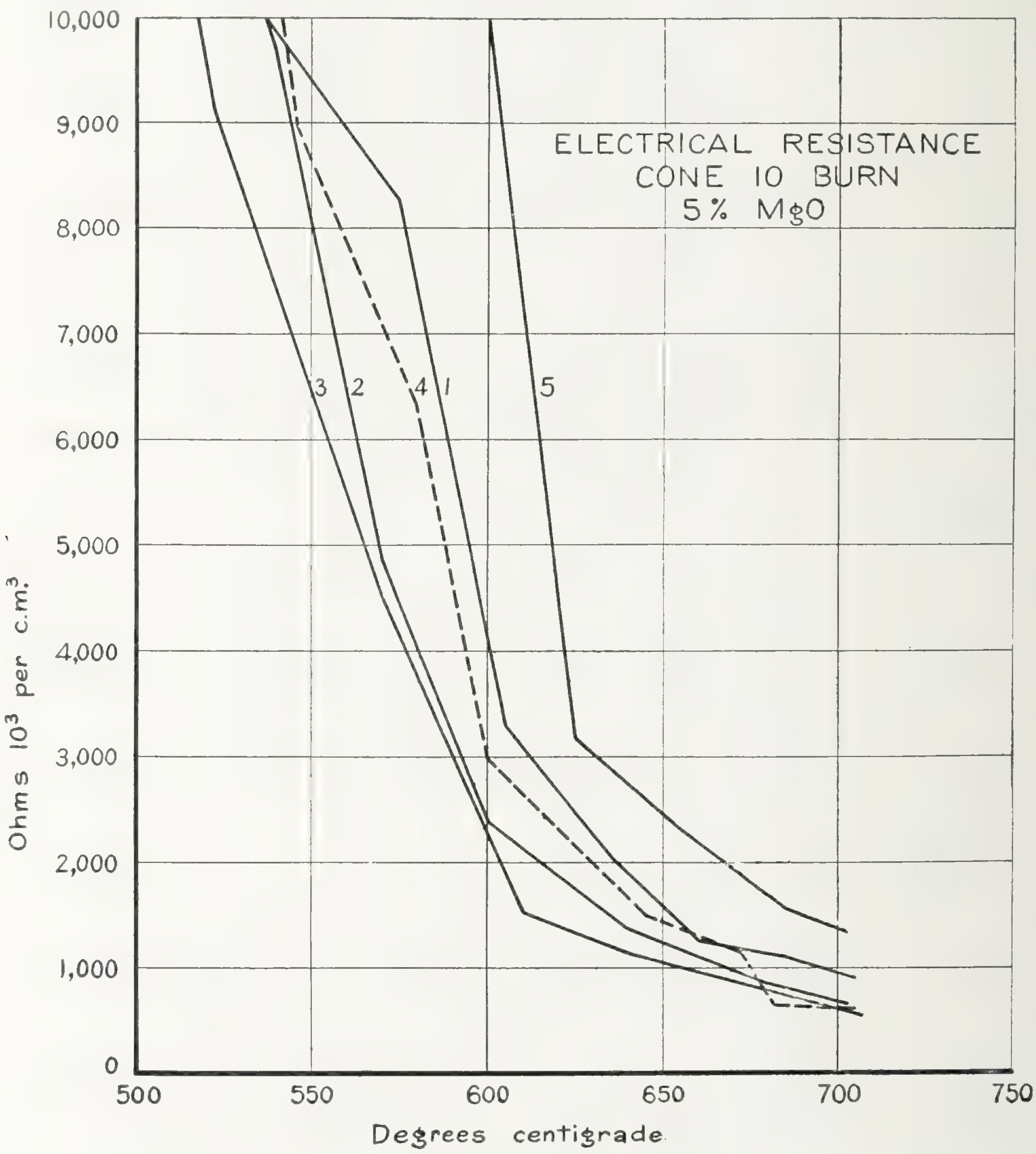


FIG. 6

compared to our value of 21×10^{-7} for body No. 6. Bodies of the type being studied would have very desirable properties if they could be melted and moulded in the fluid state as opaque glass.

ELECTRICAL RESISTIVITY

The results are given in Table No. IV and the cone 10 values are plotted in Figures 6 and 7. In the cone 10 bodies Nos. 1, 2 and 3 with

decreasing silica, resistivity decreases somewhat. Resistivity also decreased with the higher burning temperature (cone 13½).

With body No. 4 burned at cone 10 containing fused MgO and higher alumina the resistivity is higher than bodies 2 and 3 at lower temperatures but not at 700° C. At cone 13½ body No. 4 shows a definite improvement at both low temperatures and at 700° C.

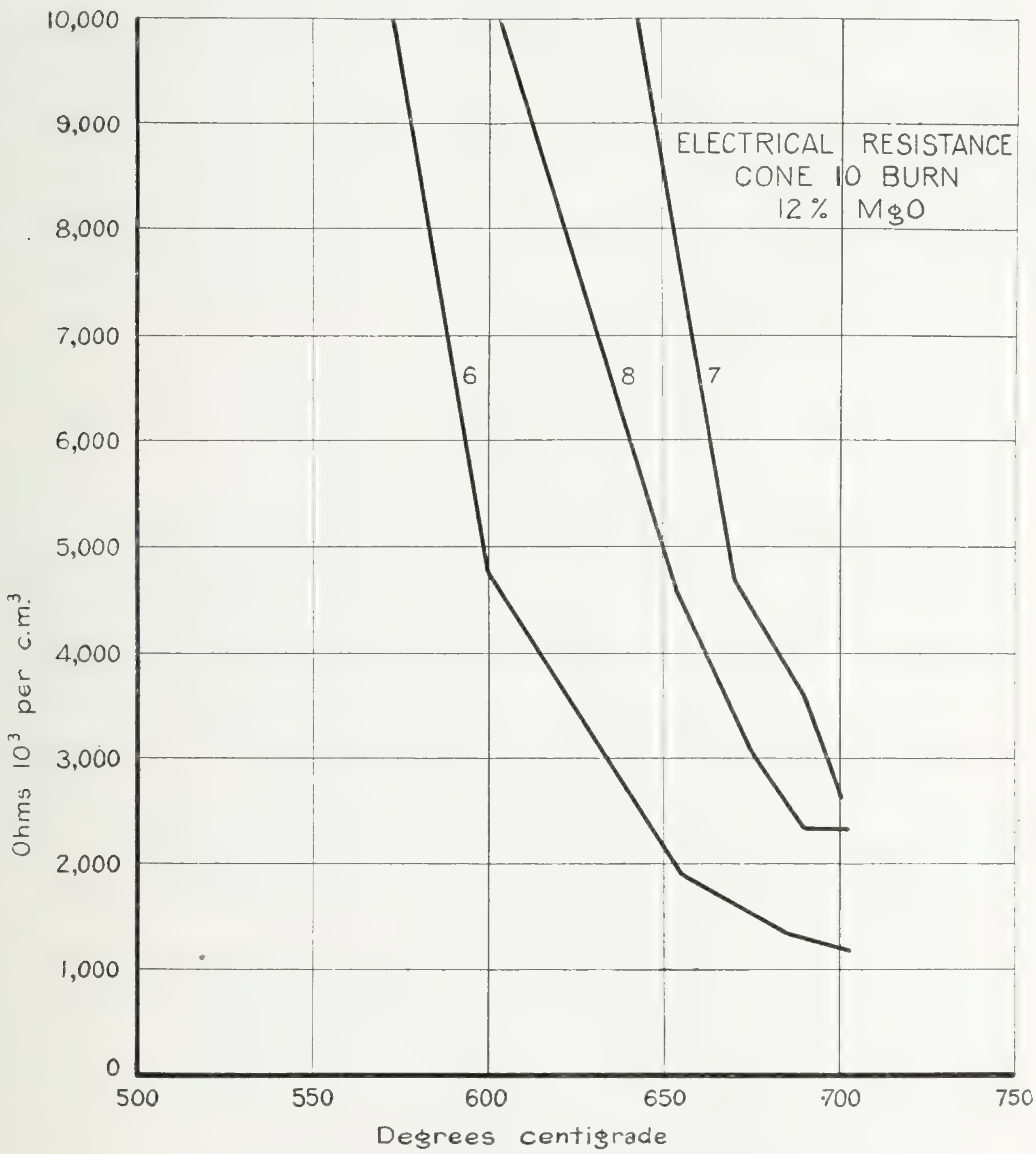


FIG 7

Body No. 5 containing talc and Al_2O_3 shows an improvement over the other bodies containing 5% MgO and at cone 13½ this body was more resistant than at cone 10. Apparently Al_2O_3 has beneficial effects when it is present in amounts above the kaolin ratio.

At cone 10, bodies 6, 7 and 8 containing 12% MgO show an increase in resistivity over the bodies containing only 5% MgO. At cone 10 body No. 7 was best while No. 3 was poorest. They stand in reverse order with the percentage porosity of the bodies as indicated below.

The degree of vitrification is a very important factor in these bodies as well as the composition. Body No. 6 was greatly improved by increased burning temperature and the resistivity at 700° C. was increased as the porosity decreased.

This increase of resistivity with increased vitrification does not hold for the entire field, as is seen in bodies 1, 2 and 3. Bodies 7 and 8 burned to cone 13½ were not tested because of melting.

While in the main the method used for making these determinations was satisfactory, the results could probably be improved by using a more sensitive set-up and using pure silver contacts instead of the alloy used for this work. This would allow a higher final temperature. Of course the use of an *AC* current would avoid polarization. The use of an *AC* current as described by R. M. King (37) should also be satisfactory.

CONCLUSIONS

Considering bodies No. 1 and 2 as being approximately the composition of present commercial bodies, others of this series are definitely better and should give better results. While increased shrinkage was obtained with a decrease in the silica content, this property can be controlled within limits by varying the proportion of raw to calcined clay. Body No. 6 burned to cone 13½ containing 12% fused MgO is doubtless the best. However, body No. 7 burned to cone 10 also containing 12% MgO but using talc as the source of the oxide, is very nearly as good and has a higher flexual strength than body No. 6 at cone 13½.

The commercial burning temperature for electrical refractory porcelain is between cone 10 and 11 so that burning to cone 13½ is not desirable. The introduction of the more expensive fused MgO to replace talc does not seem justified unless commercial failure is encountered with the bodies containing talc.

Body No. 7, however, has one disadvantage, which has been discussed previously. This is its short vitrification range which would necessitate very accurate temperature control during firing.

The $SiO_2-Al_2O_3-MgO$ ternary diagram (Fig. 1) shows that bodies No. 3 and 7 contain only clay and talc. Body No. 3 has a longer vitrification range than body No. 7 and it may be assumed that the nearer the composition approaches that of body No. 3 the longer will be the vitrification range. In approaching body No. 3, however, advantageous properties would be sacrificed.

The bodies containing fused MgO and Al_2O_3 were not carried to a

TABLE IV

Electrical Resistivity 10^3 ohms per cm^3

Body No.	Firing Temp. Cone														
1	10	Temp. °C.	0-445	475	510	540	575	605	635	660	685	705			
		Elec. Resis.	Infin.	High	24650	9875	8250	3300	2030	1235	1098	900			
1	13½	Temp. °C.	0-447	467	497	517	542	557	582	602	617	637	647	687	702
		Elec. Resis.	Infin.	High	9450	9450	4730	3155	2365	1890	1182	945	675	498	379
2	10	Temp. °C.	0-412	457	497	537	572	602	642	682	702				
		Elec. Resis.	Infin.	High	24350	9725	4860	2435	1390	885	649				
2	13½	Temp. °C.	0-493	518	528	558	578	598	613	623	633	648	693	708	
		Elec. Resis.	Infin.	High	22900	9160	9160	4580	3050	1835	1310	916	482	399	
3	10	Temp. °C.	0-447	487	527	572	612	637	687	707					
		Elec. Resis.	Infin.	High	9040	4520	1505	1130	754	565					
3	13½	Temp. °C.	0-504	514	544	564	584	599	619	659	704				
		Elec. Resis.	Infin.	High	8510	8510	2835	1700	1217	774	370				
4	10	Temp. °C.	0-486	496	521	546	576	601	626	646	676	686	706		
		Elec. Resis.	Infin.	High	22450	8980	6405	2995	2245	1498	1122	749	599		
4	13½	Temp. °C.	0-486	526	556	586	616	641	661	686	691	701			
		Elec. Resis.	Infin.	High	14870	8930	2975	2975	1784	1275	993	812			
5	10	Temp. °C.	0-542	572	602	627	657	687	702						
		Elec. Resis.	Infin.	High	9480	3160	2370	1578	1354						
5	13½	Temp. °C.	0-541	561	581	596	616	626	636	651	656	666	691	706	
		Elec. Resis.	Infin.	High	9120	9120	9120	9120	4560	4560	3040	3040	1822	1822	
6	10	Temp. °C.	0-484	514	539	574	599	629	654	684	704				
		Elec. Resis.	Infin.	High	23900	9540	4770	3185	1910	1364	1194				
6	13½	Temp. °C.	0-552	572	597	637	667	697	707						
		Elec. Resis.	Infin.	High	23800	15850	9510	9510	9510						
7	10	Temp. °C.	0-525	565	590	620	645	670	690	700					
		Elec. Resis.	Infin.	High	23650	23650	9460	4730	3638	3150					
7	13½	Temp. °C.	....	....	.....	.....	....	....	....	....					
		Elec. Resis.	....	....	.....	.....	....	....	....	....					
8	10	Temp. °C.	0-501	531	561	591	611	636	656	676	691	706			
		Elec. Resis.	Infin.	High	23150	23150	9250	4625	4625	3084	2315	2315			
8	13½	Temp. °C.	....	....	.....	.....	....	....	....	....	....	....			
		Elec. Resis.	....	....	.....	.....	....	....	....	....	....	....			

high enough temperature to judge their maximum values. These higher temperatures are not commercial at the present time.

The writers wish to acknowledge the help and suggestions of Prof. J. T. Burt-Gerrans and Prof. J. B. Ferguson of the Department of Chemistry, University of Toronto, in carrying out the work on electrical conductivity.

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THE DISTRIBUTION OF FUEL IN MULTI-CYLINDER GASOLINE ENGINES

By E. A. Allcut[1]

Introduction

A considerable amount of work has been done by numerous investigators on the subject of mixture strength and its influence on the operation of internal combustion engines. Some of the experiments described have been performed on single cylinder engines and others on multi-cylinder engines. The former give the best opportunity of studying the principles of combustion and the generation of power in a cylinder, because the number of variables is reduced to a minimum.

Tests on multi-cylinder engines involve a greater number of variables and so cannot be analysed so readily, but are more indicative of current commercial practice, and therefore can be applied more conveniently to actual conditions.

When the fuel is supplied in the gaseous form, so that intimate and uniform mixtures of fuel and air can be obtained, there is no particular reason why the results obtained on single cylinder engines should not be applied to aggregations of cylinders. The same general conditions apply to liquid fuel engines, where the oil supply to each individual cylinder is measured, but this is not so with the gasoline engine where the result finally obtained is the mean of the output of several cylinders, each working on a different mixture strength from the others.

Previous work, notably that of Ricardo[2], showed the indicated mean effective pressure in a single cylinder engine rising to a peak, when the fuel supply was 20 per cent. in excess of the theoretical quantity and a maximum economy at 15 per cent. excess air.

Berry and Kegerreis,[3] using a multi-cylinder engine, showed that the range of mixtures over which approximately full power could be obtained was quite wide and that this was not greatly affected either by changes of speed or load.

The efficiency curves, however, came to a sharp peak with a slightly weak mixture at full load, moving over to the rich side at light loads.

Tests made by Jacklin[4], showed little change in indicated mean

[1]Professor of Mechanical Engineering.

[2]*High Speed Internal Combustion Engines*, p. 54 (1931 edition).

[3]Bulletin No. 5 Purdue University, *The Carburation of Gasoline*.

[4]*Balancing Power Output in Multi-Cylinder Engines*, S.A.E. Journal, November 1929.

effective pressure for changes of air/fuel ratio from 8 to 15, on a six cylinder engine.

An investigation by the author[5], gave brake mean effective pressures that were nearly constant, over a wide range of mixture strengths at different speeds (Fig. 1).

FIG. 1.

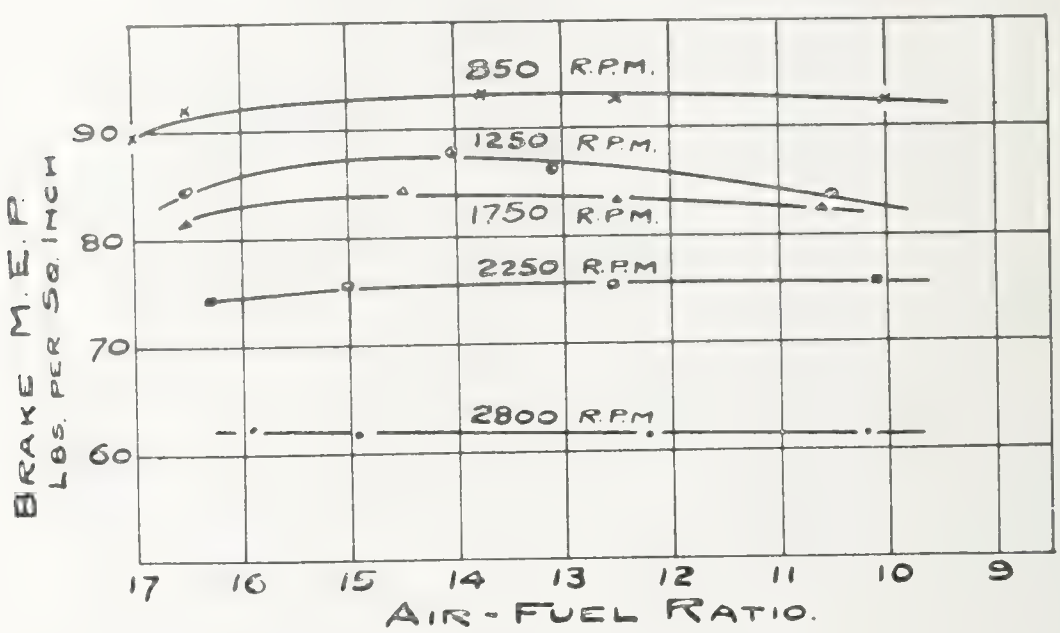

Fig. 1. Variation of brake M.E.P. with mixture strength in author's tests (1928) on six cylinder engine.

Janeway[6] states that the leanest air/fuel ratio for maximum power appears to be "inherent for conventional type engines and independent of combustion chamber variables", so that the differences noted above may fairly be attributed to lack of uniformity in the fuel supply to different cylinders.

The characteristics desirable in the fuel-air mixture leaving the carburettor, according to Taub[7] are:

1. Atomisation or division of fuel into small drops.
2. Homogeneity or good mixing.
3. Avoidance of precipitation or condensing in manifold.
4. Proper direction and division of air flow.
5. Correct application of heat as regards (*a*) quantity, (*b*) temperature.

It is unnecessary here to discuss these factors, as this has been done adequately elsewhere, but they serve to indicate the difficulty of distributing a wet mixture evenly, among the various cylinders. The inertia of fuel drops, which makes them harder to start and stop than is the case with the air which should accompany them, changes of

[5]Engineering Journal, November 1928.
[6]S.A.E. Journal, April 1930, p. 464.
[7]S.A.E. Journal, April 1930, p. 454.

pressure and velocity due to irregularity of impulses, sudden bends or changes of section and reversals of flow are all important factors in distribution.[8] The firing order, which determines the direction and sequence of the reversals of flow, evidently affects the distribution of fuel and so probably influences the form of the power curve.

Many of the tests cited, including those herein described, were made on the basis of brake mean effective pressure only and it is therefore necessary to enquire in what manner the mechanical efficiency varies with mixture strength. Little information is available on this point on account of the difficulty of obtaining reliable high speed indicator diagrams from a number of cylinders.

Berry and Kegerreis[9] obtained the indicated horse power by motoring the engine round and adding to the brake horse power the friction horse power thus obtained. Two sets of results at approximately full load are given in Fig. 2, showing a change of mechanical efficiency of

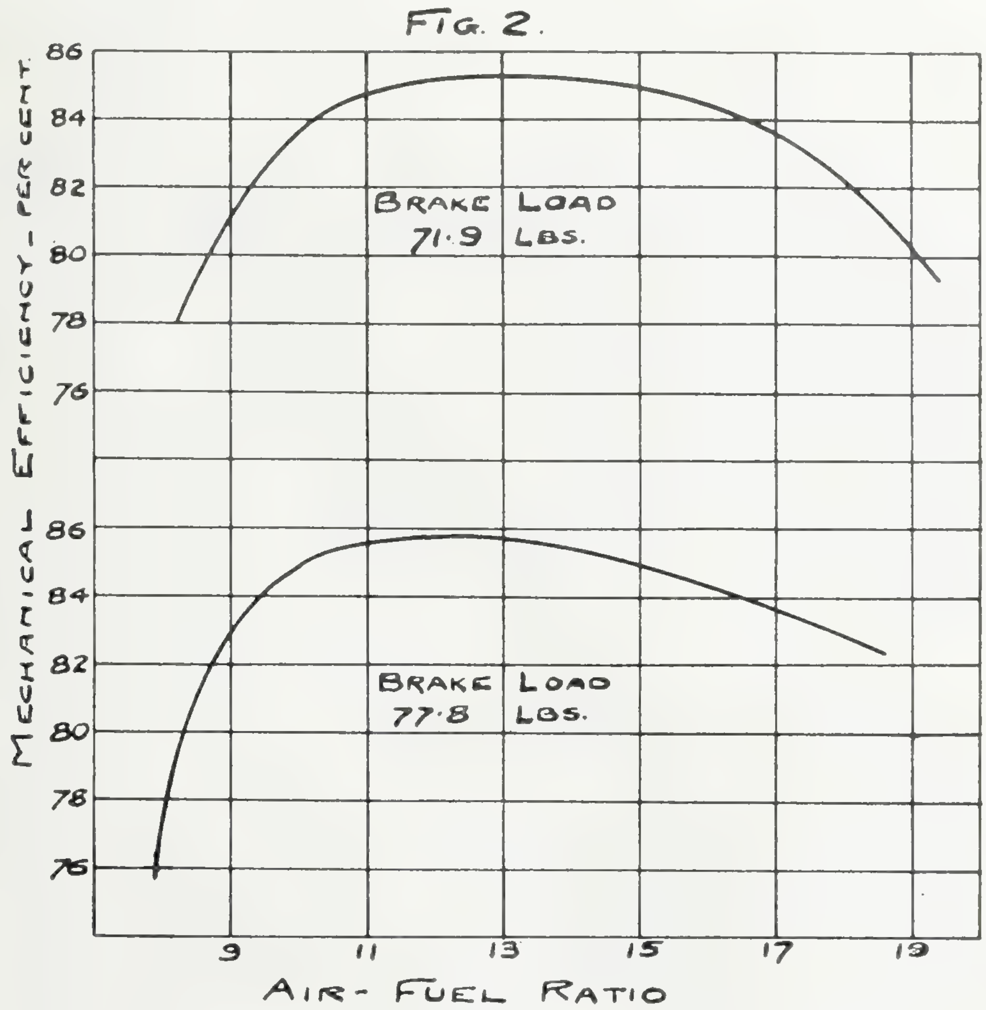

Fig. 2. Mechanical efficiencies as obtained by Berry and Kegerreis.

from 82 to 85 per cent. and 83 to 86 per cent. between the limits of 9 and 16 for the air/fuel ratio. The results obtained by H. M. Jacklin[10] are

[8]See Zucrow—S.A.E. Journal, October 1929, for a full discussion of these factors.

[9]*Loc. cit.*

[10]S.A.E. Journal, March 1928.

shown in Fig. 3. These tests were performed on a small single cylinder experimental engine arranged for multiple ignition. The curves in Fig. 3 are plotted from the figures obtained with a single spark plug, at different angles of spark advance. These two sets of curves are inclined in opposite directions and their mean, with normal spark advance, would be practically constant. For these reasons, it is assumed that the curve of brake mean effective pressure should vary approximately in the same manner as that for indicated mean effective pressure, as the mixture changes.

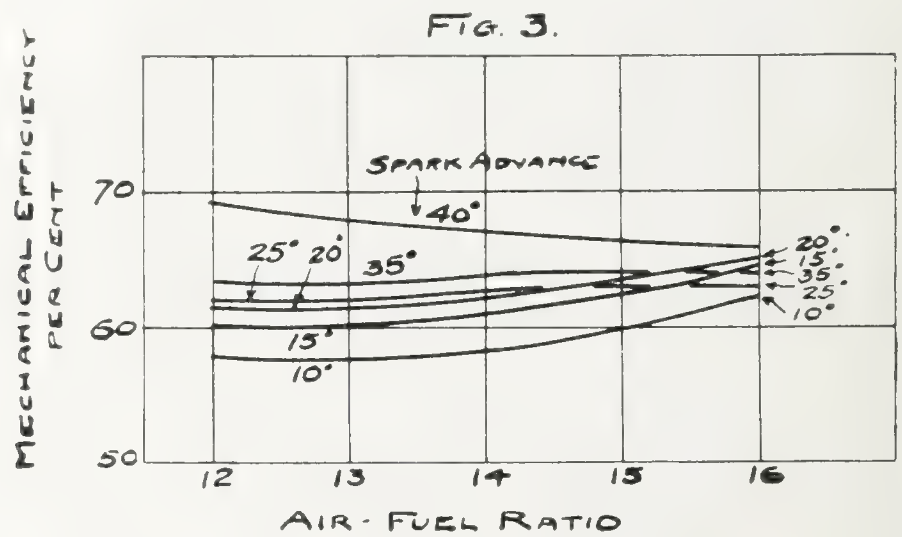

Fig. 3. Mechanical efficiencies with different angles of spark advance according to Jacklin.

The quantities of air and fuel actually used by the engine in each test are fairly easy to measure by orifice box and weighing machine, respectively, but the quantities consumed by each cylinder cannot be determined with the same facility. A comparison of the methods of obtaining the air/fuel ratio from the analyses of exhaust gases and the corresponding figures obtained by direct measurement, is given by Best,[11] but his conclusions are open to the objection that, with the multi-cylinder engine used, it is not reasonable to expect that the average figures obtained will agree as closely as those from a single cylinder engine. Petze, in the discussion of this paper, states that there is a difference of 1½ to 3 per cent. between the calculated and measured ratios for a single cylinder engine, the orifice method giving the higher figure. He concludes that the two methods give results which check within the limits of experimental error.

Formulae for calculating the mixture ratio from the exhaust gas analysis are given by Lockwood[12] and by O. C. Bridgman.[13] After examining the various factors indicated above, it was considered that

[11]S.A.E. Journal, November 1929, p. 532.
[12]S.A.E. Journal, November 1927.
[13]S.A.E. Journal, September 1928.

analyses of the exhaust gases taken from individual cylinders, gave a fairly close indication of the air/fuel ratio existing in those cylinders. If, therefore, there is a considerable variation in mixture strength in the various cylinders, it is evident that the power or brake mean effective pressure curves can have any form, depending on manifold conditions, and so far from having a maximum point with rich mixtures, may actually have a peak anywhere in the mixture range, or may be constant. In the following work, therefore, the form of the power curve is calculated, that would be anticipated from the mixture strengths existing in each cylinder.

DESCRIPTION OF APPARATUS

Four engines were available, for the purpose of this investigation, including one with four cylinders, two with six and one with eight. The four cylinder engine was of the tractor type, forming part of the existing laboratory equipment of the University of Toronto and was coupled to a Sprague electric dynamometer. The other engines were loaned

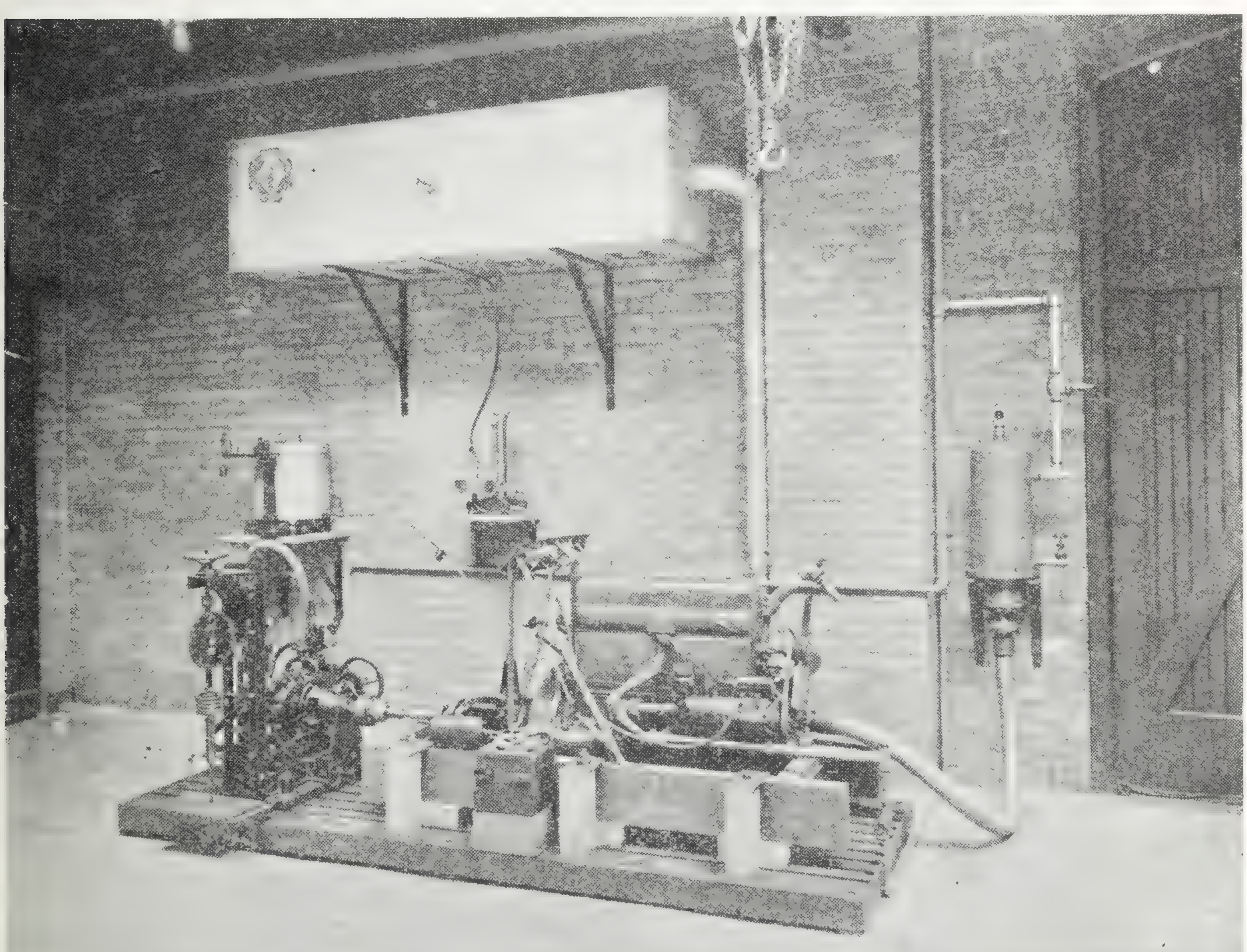

Fig. 4. Test layout at University of Toronto including orifice box, dynamometer and fuel tank.

by the manufacturers and were of the ordinary commercial automobile type. They, in turn, were coupled to a Heenan and Froude dynamometer, from the readings of which the brake horse power was calculated. All speeds were observed by means of a speed counter and stop watch. The fuel tank was placed on a weighing machine and readings of fuel consumption were made at frequent intervals during each test by observing the rise of the weighing machine steelyard. The times were taken by stop watch and the tests were of 30 to 40 minutes duration. The air consumption was calculated from the observed depressions in a large box, provided with orifices of various sizes. The depression in this box was measured with an accuracy of 1/1000 inch of water or better, by means of the differential gauge shown on the photograph (Fig. 4) below the orifice box and further illustrated by Fig. 5. The cooling water losses in each test, were obtained by weighing the quantity of cooling water used and its inlet and outlet temperatures. Readings of temperatures, pressures, speeds, etc., were taken every three or five minutes in each test. The exhaust temperatures were observed by

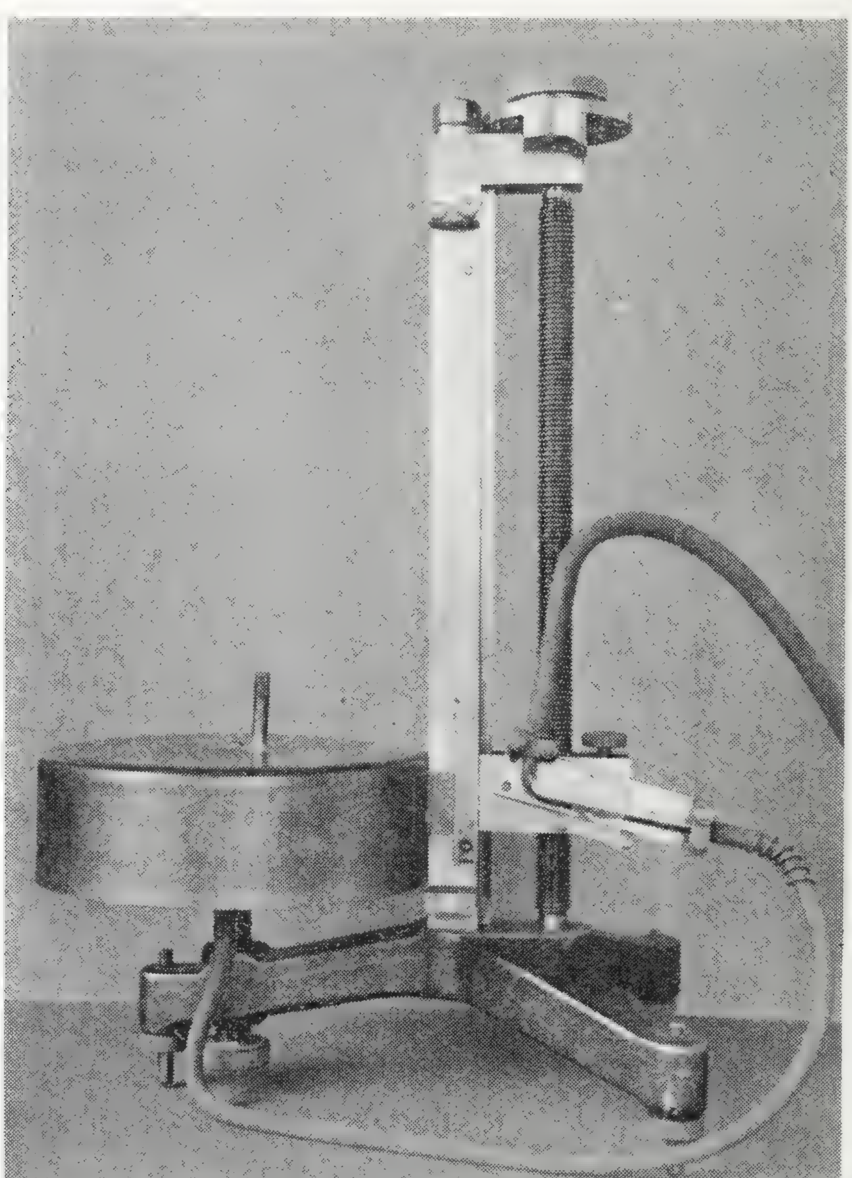

Fig. 5. Differential gauge used for indicating pressure differences in orifice box.

means of a calibrated thermocouple. Samples of exhaust gas from each cylinder and also from the exhaust pipe, were taken in all tests and were analysed by means of an Orsat apparatus. In the case of the eight cylinder engine, however, the intermediate cylinders were connected in pairs to the exhaust manifold so that, apart from cylinders 1 and 8 it was impracticable to get separate samples. The curves relating to this engine therefore have the analyses for pairs of cylinders indicated by single points. It was impossible also, to connect the air box to the four cylinder engine, so that the air/fuel ratios for this engine are approximate only and are included for comparative purposes. The analyses from individual cylinders, however, show how the mixture was distributed. No test was started until the engine had been warmed up to its final temperature and the regularity of fuel consumption and constancy of the exhaust and cooling water temperatures, were taken as indications of constant conditions during the tests.

The characteristics of the engines were as follows:

Engine	A	B	C	D
Number of cylinders	6	6	8	4
Bore—inches	3	3 7/16	2 3/4	5
Stroke—inches	4 1/2	4 5/8	4	6 1/2
Valves	Over head	Over head	Over head	Side
Firing order	142635	142635	16258374	1342
Compression ratio	......	4.5	5.27	3.6

The form of manifold for each engine is shown in Figs. 6, 7 and 8.

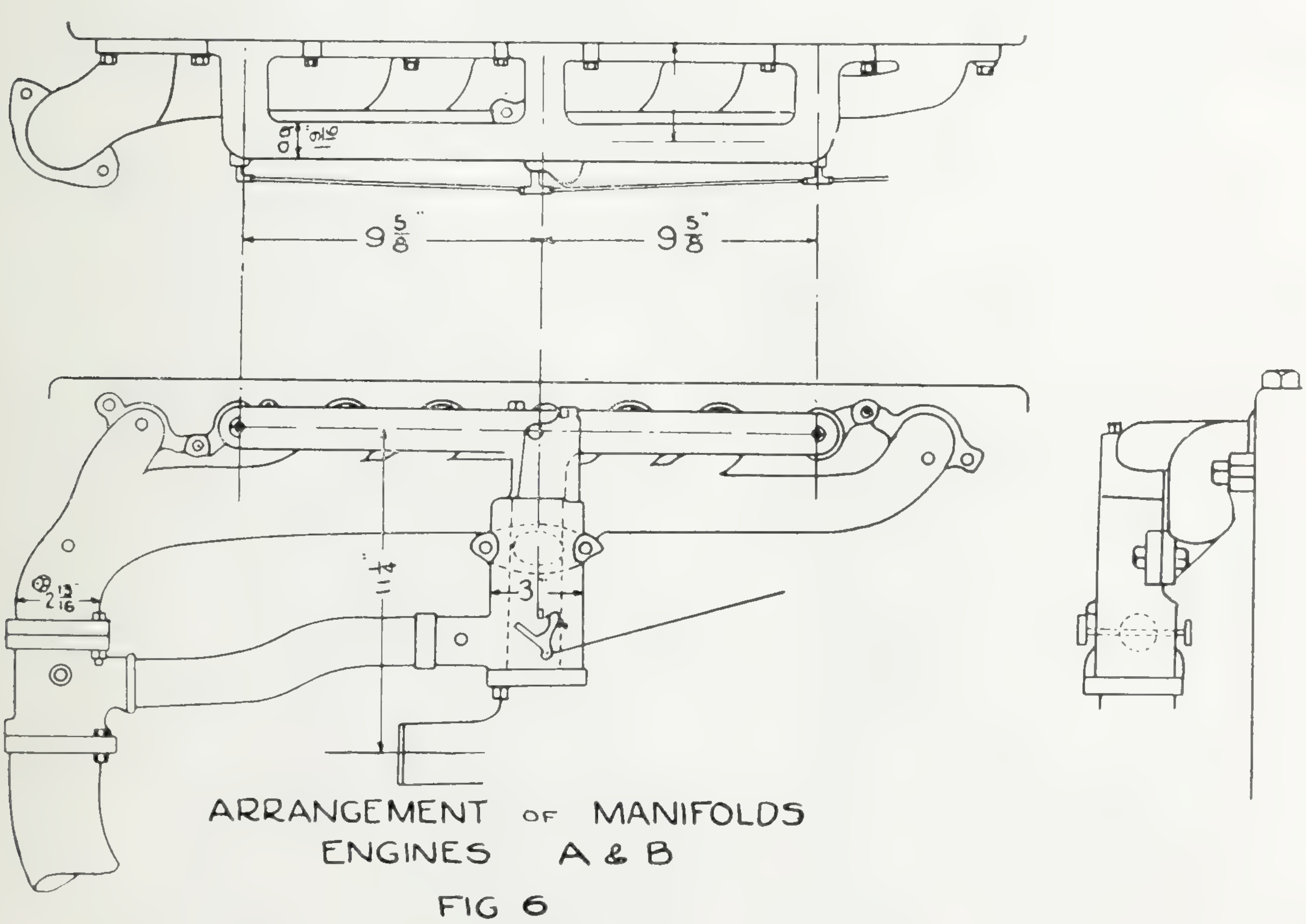

Fig. 6. Inlet and exhaust manifolds of Engines "A" and "B".

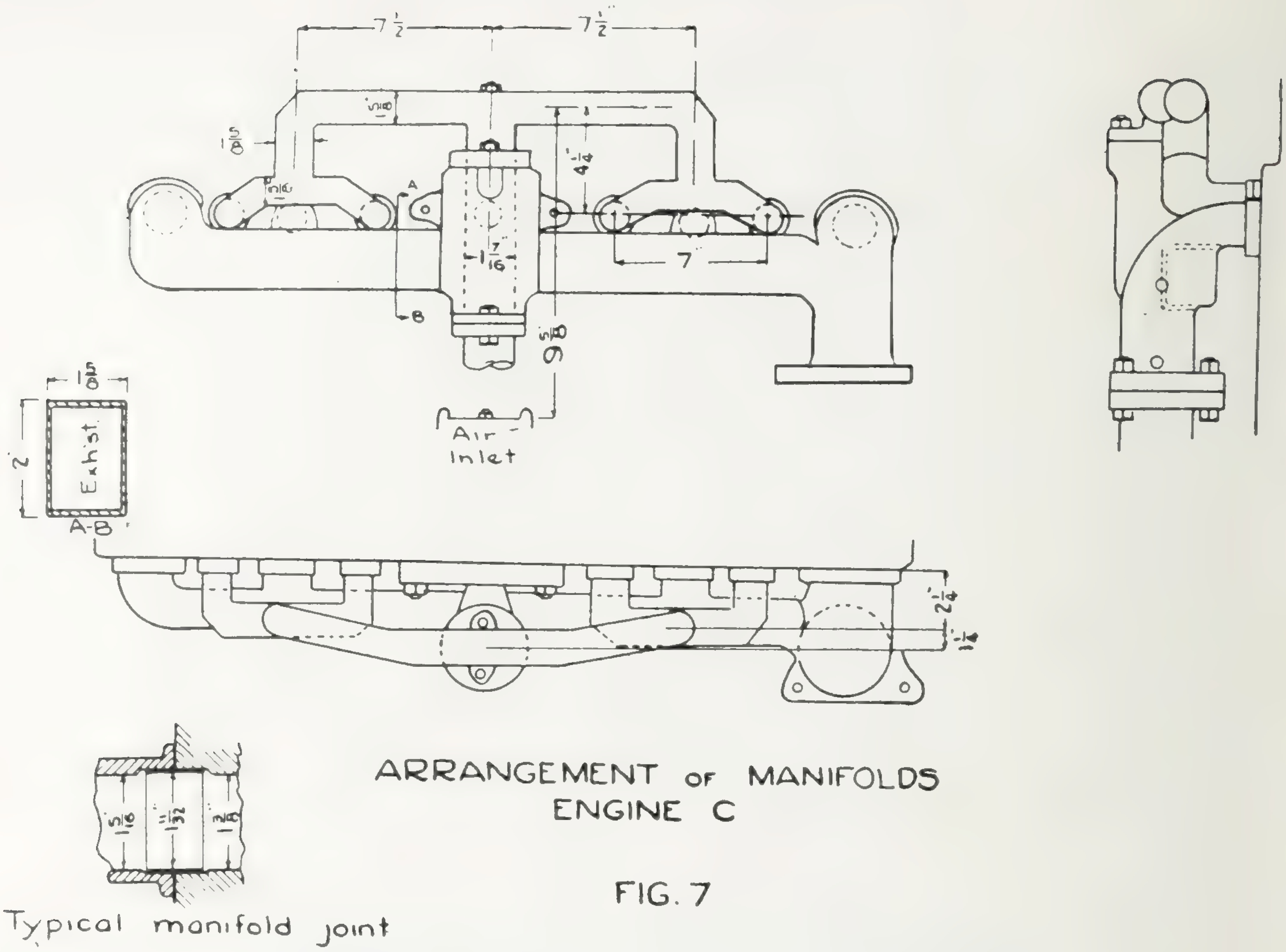

Fig. 7. Inlet and exhaust manifolds of Engine "C".

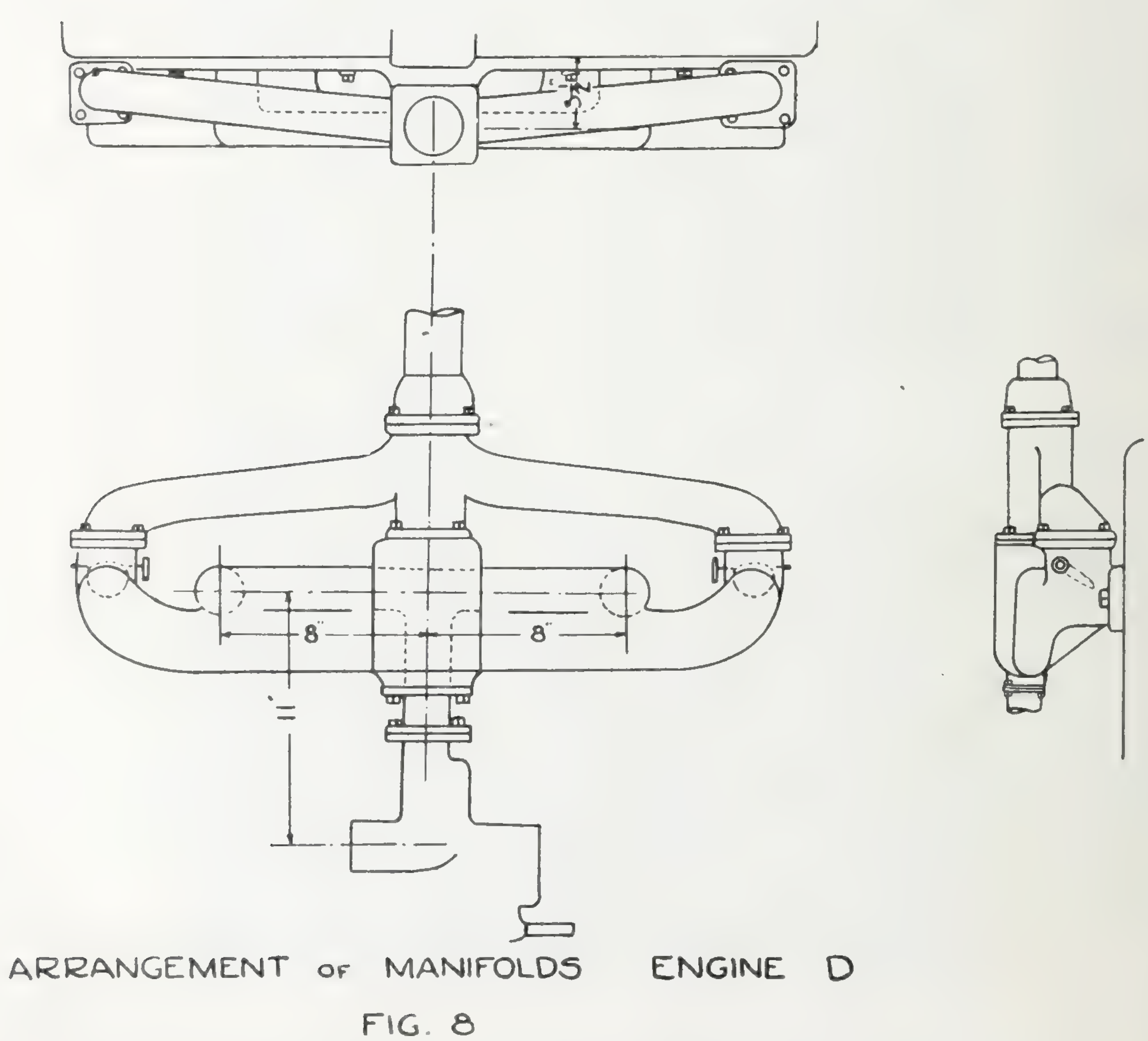

Fig. 8. Inlet and exhaust manifolds of Engine "D".

Test Procedure

(*a*) *Engine "A"*—Two series of tests were made at approximately 800 and 1200 revolutions per minute respectively. The throttle was fully open and the spark advanced to give the best power in each test, but the carburettor was adjusted between each pair of tests to vary the mixture strength. Thus, tests 1 and 2 were made with the same carburettor setting, as also were tests 3 and 4, 5 and 6, 7 and 8. It will be noted from Table 1 that the mixture becomes weaker as the speed is increased.

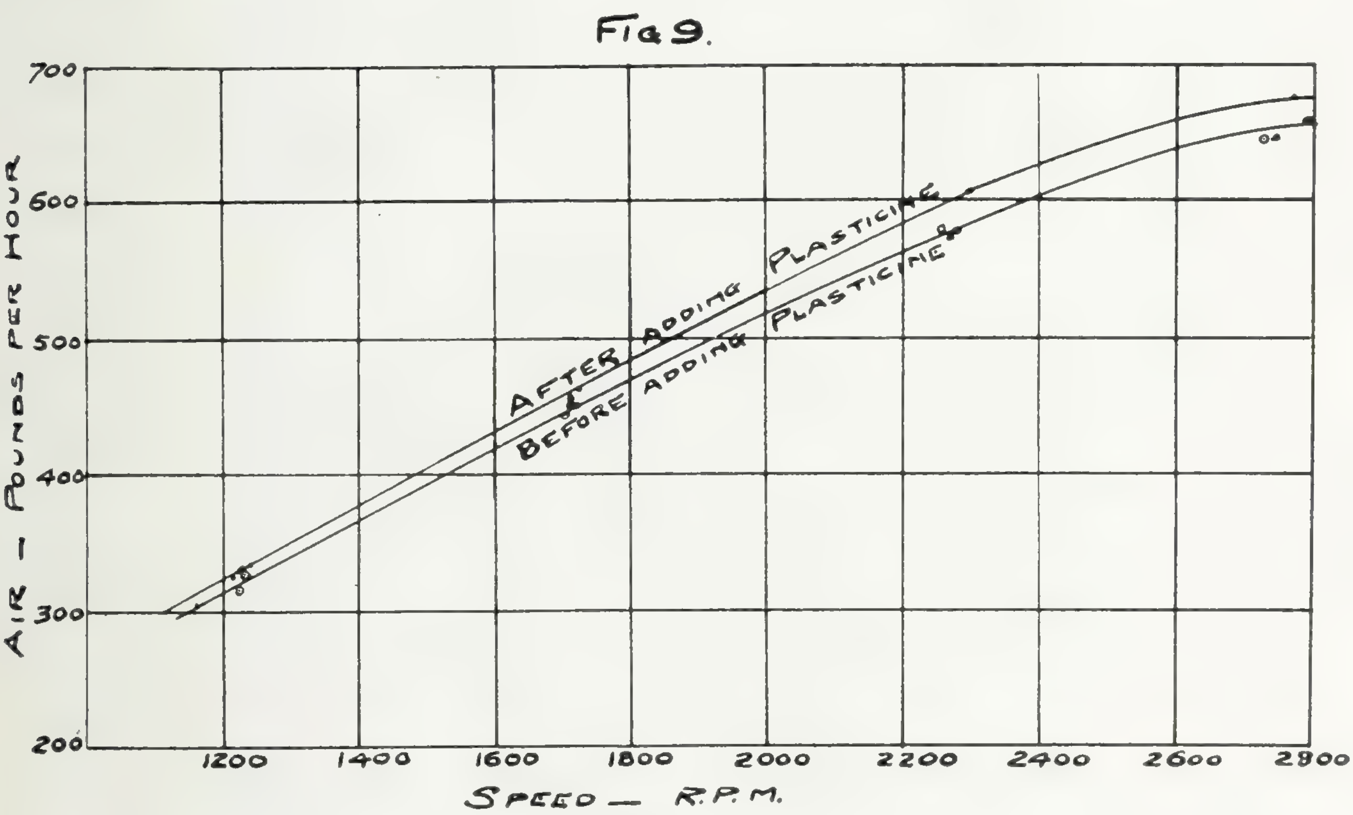

Fig. 9. Air consumption curves before and after sealing orifice box.

(*b*) *Engine "B"*—The procedure in this case was to make a series of tests, indicated respectively as "BA", "BB", "BC", "BD", and "BE" in Tables 3 to 8, keeping the same carburettor setting and varying the speed for each series and here the same tendency is observed as in the case of Engine "A". The number and size of orifices used in the air box were varied at different speeds, to avoid excessive pressure depressions in the box. These are shown in Tables 3 to 9 under the heading "Air Gauge", and are expressed in inches of water below atmospheric pressure. Some leakage into the box was discovered and therefore the joints were covered with plasticine, the extent of the correction being shown in Fig. 9. The figures given in all tables are corrected for this, where necessary.

(*c*) *Engine "C"*—Two series of tests were made, as in the case of Engine "A", at 1200 and 1800 revolutions per minute respectively. It was found impossible to run at higher speeds, on account of spark plug trouble, but in other respects, the same methods were employed

as in the previous engines. This engine was equipped with a "Schebler" carburettor, but further tests were made with a "Solex" carburettor, to determine whether the distribution was affected by the kind of carburettor used. Also, an attempt was made (Table 10, Test M14) to run with four carburettors, connected to adjacent pairs of cylinders. The running, however, was not steady under these conditions and no further test was made with multiple carburettors.

(*d*) *Engine "D"*—The speed was kept constant in this case, by means of a governor connected to the throttle, variations of speed being obtained by changing the tension of the governor spring. The engine was loaded to its maximum torque in each test and was run at its highest speed. No orifice box was available in this instance, so that the mixture strengths given in Table 12 are approximate only, being deduced from the exhaust gas analyses.

TEST RESULTS

The results obtained are given in Tables 1 to 13 and are illustrated by curves in Figures 10 to 32. The fuel used in all tests was gasoline from the same tank, having a higher calorific value of 19,000 B.T.U. per pound and a specific gravity of .783. The fractional distillation curve for this fuel is given in Bulletin No. 8, Section 10 (Fuel "A"), published by the School of Engineering Research, University of Toronto. The thermal efficiencies and volumetric figures for fuel consumption are calculated from these average figures. The air temperatures given in the tables, are those near the engine, so that they represent approximately the temperatures of the air entering the carburettor. The relative humidity of the air in the laboratory and its barometric pressure are also stated. The exhaust and radiation losses were obtained by difference and were not measured separately.

(*a*) *Volumetric efficiency*—Fig. 10 shows in what way and to what extent the volumetric efficiency falls as the speed increases, in the case

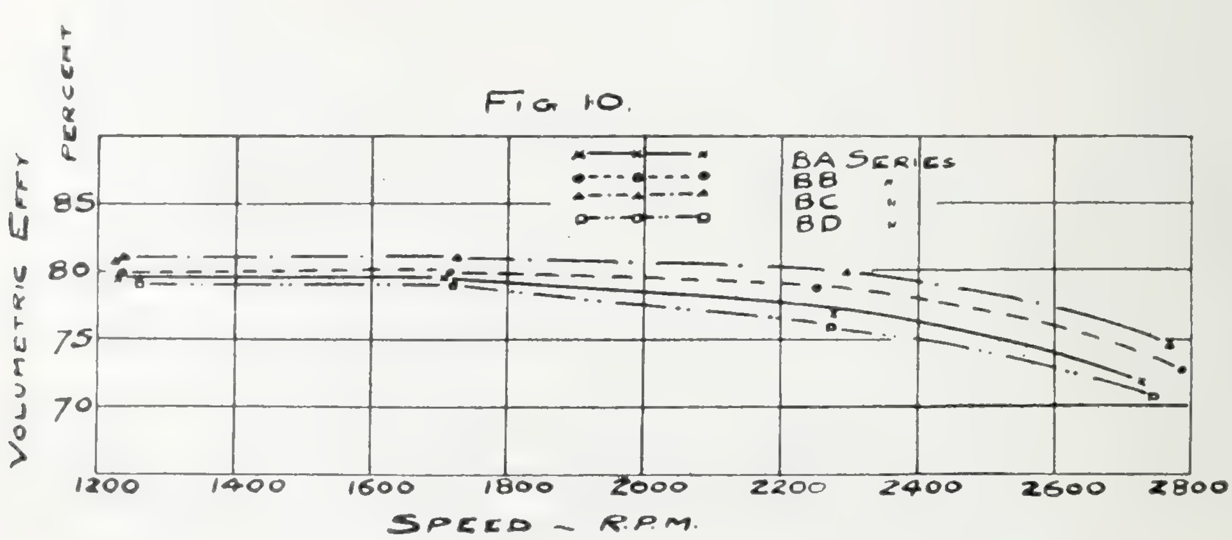

Fig. 10. Volumetric efficiencies of engine "B" at different speeds.

of Engine "B". The efficiencies of engines "A", "B", and "C" are plotted on a basis of air-fuel ratio (that is, pounds of air used per pound of fuel supplied to the engine) in Fig. 11. The points for the two lower speeds in engines "A" and "B" follow approximately the same curve, showing that this change of speed has little influence on the volumetric efficiency. All of the curves for these two engines, however, show a distinct fall of volumetric efficiency as the mixture increases in richness, in spite of the fact that the temperature of the exhaust gases falls rapidly within this range of mixture strength.

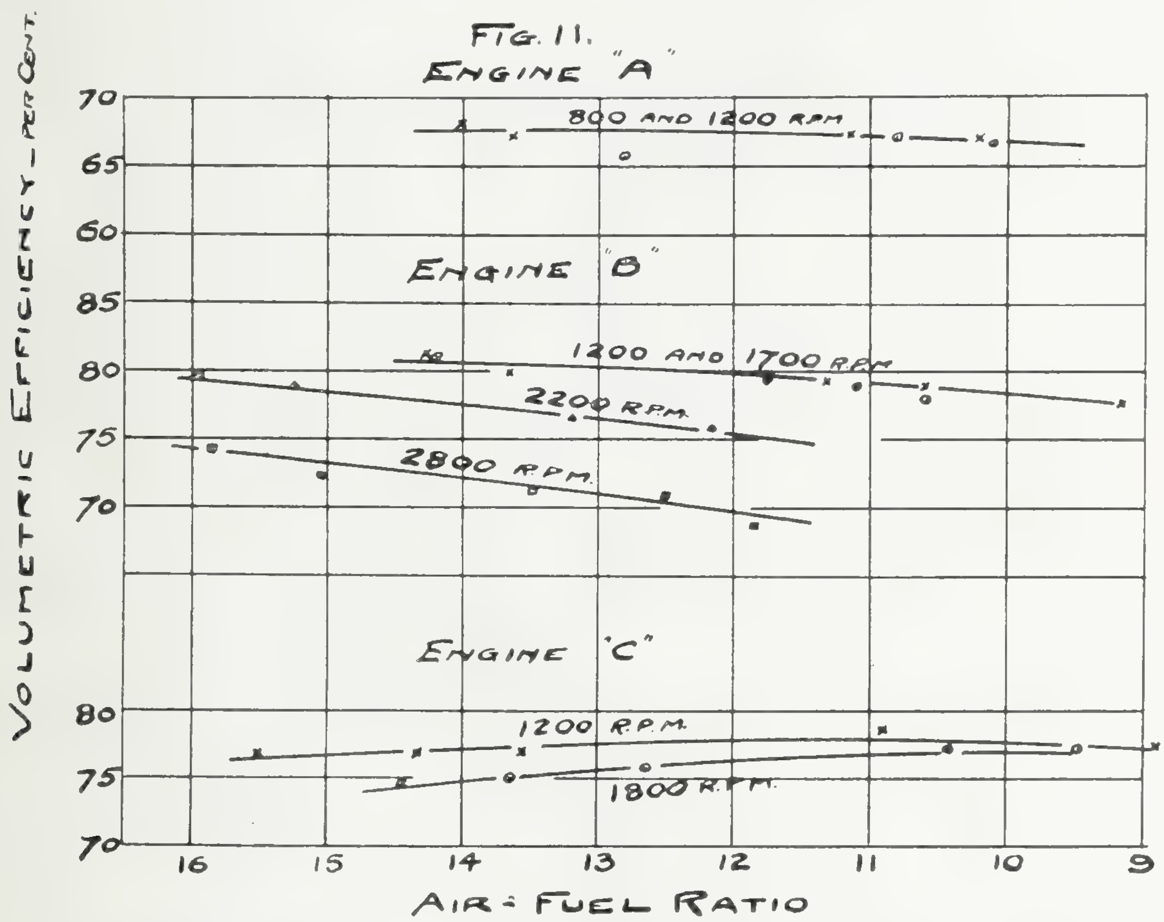

Fig. 11. Volumetric efficiencies of engine "A", "B" and "C" at different mixture strengths.

Engine "C" on the other hand, shows a definite and progressive increase of volumetric efficiency as the mixture becomes richer. Variations of pressure in the air box do not appear to be sufficient to account for this discrepancy.

(*b*) *Exhaust temperature*—The manner in which the exhaust temperature changes in engines "A", "B" and "C" for variable mixture strengths, is shown in Figs. 12 to 14. In all tests, there is a definite fall of temperature with increasing richness and each curve appears to reach a maximum value at about 15 pounds of air per pound of fuel. This is approximately the amount of air required theoretically to burn completely all the combustibles in the fuel and is generally called the chemically correct mixture. The curves for engine "B" (Fig. 13) indicate a tendency for the peak to move toward the rich side as the speed decreases.

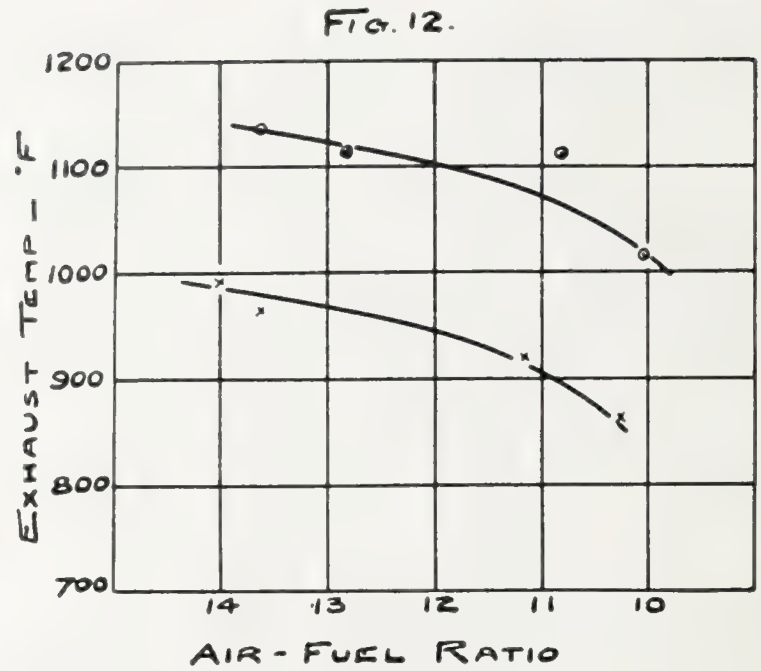

Fig. 12. Exhaust temperatures—Engine "A".

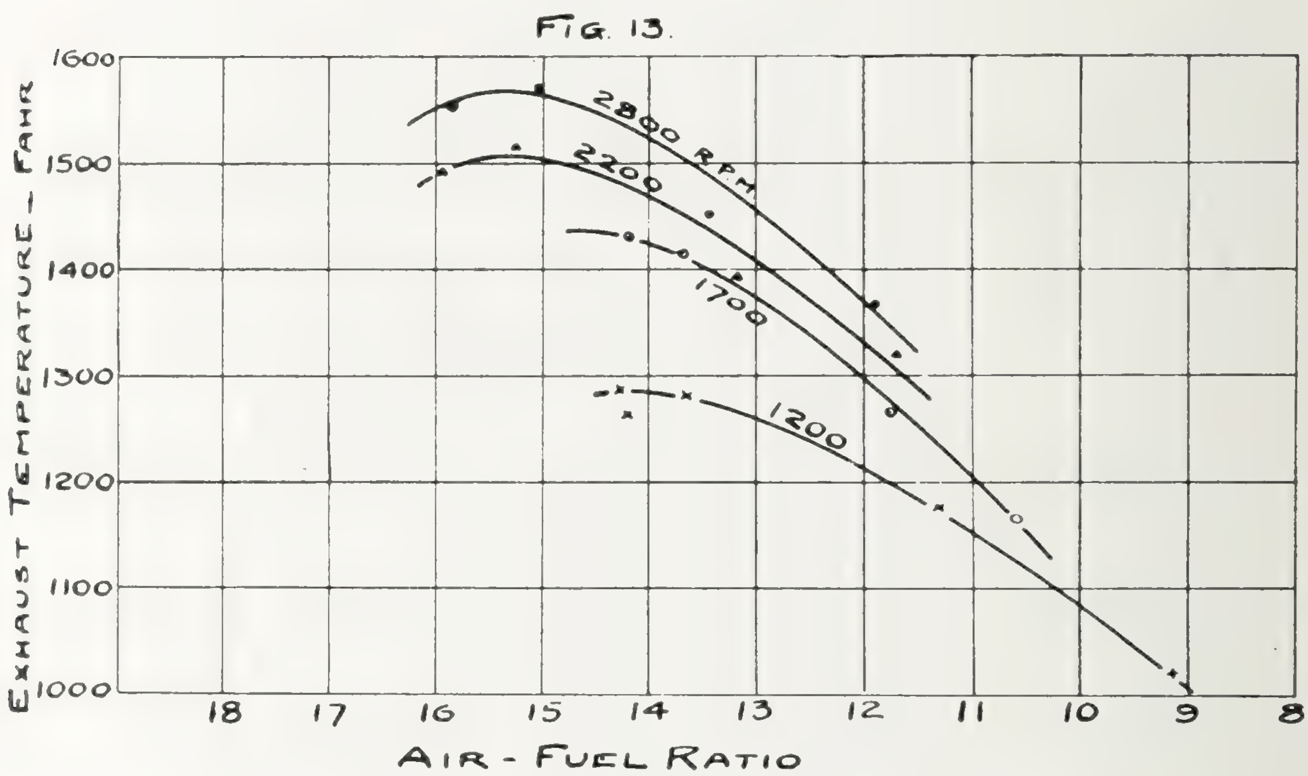

Fig. 13. Exhaust temperatures—Engine "B".

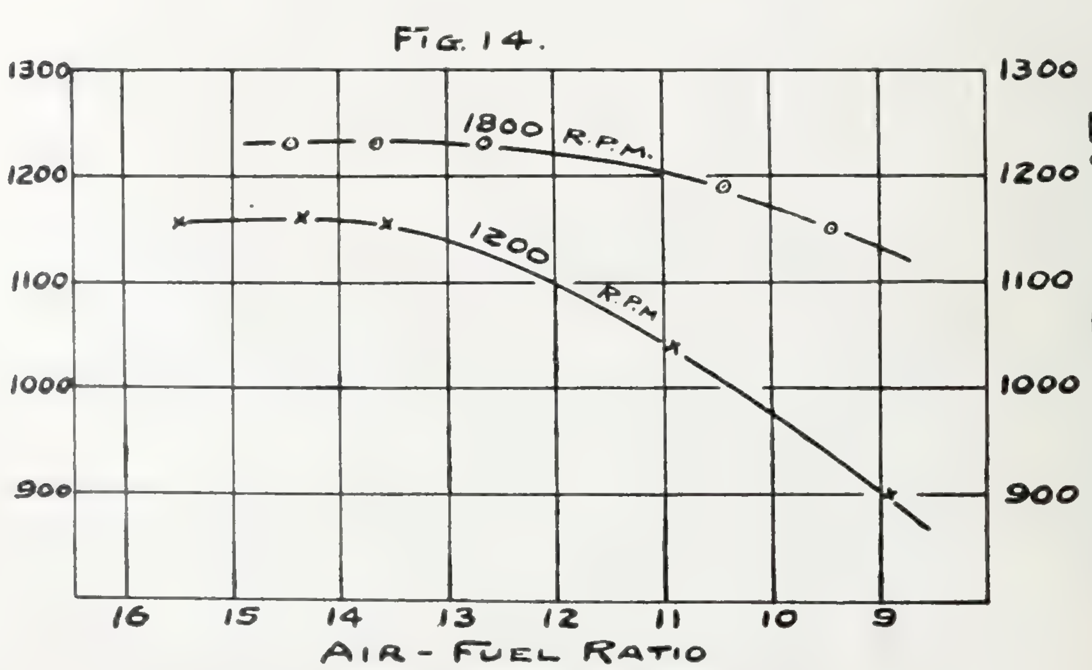

Fig. 14. Exhaust temperatures—Engine "C".

(*c*) *Heat carried away by the Cooling Water*—In the first place, the cooling water loss per unit of power developed, was plotted against mixture strength, but no definite relationship was obtained. When this loss was expressed as a percentage of the heat in the fuel supplied, however, the results fell into line and are shown in Figs. 15 to 18. From these, it is evident that the percentage of the heat in the fuel supplied that flows through the cylinder walls, falls rapidly as the mixture becomes richer and again has its maximum value at about the chemically correct mixture of 15 to 1. The actual proportion lost in this way becomes less as the speed increases, up to a certain point. Fig. 16 seems to indicate that there is a limit to this tendency, as the curves for 2200 and 2700 revolutions per minute are close together and cross each other.

(*d*) *Distribution of Mixture to the Various Cylinders*—This is indicated by the analyses of the samples of exhaust gas taken from each cylinder, and is illustrated by Figs. 19 to 26. In some cases, both the percentages of carbon dioxide and of carbon monoxide obtained from each cylinder are plotted, but in most instances this was not necessary, as the one is generally inverse of the other. The arrangement of the exhaust manifold in Engine "C" made it impossible to take separate samples from the intermediate cylinders 2 to 7 and therefore these were taken in pairs, as stated above. The cylinders were numbered from the fan end and toward the fly wheel end of each engine. Engine "A" (Fig. 19 and Table 2) shows a definite tendency for rich mixtures to collect in cylinders 2 and 5, this irregularity being more marked at the lower speed. As the mixture becomes weaker (c.f. Tests 2, 4, 6, 8) the CO_2 output of the various cylinders tends to become more even. It will be noted that cylinders 2 and 5 are symmetrical in the firing order (142635) and this may have some influence on the result, but curiously enough, Engine "B" which has a similar manifold and the same firing order does not show the same tendency.

The analyses for Engine "B" (Figs. 20 to 23 and Table 8) generally show that rich mixtures are supplied to the two end cylinders 1 and 6. This is probably due to the greater inertia of the larger fuel drops, which tend to collect at the ends of the manifold. Here, again, the CO_2 curves tend to straighten out as the mixture becomes weaker and also as the speed increases. The reason for the occasional discrepancies in cylinder No. 6 is not apparent as the analyses were repeated and found to be correct.

The analyses from Engine "C" show the same tendencies, but to a greater extent and here there are no exceptions, as the curves are all consistent. The high percentages of carbon monoxide obtained from some cylinders, with rich mixtures, are well shown in Fig. 24 and Table 11. It is probable that the inclination of the throttle valve accounts for the

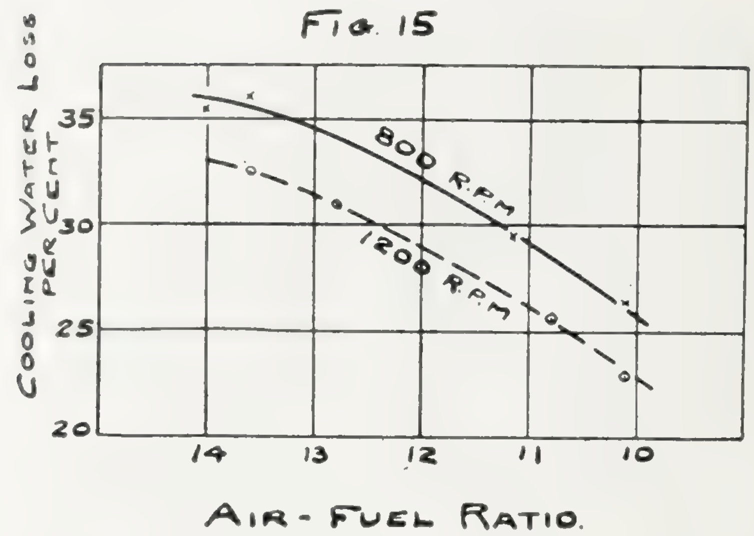

Fig. 15. Loss of heat to cooling water, Engine "A"—expressed as a percentage of the heat in the fuel.

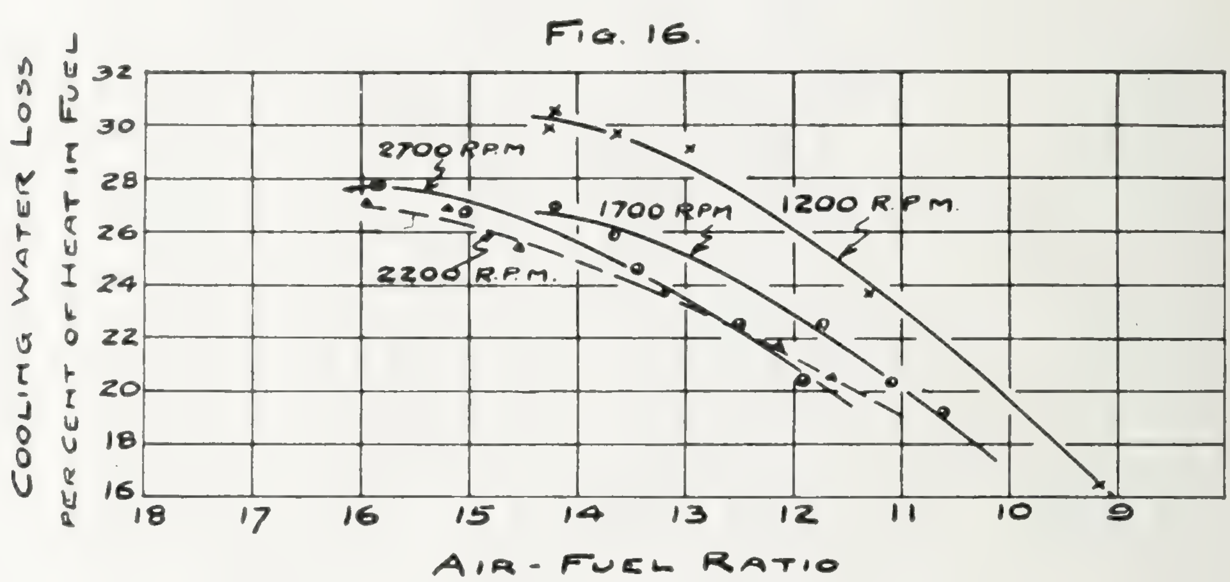

Fig. 16. Cooling water loss—Engine "B".

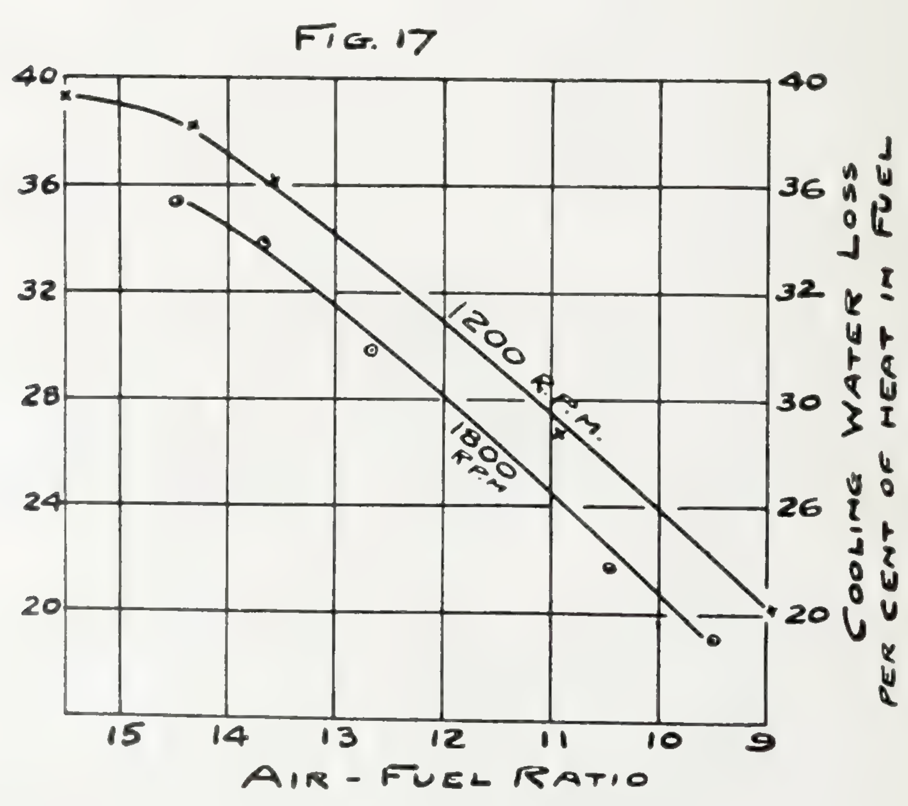

Fig. 17. Cooling water loss—Engine "C".

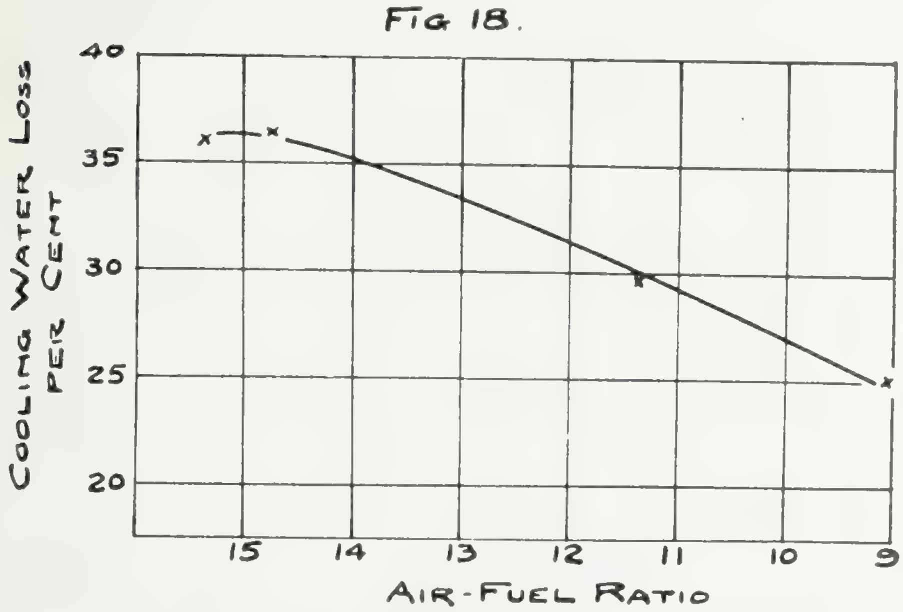

Fig. 18. Cooling water loss—Engine "D".

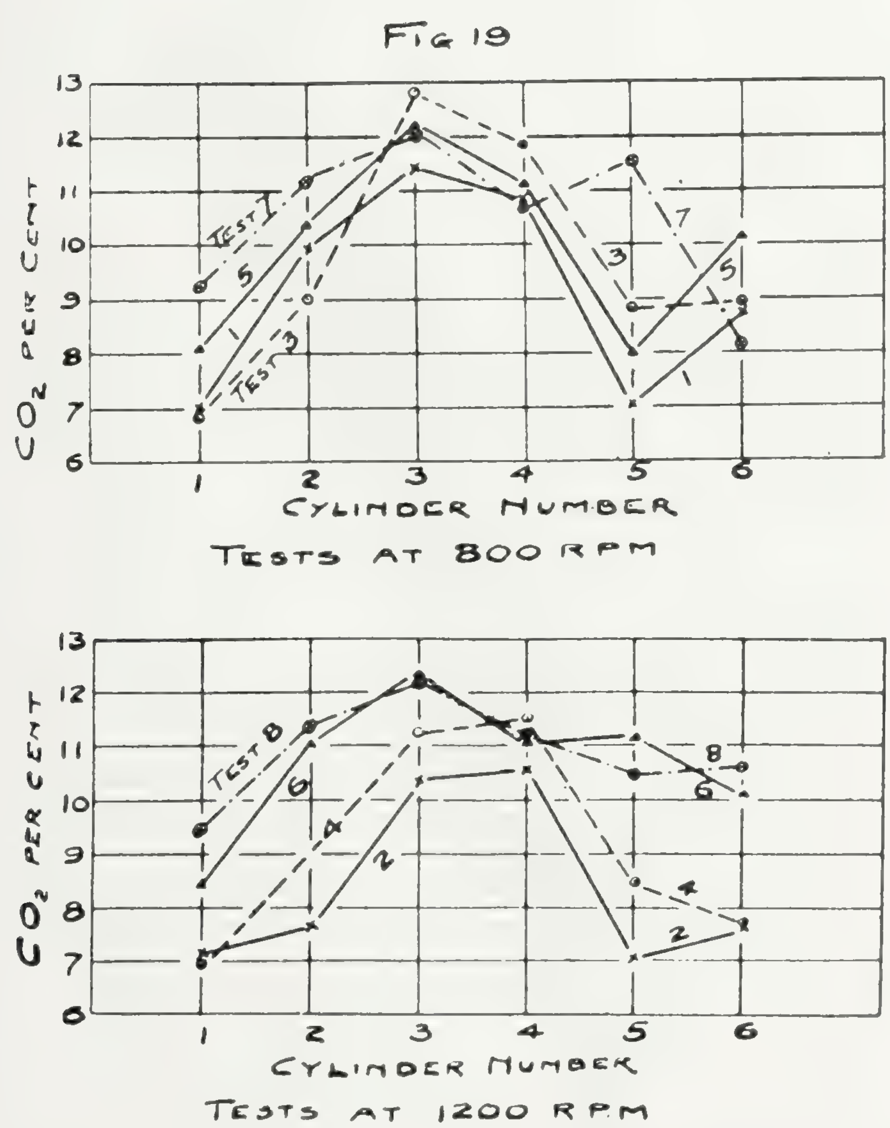

Fig. 19. Distribution of fuel to cylinders—Engine "A".

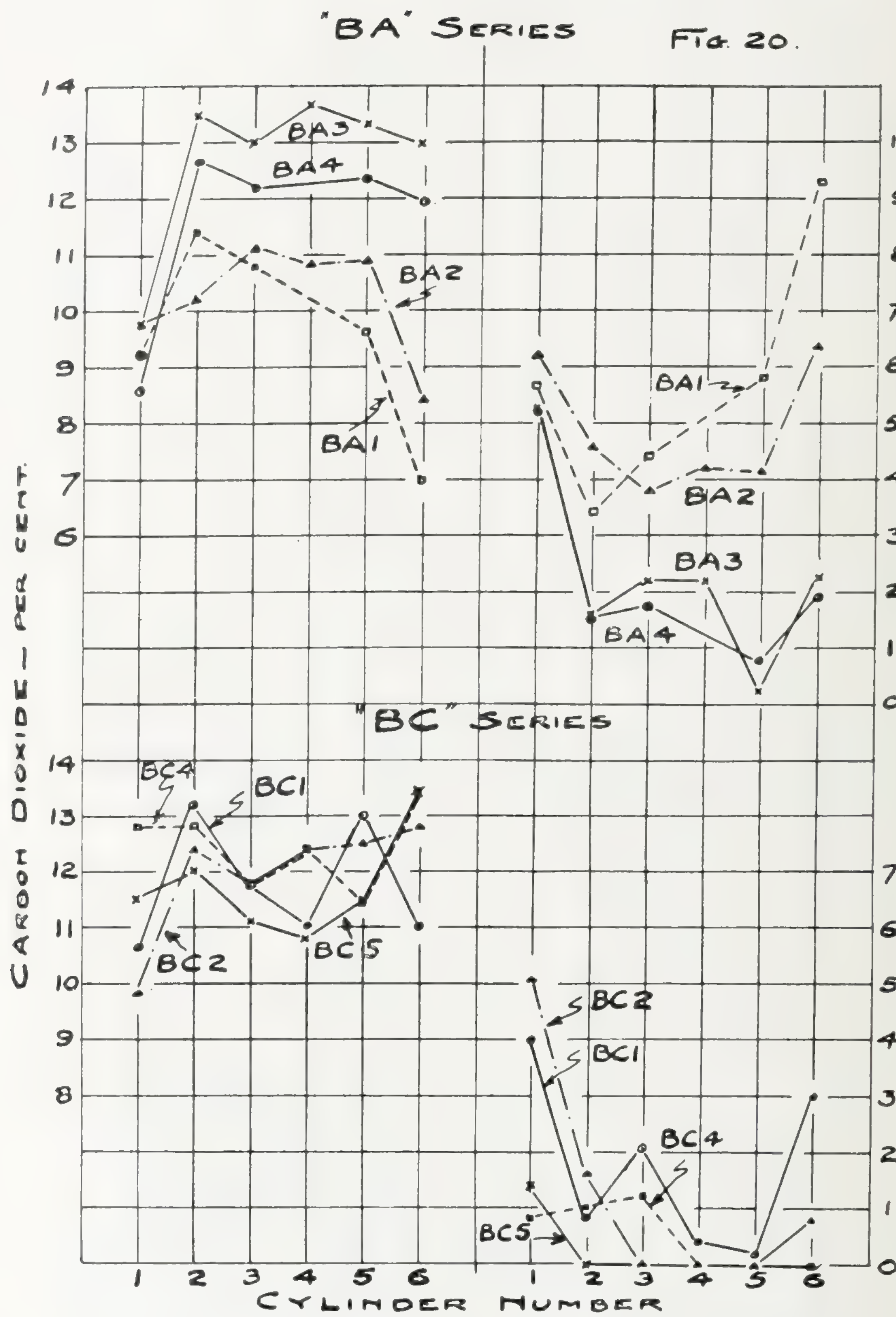

Fig. 20. Distribution of fuel to cylinders—Engine "B".

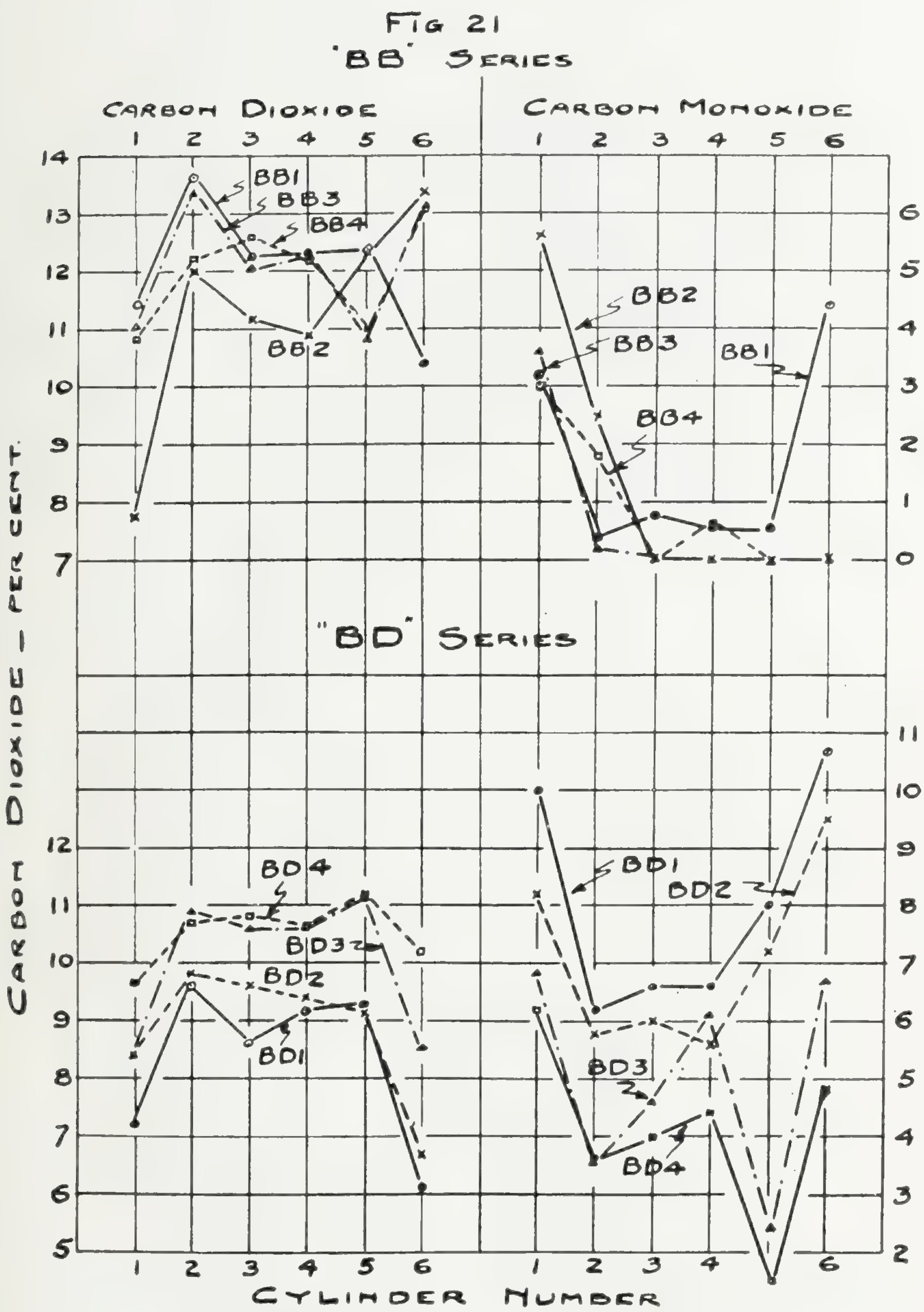

Fig. 21. Distribution of fuel to cylinders—Engine "B".

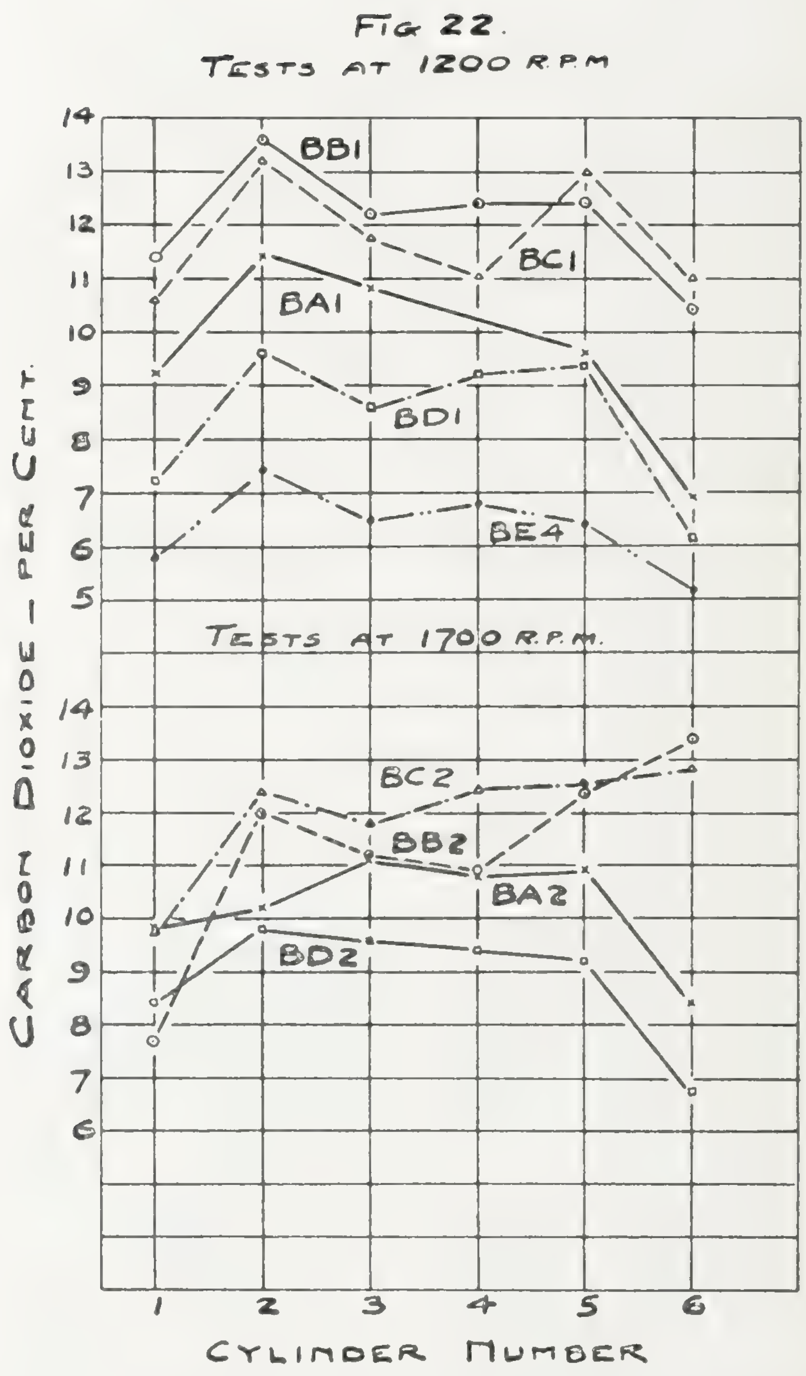

Fig. 22. Distribution of fuel to cylinders—Engine "B".

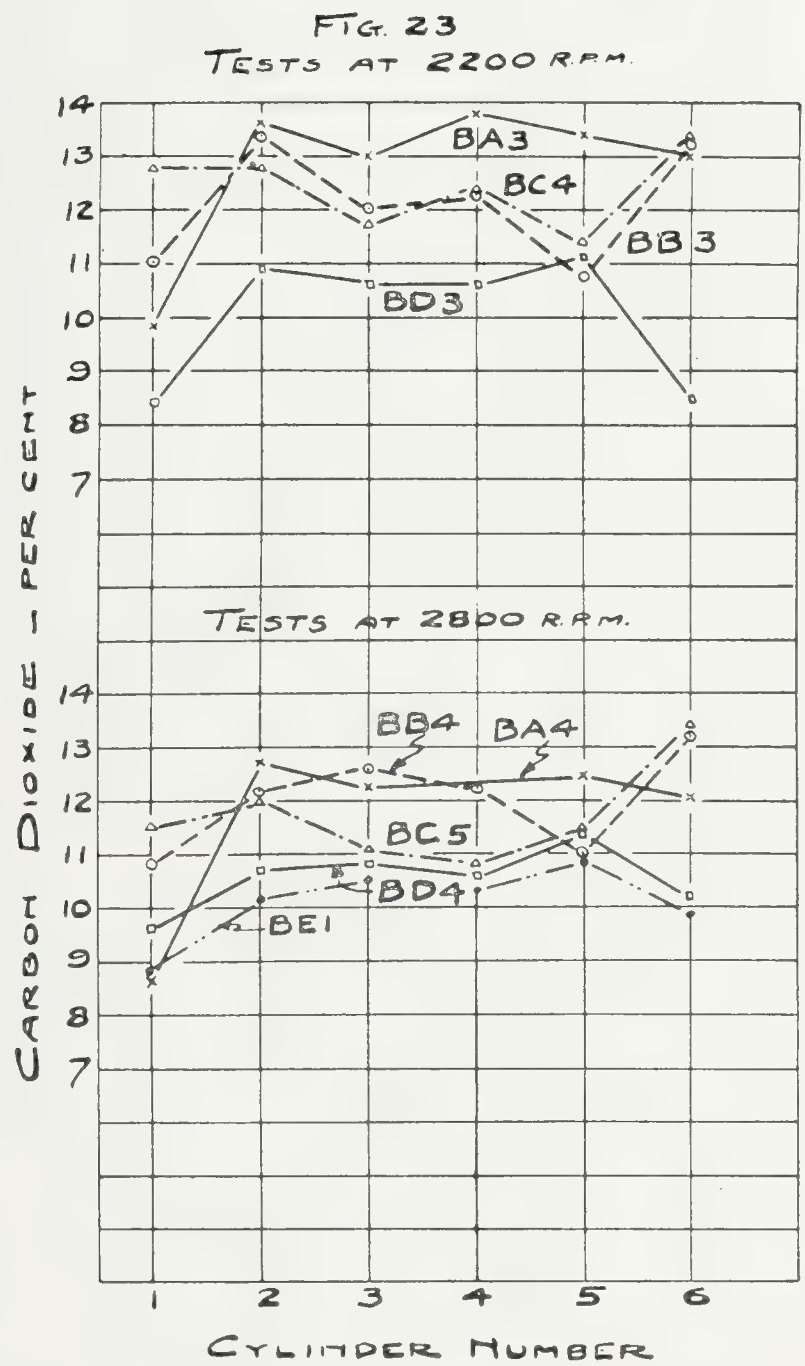

Fig. 23. Distribution of fuel to cylinders—Engine "B".

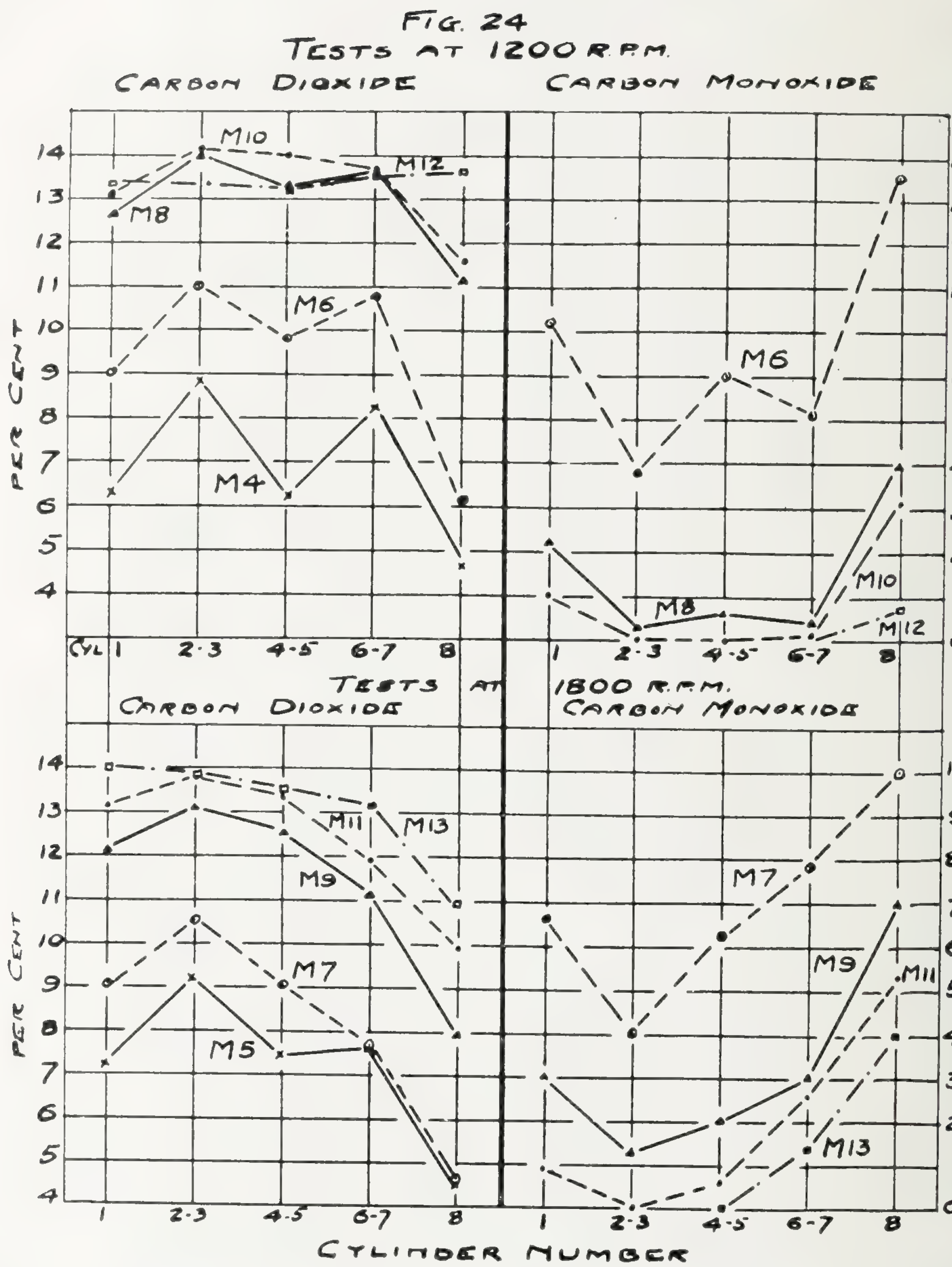

Fig. 24. Distribution of fuel to cylinders—Engine "C" (Schebler carburettor).

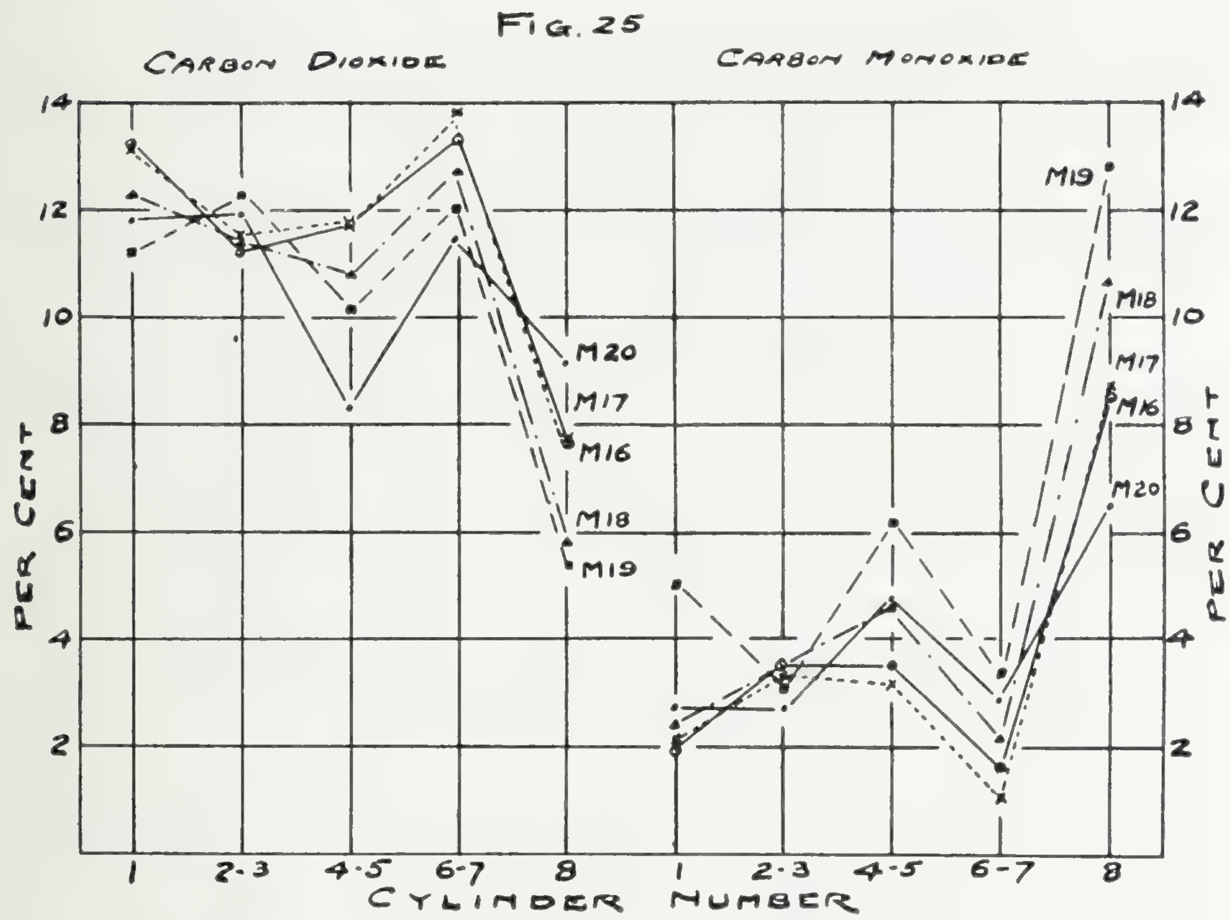

Fig. 25. Distribution of fuel—Engine "C" (Solex carburettor).

richer mixtures obtained in cylinders 4 to 8 by deflecting the larger fuel drops to that side of the manifold. Attention is called to the symmetrical form of the intake manifold (Fig. 7), all cylinders being equidistant from the carburettor, but in spite of this, the distribution is still bad when the mixture is rich.

The curves obtained with the Solex carburettor are given in Fig. 25 and are similar in form to the others, so that apparently the carburettor has little or no influence on these phenomena.

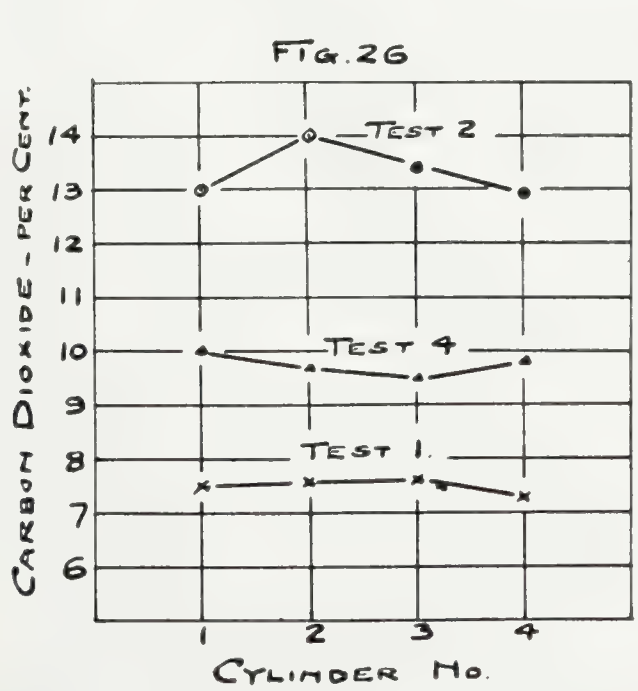

Fig. 26. Distribution of fuel to cylinders—Engine "D".

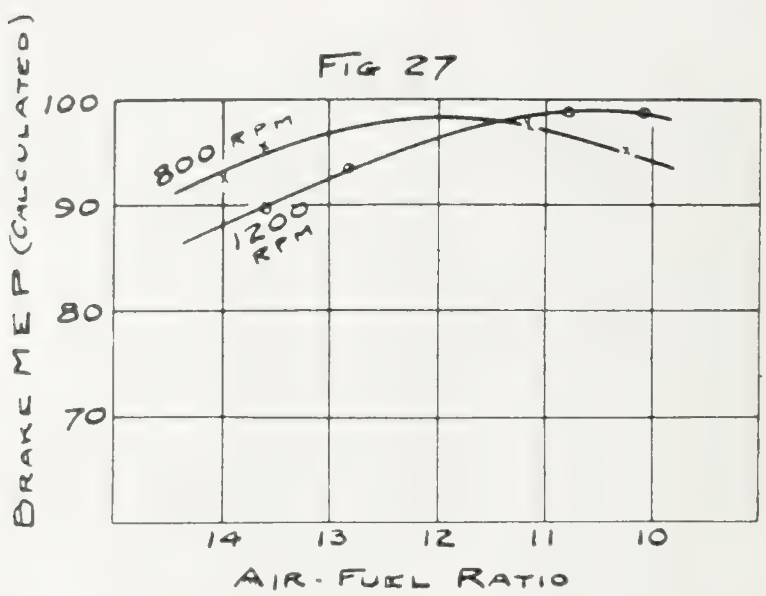

Fig. 27. Brake M.E.P. (calculated)—Engine "A".

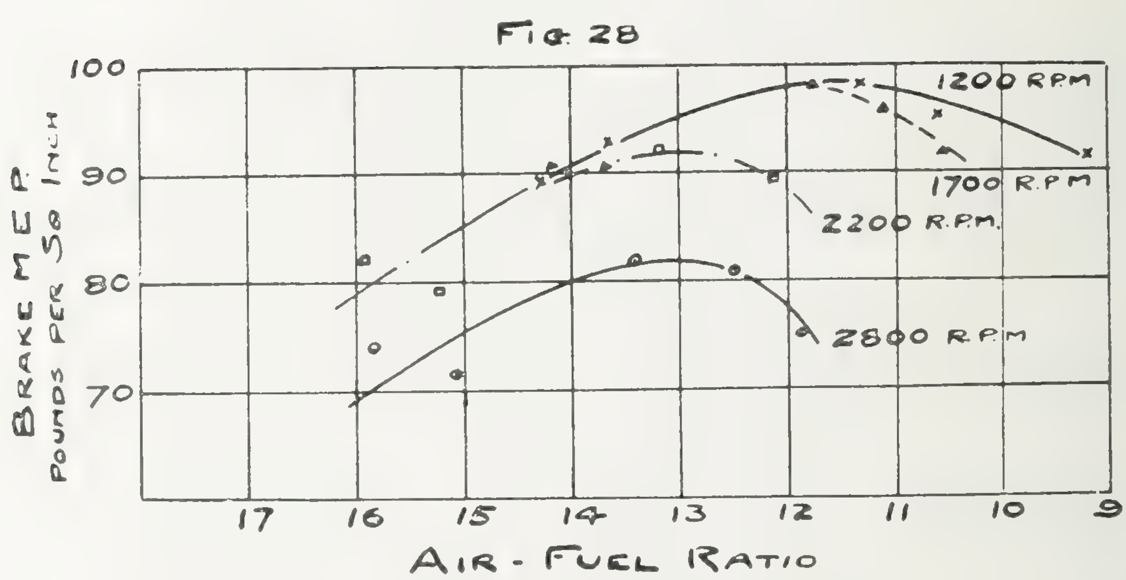

Fig. 28. Brake M.E.P. (observed)—Engine "B".

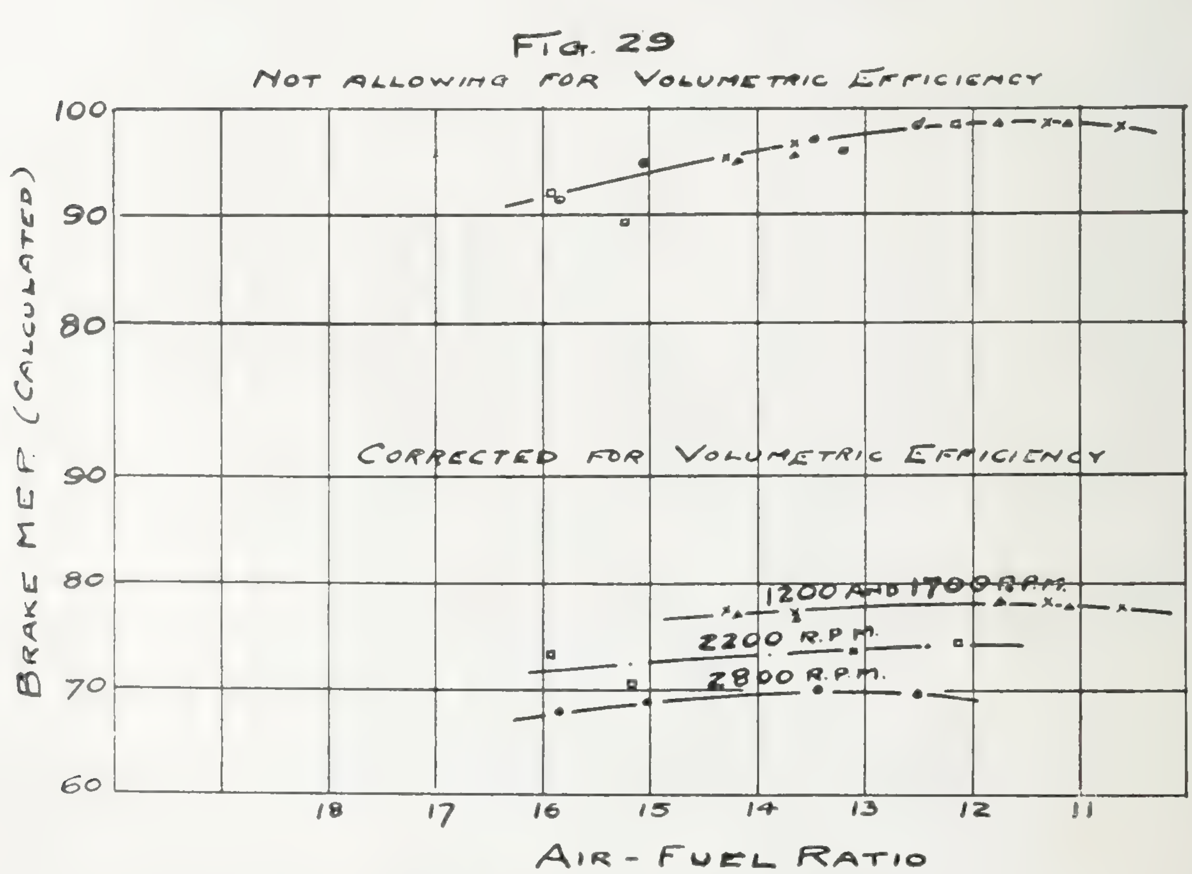

Fig. 29. Brake M.E.P. (calculated)—Engine "B".

The analyses from Engine "D" (Fig. 26 and Table 13) show that the distribution in this low speed engine was very uniform, even with rich mixtures.

(*e*) *Thermal Efficiencies*—All of the engines tested, gave maximum efficiencies at the weakest mixtures used, thus confirming the results of previous experimenters, as indicated in the introduction.

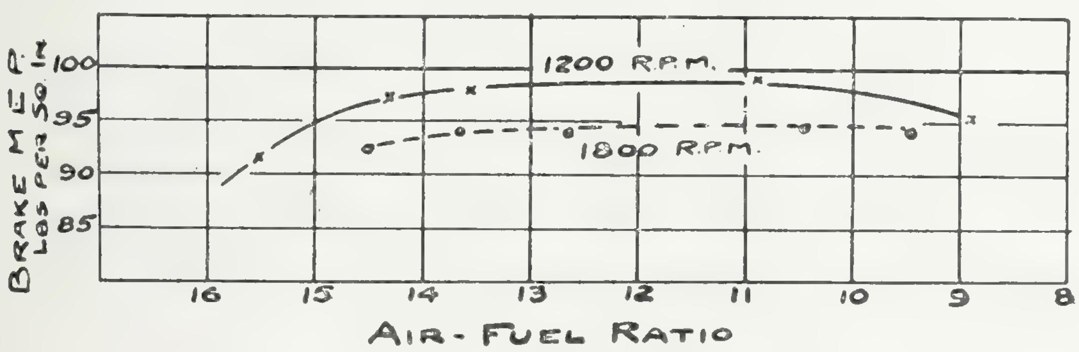

Fig. 30. Brake M.E.P. (observed)—Engine "C".

(*f*) *Mean Effective Pressure*—Engine "A" gave its maximum brake mean effective pressure at a mixture strength of about 11 to 1, as obtained from the actual test results. Fig. 27 shows the form of curve calculated from the gas analyses obtained from the various cylinders. The figures given in this curve for the B.M.E.P. are purely arbitrary,

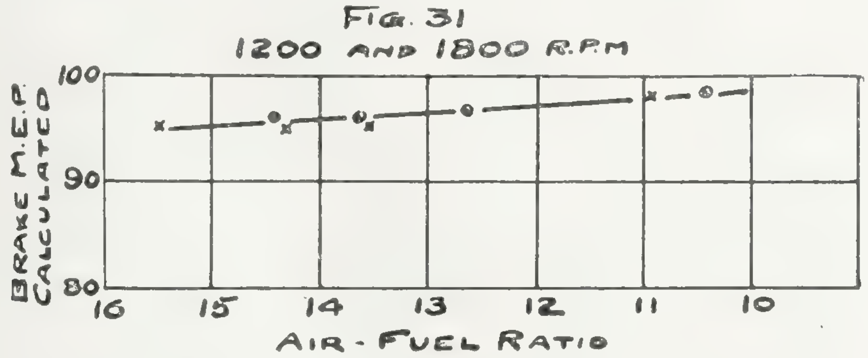

Fig. 31. Brake M.E.P. (calculated)—Engine "C".

but the curves have their maxima also at about an air/fuel ratio of 11 to 1. Fig. 28 gives the observed values for Engine "B", the highest point moving toward the weak side as the speed increases. The calculated curve (Fig. 29) is the same for all speeds, if changes of volumetric efficiency are not allowed for, but on multiplying these values by the

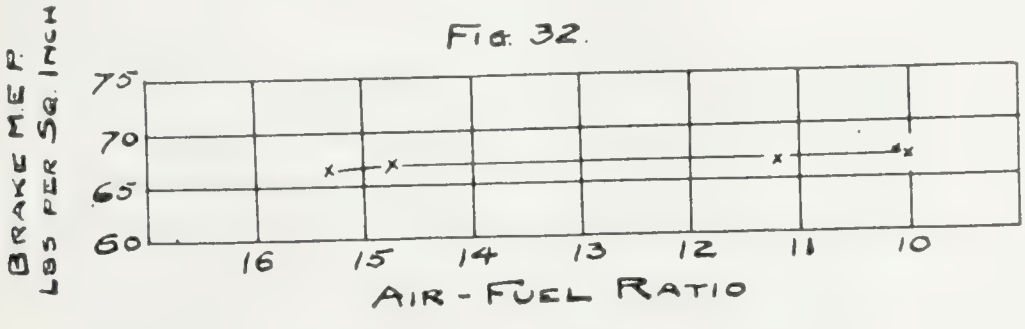

Fig. 32. Brake M.E.P. (observed)—Engine "D".

volumetric efficiency for each test, the lower series of curves is obtained. Here the fall of volumetric efficiency with the richer mixtures and higher speeds, produces the change in peak position mentioned above. Engine "C" gave a very flat curve at both speeds (Fig. 30) with little change in B.M.E.P. from 10 to 14 pounds of air per pound of fuel. The calculated curve (Fig. 31) was practically the same for both speeds, being a straight line with no maximum within the range of mixture strengths investigated. This curve was not corrected for changes of volumetric efficiency, which would separate the results for the two speeds, without changing materially the form of the curves.

The observed curve for Engine "D" is given in Fig. 32, but the number of tests for which complete analyses were available, was too small to justify a calculated curve. It will be noted that the curve actually obtained is quite flat over a wide range.

CONCLUSION

Most of the conclusions to be derived from these experiments are contained in the previous pages, but there is one aspect of the situation that calls for further comment. It is evident that the power curve need not, and in many cases does not, rise to a sharp peak on the rich side of the mixture and thus there is no great sacrifice of power in working at, or near, the chemically correct mixture strength. It is recognized that, for easy starting and rapid acceleration, rich mixtures may be necessary, but these conditions only exist for a comparatively short time and can be handled manually.

The danger and extent of the carbon monoxide hazard is well known. Extensive and costly investigations were made in connection with the ventilation of the Holland tunnel under the Hudson River at New York and it was found that, on a grade of 3 per cent., the average volume of carbon monoxide evolved per car mile was 5.5 cubic feet[14] and several committees have been formed to study the effect of this on atmospheric pollution generally. Several attempts also, have been made by means of catalysts, and in other ways, to make the exhaust gases innocuous, but so far, without great success. Evidently, both from the standpoint of safety and from that of economy, it is advisable to work at, or near, the chemically correct mixture and so to avoid as far as possible the production of carbon monoxide. A discussion of this subject by Tait[15] contains the following statement: "If fuel economy is to be obtained at the same time as maximum power is developed, it is necessary . . . that this mixture should be distributed equally and with uniformity to

[14]S.A.E. Journal, September 1928, p. 316.
[15]Proc. Inst. A.E., April 1928, p. 780.

all cylinders". The foregoing analyses show how uneven the distribution is in the ordinary commercial automobile engine, particularly at low speeds, but they indicate also, that more even distribution is obtained generally with the weaker mixtures. It was intended, originally, to continue the present series of tests at different throttle settings, but circumstances did not permit. However, Tait[16] found on a four cylinder engine that, "with the exception of the tests at full open throttle, a large variation in the mixture strength appeared to cause a relatively small alteration in the air consumption".

Large cities are now taking up seriously the problem of smoke prevention and it is inevitable that, with the ever increasing density of motor traffic, the more deadly and insidious danger of carbon monoxide pollution will receive increasing attention. For this reason, if for no other, the question of greater efficiency and uniformity of fuel distribution in multi-cylinder engines must receive greater attention from automobile engines and manufacturers.

This research was made under the auspices of the School of Engineering Research of the University of Toronto. The author's thanks are due to the General Motors Corporation and the Marmon Motor Co., also to Mr. E. L. Bowerman, B.A.Sc. for his assistance in making and checking the necessary calculations.

[16] *Loc. cit.*, p. 793.

TABLE 1—ENGINE A

Test No.	1	2	3	4
Date	June 21	June 21	June 24	June 24
Air Temp. ° F.	81.2	84	80.5	84
Exhaust temp. ° F. (from engine)	861	1015	918	1110
Fuel per hour—lbs.	13.65	20.4	12.95	20.4
gals.	1.848	2.77	1.755	2.77
Cooling water per hour—lbs.	811	1033	846	1095
" " inlet temp. ° F.	56.9	53.7	58.7	57
" " outlet temp. ° F.	141.6	140	144.5	147.6
" " temp. rise	84.7	86.3	85.8	90.6
" " BTU per hour	68600	89100	72500	99400
Brake load—net lbs.	45.9	43.5	48.77	50.85
Brake MEP lbs/sq. in.	68.4	64.6	72.4	75.5
" " torque lbs. ft.	86.2	81.8	91.40	95.40
Speed—r.p.m.	820	1217	840.6	1290
Brake horse power	13.42	18.9	14.62	23.4
Fuel per BHP hour—lbs.	1.02	1.079	.886	.872
gals.	.1382	.1461	.120	.118
Thermal effy. per cent. on BHP	13.14	12.41	15.11	15.36
Cooling loss BTU per BHP hour	5120	4720	4950	4250
Cooling loss per cent.	26.40	22.95	29.40	25.60
Exhaust and radiation %	60.46	64.64	55.49	59.04
Air Gauge—"water	1.478	3.27	1.57	3.75
Air—lbs/hour	140	206	144.5	220
$\frac{\text{Air}}{\text{Gas}}$	10.25	10.1	11.15	10.8
Vol. effy.—%	66.9	66.7	67.3	67.1
Oil temp. in crankcase—° F.	138	160	134	160
Barometer—ins. Hg.	29.60	29.60	29.78	29.78
Exhaust press.—ins. mercury	1.0	2.0	1.1	2.3
Relative humidity—%	73	69	69	62

TABLE 1—ENGINE A—*continued*

Test No.	5	6	7	8
Date	June 25	June 25	June 27	June 27
Air Temp. ° F	80	81	75	79
Exhaust temp. ° F. (from engine)	964	1112	993	1136
Fuel per hour—lbs	10.46	16	10.85	15.6
gals	1.418	2.17	1.47	2.11
Cooling water per hour—lbs	847	1209	1036.5	985
" " inlet temp	56.6	54.5	56.5	54.7
" " outlet temp	141.2	132.3	126.8	152.1
" " temp. rise	84.6	77.8	70.3	97.4
" " BTU per hour	71650	94000	72800	96000
Brake load—net lbs	42.1	41.4	41.0	43.5
Brake MEP lbs/sq. in	62.6	61.6	60.9	64.6
" " torque lbs. ft	79.20	77.9	77.0	81.9
Speed—r.p.m	837	1227	865	1228
Brake horse power	12.6	18.15	12.65	19.1
Fuel per BHP—lbs	.83	.882	.857	.818
gals	.1122	.1194	.1161	.111
Thermal effy. per cent. on BHP	16.15	15.20	15.61	16.38
Cooling loss BTU per BHP hour	5685	5180	5750	5030
Cooling loss per cent	36.10	30.90	35.30	32.40
Exhaust and radiation—%	47.75	53.90	49.09	51.22
Air Gauge—"water	1.535	3.24	1.72	3.43
Air—lbs/hour	142.5	205	152	212
$\frac{\text{Air}}{\text{Gas}}$	13.62	12.82	14	13.6
Vol. effy.—%	67.0	65.7	68	67.4
Oil temp. in crankcase—° F	132	150	..	....
Barometer—ins. Hg	29.58	29.58	29.75	29.75
Exhaust press.—ins. mercury	1.1	1.9	1.1	1.9
Relative humidity—%	66	62	...	...

TABLE 2—ENGINE A

EXHAUST GAS ANALYSES

Series	Air / Fuel	Cyl. No.	CO_2	O_2	CO	Series	Air / Fuel	Cyl. No.	CO_2	O_2	CO
1	10.25	1	7.0	4.9	2.5	5	13.62	1	8.1	8.0	1.0
		2	9.9	1.6	5.4			2	10.4	2.4	3.3
		3	11.4	0.7	3.1			3	12.2	3.1	0.1
		4	10.8	2.8	1.9			4	11.1	4.3	0.4
		5	7.0	1.7	7.4			5	8.0	9.1	0.0
		6	8.7	3.8	3.4			6	10.1	5.6	0.5
2	10.10	1	7.2	3.6	5.3	6	12.82	1	8.5	5.2	3.4
		2	7.7	1.9	6.3			2	11.1	1.7	3.2
		3	10.4	0.8	4.0			3	12.4	2.2	0.6
		4	10.6	2.2	3.5			4	11.1	5.0	0.0
		5	7.1	1.5	8.5			5	11.2	2.7	2.0
		6	7.6	3.9	5.3			6	10.1	5.5	0.9
3	11.15	1	6.8	7.0	3.2	7	14.0	1	9.2	6.9	0.0
		2	9.0	1.1	5.6			2	11.1	2.2	2.4
		3	12.8	0.5	1.8			3	12.0	3.2	0.0
		4	11.8	2.3	0.8			4	10.7	5.4	0.0
		5	8.8	1.2	6.3			5	11.5	2.4	1.4
		6	8.9	3.9	3.1			6	8.1	8.6	0.0
4	10.80	1	7.0	5.1	6.1	8	13.6	1	9.5	4.6	2.6
		2	...	...	...			2	11.4	2.3	2.4
		3	11.3	0.7	4.3			3	12.2	2.5	0.7
		4	11.5	2.2	2.5			4	11.3	4.2	0.3
		5	8.5	1.7	7.4			5	10.5	2.8	2.8
		6	7.7	4.2	5.4			6	10.6	4.7	1.1

TABLE 3—ENGINE B

Test No.	BA 1	BA 2	BA 3	BA 4
Date	Dec. 27	Dec. 27	Dec. 28	Dec. 28
Air Temp. ° F	72.5	74	71	73.5
Exhaust Temp. ° F	1175	1265	1390	1450
Fuel per hour—lbs	28.25	37.8	43.95	48
gals	3.83	5.12	5.96	6.50
Cooling water per hour—lbs	1019	1263	1536	1662
" " inlet temp	44.3	42	42.4	42
" " outlet temp	167.9	169.1	171.1	177.5
" " temp. rise	123.6	127.1	128.7	135.5
" " BTU per hr	126000	160800	197500	225000
Brake load—net lbs	88.7	89.17	83.26	73.79
Brake MEP lbs/sq. in	98	98.2	92.0	81.4
" " torque lbs. ft	166.9	168	156.8	138.9
Speed—r.p.m	1228	1707	2279	2729
Brake horse power	38.91	54.34	67.79	71.9
Fuel per BHP hour lbs	.727	.698	.648	.669
gals	.0984	.0945	.0878	.0906
Thermal effy. per cent	18.42	19.16	20.60	20.00
Cooling loss BTU per BHP hour	3240	2970	2910	3130
Cooling loss per cent	23.50	22.40	23.60	24.60
Exhaust and radiation %	58.08	58.44	55.80	55.40
Air gauge—"water	1.52	2.94	1.23	1.545
Air—lbs/hour	320	444	579	646
$\frac{\text{Air}}{\text{Gas}}$	11.31	11.75	13.2	13.45
Vol. Effy.—%	79.3	79.3	76.5	71.8
Oil Temp. in crankcase ° F	....	....	....	....
Barometer—ins	29.61	29.62	29.77	29.74
Exhaust pressure—ins. mercury	1.0	1.5	2.6	2.5
Relative humidity—%	56	50	47	49

TABLE 4—ENGINE B

Test No.	BB 1	BB 2	BB 3	BB 4
Date	Dec. 30	Dec. 30	Dec. 30	Dec. 30
Air Temp. ° F	66.5	67.5	70	72.5
Exhaust Temp. ° F	1281	1416	1515	1571
Fuel per hour—lbs	23.9	33.0	38.0	43.95
gals	3.24	4.48	5.15	5.96
Cooling water per hour lbs	1051	1260.5	1534	1735
" " inlet temp	41.9	41	41	41
" " outlet temp	170.1	169.6	167.5	168.4
" " temp. rise	128.2	128.6	126.5	127.4
" " BTU per hour	134900	162200	194000	221000
Brake load—net lbs	84.3	81.9	71.7	64.9
Brake MEP lbs/sq. in	93.0	90.5	79.2	71.6
" " torque lbs. ft	158.6	154.0	134.9	122.1
Speed—r.p.m	1236	1712	2253	2792
Brake horse power	37.2	50.1	57.7	64.7
Fuel per BHP hour lbs	.643	.659	.659	.678
gals	.0872	.0892	.0892	.0919
Thermal effy. per cent	20.80	20.30	20.30	19.72
Cooling loss BTU per BHP hour	3630	3240	3360	3420
Cooling loss per cent	29.65	25.8	26.85	26.55
Exhaust and radiation %	49.55	53.90	52.85	53.73
Air Gauge—"water	1.56	3.02	1.251	1.63
Air—lbs/hour	326	451	580	660
$\frac{\text{Air}}{\text{Gas}}$	13.65	13.68	15.26	15.05
Vol. Effy.—%	79.8	79.8	78.7	72.3
Oil Temp. in crankcase ° F	....	....	....	....
Barometer—ins	29.36	29.37	29.30	29.32
Exhaust pressure—ins. mercury	.725	1.4	2.35	2.75
Relative humidity—%	53	53	47	51

TABLE 5—ENGINE B

Test No.	BC 1	BC 2	BC 3	BC 4	BC 5
Date	Dec. 31	Dec. 31	Dec. 31	Dec. 31	Dec. 31
Air Temp. ° F	67	72.5	72.5	72	75.7
Exhaust Temp. ° F	1286	1429	1260	1490	1551
Fuel per hour—lbs	23.4	32.5	22.8	38.0	42.75
gals	3.175	4.4	3.09	5.15	5.81
Cooling water per hour lbs	1020	1301	1033.5	1519	1771
" " inlet temp	43	42.5	42.3	43.4	42.0
" " outlet temp	172.6	170.5	171.0	171.6	169.2
" " temp. rise	129.6	128.0	128.7	128.2	127.2
" " BTU per hour	132200	166600	133000	194500	225500
Brake load—net lbs	80.7	82.2	82.3	74.2	67.6
Brake MEP lbs/sq. in	89.3	90.8	90.8	82.1	74.7
" " torque lbs. ft	155.22	154.6	155	139.8	127.2
Speed—r.p.m	1237	1726	1218	2297	2771
Brake horse power	35.7	50.7	35.8	60.8	66.9
Fuel per BHP hour lbs	.656	.642	.637	.624	.638
gals	.0888	.0869	.0863	.0846	.0865
Thermal effy. per cent	20.40	20.85	21.00	21.45	20.99
Cooling loss BTU per BHP hour	3710	3280	3715	3195	3370
Cooling loss per cent	29.75	26.90	30.70	26.95	27.80
Exhaust and radiation %	49.85	52.25	48.30	51.60	51.21
Air Gauge—"water	1.64	3.17	1.558	1.355	1.712
Air lbs/hour	334	462	324	607	678
$\frac{\text{Air}}{\text{Gas}}$	14.28	14.2	14.22	15.95	15.85
Vol. Effy.—%	81.2	81	80.7	79.8	74.4
Oil Temp. in crankcase ° F	....	..	....	....	....
Barometer—ins	29.64	29.72	29.71	29.76	29.76
Exhaust pressure—ins. mercury	.81	1.5	0.8	2.50	2.77
Relative humidity—%	51	46.7	46.7	44	46

TABLE 6—ENGINE B

Test No.	BD 1	BD 2	BD 3	BD 4
Date	Jan. 2	Jan. 2	Jan. 2	Jan. 2
Air Temp. ° F	67.8	71	70	71.25
Exhaust Temp. ° F	....	..	..	.....
Fuel per hour—lbs.	31.25	40.5	47.2	51.5
gals.	4.24	5.49	6.38	6.98
Cooling water per hour lbs.	964	1230	1566	1722
" " inlet temp.	43.1	42.3	42.0	42.0
" " outlet temp.	170.1	168.8	166.3	168.1
" " temp. rise	127.0	126.5	124.3	126.1
" " BTU per hour	122200	155800	194700	217800
Brake load—net lbs.	86.9	86.8	81.2	72.9
Brake MEP lbs/sq. in.	95.9	95.9	89.6	80.5
" " torque lbs. ft.	163.4	163.1	152.4	137
Speed—r.p.m.	1260	1719	2271	2745
Brake horse power	39.1	53.5	65.7	71.5
Fuel per BHP hour lbs.	.801	.761	.718	.721
gals.	.1085	.1031	.0974	.0978
Thermal effy. per cent	16.72	17.58	18.68	18.56
Cooling loss BTU per BHP hour	3130	2920	2960	3050
Cooling loss per cent	20.60	20.20	21.70	22.25
Exhaust and radiation %	62.68	62.22	59.62	59.19
Air Gauge—"water	1.618	2.99	1.20	1.52
Air lbs/hour	332	450	573	643
$\frac{\text{Air}}{\text{Gas}}$	10.6	11.1	12.15	12.50
Vol. effy.—%	79	78.9	75.9	70.7
Oil Temp. in crankcase ° F.	..	....	....	....
Barometer—ins.	29.70	29.70	29.70	29.70
Exhaust pressure—ins. mercury	.79	1.44	2.32	2.53
Relative humidity	59	57.3	56.0	58.5

TABLE 7—ENGINE B

Test No.	BE 4	BE 3	BE 2	BE 1
Date	.....	.....	.....	Feb. 20
Air Temp. ° F	71	71	71	71
Exhaust Temp. ° F. (from engine)	1020	1165	1322	1367
Fuel per hour—lbs	35.1	39.8	51.5	54.0
gals	4.76	5.39	6.98	7.3
Cooling water per hour—lbs	894	1031.4	1626.6	1617
" " inlet temp. ° F	39	39	39	39
" " outlet temp. ° F	160.8	178.5	162	168
" " temp. rise	121.8	139.5	123	129
" " BTU per hour	108800	143900	200000	208000
Brake load—net lbs	82.82	83.2	74.35	67.64
Brake MEP lbs/sq. in	91.3	91.9	82	74.7
" " torque lbs. ft	155.60	156.20	139.80	127.60
Speed—r.p.m	1249	1629.8	2513.5	2823
Brake horse power	36.96	48.43	66.73	68.28
Fuel per BHP hour lbs	.950	.822	.772	.790
gals	.1288	.1113	.1048	.1070
Thermal effy. per cent. on BHP	14.10	16.32	17.35	16.96
Cooling loss BTU per BHP hour	2950	2975	3000	3045
Cooling loss per cent	16.32	19.04	20.45	20.25
Exhaust and radiation—%	69.58	64.64	62.20	62.79
Air Gauge—"water	1.52	.644	1.334	1.507
Air lbs/hour	321	422	602	642
$\frac{\text{Air}}{\text{Gas}}$	9.15	10.59	11.69	11.89
Vol. effy.—%	77.3	77.9	72.2	68.5
Oil temp. in crankcase—° F	....	....	....	....
Barometer—ins. Hg	29.818	29.818	29.818	29.818
Exhaust pressure—ins. mercury	1.0	1.2	2.4	3.45
Relative humidity—%	...	...	...	52

TABLE 8—ENGINE B

EXHAUST GAS ANALYSES

Series	Air/Fuel	Cyl. No.	CO_2	O_2	CO	Series	Air/Fuel	Cyl. No.	CO_2	O_2	CO
BA 1	11.31	1	9.2	0.4	5.7	BB 1	13.65	1	11.4	0.6	3.2
		2	11.4	0.4	3.4			2	13.6	0.8	0.4
		3	10.8	0.2	4.4			3	12.2	0.8	0.8
		4	....	...	...			4	12.4	0.9	0.6
		5	9.6	0.2	5.8			5	12.4	1.0	0.6
		6	6.9	0.3	9.3			6	10.4	0.4	4.4
BA 2	11.75	1	9.8	0.3	6.2	BB 2	13.68	1	7.7	0.3	5.6
		2	10.2	0.4	4.6			2	12.0	0.6	2.4
		3	11.1	0.3	3.8			3	11.2	2.2	0.0
		4	10.8	0.3	4.2			4	10.9	0.7	0.0
		5	10.9	0.2	4.1			5	12.4	0.8	0.0
		6	8.4	0.5	6.4			6	13.4	0.4	0.0
BA 3	13.2	1	9.8	0.7	5.3	BB 3	15.26	1	11.0	1.8	3.6
		2	13.6	0.2	1.6			2	13.4	0.8	0.2
		3	13.0	0.4	2.2			3	12.0	1.6	0.0
		4	13.8	0.2	2.2			4	12.3	3.3	0.0
		5	13.4	0.7	0.2			5	10.7	2.7	0.0
		6	13.0	0.1	2.3			6	13.2	1.6	0.0
BA 4	13.45	1	8.6	0.4	5.7	BB 4	15.05	1	10.8	0.8	3.0
		2	12.7	0.3	1.6			2	12.2	0.3	1.8
		3	12.2	0.2	1.8			3	12.6	2.4	0.0
		4	....	...	...			4	12.2	0.8	0.6
		5	12.4	0.9	0.8			5	11.0	2.2	0.0
		6	12.0	0.5	1.9			6	13.2	1.4	0.0
BC 1	14.28	1	10.6	1.6	4.0	BD 1	10.6	1	7.2	1.4	10.0
		2	13.2	1.2	0.8			2	9.6	0.4	6.2
		3	11.7	1.6	2.1			3	8.6	0.6	6.6
		4	11.0	3.0	0.4			4	9.2	0.8	6.6
		5	13.0	1.6	0.2			5	9.3	0.0	8.0
		6	11.0	1.2	3.0			6	6.1	2.2	10.7

TABLE 8—ENGINE B—*continued*

Series	Air/Fuel	Cyl. No.	CO_2	O_2	CO	Series	Air/Fuel	Cyl. No.	CO_2	O_2	CO
BC 2	14.20	1	9.8	0.6	5.1	BD 2	11.10	1	8.4	0.6	8.2
		2	12.4	1.0	1.6			2	9.8	1.0	5.8
		3	11.8	2.6	0.0			3	9.6	0.6	6.0
		4	12.4	2.0	0.0			4	9.4	1.0	5.6
		5	12.5	1.3	0.0			5	9.2	0.2	7.2
		6	12.8	0.8	0.8			6	6.7	0.6	9.5
BC 4	15.95	1	12.8	1.0	0.8	BD 3	12.15	1	8.4	1.2	6.8
		2	12.8	1.4	1.0			2	10.9	1.7	3.6
		3	11.8	3.0	1.2			3	10.6	0.6	4.6
		4	12.4	3.0	0.0			4	10.6	0.5	6.1
		5	11.4	3.0	0.0			5	11.1	0.7	2.4
		6	13.4	1.8	0.0			6	8.5	0.3	6.7
BC 5	15.85	1	11.5	0.8	1.4	BD 4	12.50	1	9.6	0.0	6.2
		2	12.0	2.8	0.0			2	10.7	0.5	3.6
		3	11.1	4.6	0.0			3	10.8	0.4	4.0
		4	10.8	2.6	0.0			4	10.6	0.4	4.4
		5	11.5	2.4	0.0			5	11.2	0.5	1.4
		6	13.4	1.8	0.0			6	10.2	1.4	4.8
BE 4	9.15	1	5.8	0.2	11.8	BE 3	10.59	All	8.1	0.3	8.8
		2	7.4	0.2	9.9						
		3	6.5	0.2	11.1						
		4	6.8	0.2	10.2						
		5	6.4	0.6	10.1						
		6	5.2	0.2	12.4						
BE 2	11.69	All	9.9	0.2	6.8						
BE 1	11.89	1	8.8	0.6	7.6						
		2	10.2	0.2	4.9						
		3	10.6	0.2	4.9						
		4	10.4	0.3	5.0						
		5	10.9	0.5	3.9						
		6	9.8	0.5	6.7						

TABLE 9—ENGINE C

SCHEBLER CARBURETTOR

Test No.	M 4	M 5	M 6	M 7	M 8
Date	July 22	July 22	July 29	July 29	July 31
Alr Temp. ° F	86	88	81	84	80
Exhaust Temp. ° F	900	1150	1040	1190	1155
Fuel per hour—lbs	25.9	35.6	20.9	31.8	16.6
gals	3.51	4.82	2.83	4.32	2.25
Cooling water per hr. lbs	1150	1398	1408	1350	1243
" " inlet temp. ° F	60.6	60.1	62.9	60.7	61.9
" " outlet temp. ° F	147.7	152.6	138.6	158	154
" " temp. rise ° F	87.1	92.5	75.7	97.3	92.1
" " BTU per hour	100000	129000	106500	131500	114200
Brake load—net lbs	63.6	62.7	66.15	63.0	65.5
Brake MEP lbs/sq. in	95.2	94.0	99.0	94.5	97.6
" " torque lbs. ft	119.5	118	124.3	118.9	123
Speed—r.p.m	1219	1811	1200	1790	1208
Brake horse power	27.7	40.7	28.4	40.4	28.2
Fuel per BHP hour lbs	.936	.877	.737	.788	.591
gals	.1269	.1189	.0998	.1069	.0801
Thermal effy. per cent	14.30	15.30	18.19	17.00	22.65
Cooling loss BTU per BHP hr.	3610	3180	3750	3260	4060
Cooling loss per cent	20.25	19.10	26.75	21.75	36.10
Exhaust and radiation %	65.45	65.60	55.06	61.25	41.25
Air Gauge—"water	4.04	3.86	.781	1.66	.754
Air lbs/hour	230	337	228	332	225
$\frac{\text{Air}}{\text{Gas}}$	8.9	9.46	10.9	10.42	13.55
Vol. effy.—%	77.6	77.2	78.8	77.4	77.0
Oil temp. in crankcase ° F	....	....	....	....	....
Barometer—ins	29.96	29.96	29.63	29.63	29.66
Exhaust pressure—ins. merc	.....	.....	.....	.....	.....
Relative humidity—%	51	49	69	63	62

TABLE 9—ENGINE C—*continued*

Test No.	M 9	M 10	M 11	M 12	M 13
Date	July 31	Aug. 2	Aug. 2	Aug. 6	Aug. 6
Air temp. ° F	84	76	80	74.5	76
Exhaust temp. ° F	1230	1160	1230	1155	1230
Fuel per hour—lbs	25.8	15.9	23.9	14.75	22.55
gals	3.49	2.155	3.24	1.999	3.055
Cooling water per hr. lbs	1469	1260	1520	1135.5	1503
" " inlet temp. ° F	56.7	62.37	58.5	63	60.3
" " outlet temp. ° F	156.3	154.5	159.6	160.3	160.8
" " temp. rise ° F	99.6	92.13	101.1	97.3	100.5
" " BTU per hour	146000	116000	153800	110300	151000
Brake load—net lbs	62.8	65	63	61.4	61.8
Brake MEP lbs/sq. in	93.9	97.1	94.0	91.5	92.5
" " torque lbs. ft	117.8	122.1	118.1	115.2	116.2
Speed—r.p.m	1791	1203	1784	1214	1787
Brake horse power	40.2	27.95	40.1	26.6	39.5
Fuel per BHP hour lbs	.642	.571	.596	.555	.571
gals	.0869	.0774	.0807	.0752	.0774
Thermal effy. per cent	20.85	23.40	22.40	24.20	23.45
Cooling loss BTU per BHP hr.	3640	4150	3840	4140	3830
Cooling loss per cent	29.85	38.20	33.85	39.30	35.30
Exhaust and radiation %	49.30	38.40	43.75	36.50	41.25
Air gauge—"water	1.60	.765	1.580	.770	1.580
Air lbs/hour	326	228	326	229	326
$\frac{\text{Air}}{\text{Gas}}$	12.65	14.33	13.65	15.5	14.45
Vol. effy.—%	75.8	76.8	75.0	76.9	74.8
Oil temp. in crankcase ° F	....	161	181	161	181
Barometer—ins	29.64	29.86	29.86	29.72	29.72
Exhaust pressure—ins. mer	.....	.....	.....	.....	.....
Relative humidity—%	63	64	62	57	57

TABLE 10—ENGINE C

SOLEX CARBURETTOR

No. of carburettors	4	1	1	1	1
Test No	M 14	M 16	M 18	M 19	M 20
Date	Aug. 16	Aug. 19	Aug. 21	Aug. 21	Aug. 22
Air temp. ° F	72	80	74	76	78
Exhaust temp. ° F. (from engine)	678	1140	1120	1070	1145
Fuel per hour—lbs	22.8	17.15	18.8	21.6	14.5
gals	3.09	2.32	2.55	2.92	1.96
Cooling water per hr. lbs	745	1560	....	....	1012
" " inlet temp. ° F	64.1	61.5	62.9	60.3	60.6
" " outlet temp. ° F.	130.1	132	144.2	134	162.4
" " temp. rise ° F	75.0	70.5	81.3	73.7	101.8
" " BTU per hour	53800	110000	....	....	103000
Brake load—net lbs	32.14	65.3	65.9	67.5	60.9
Brake MEP lbs/sq. in	48	97.6	98.4	100.5	90.9
" " torque lbs. ft	60.4	123.0	124.0	127.0	114.6
Speed—r.p.m	1255	1231	1221	1211	1177
Brake horse power	14.4	28.8	28.7	29.2	25.6
Fuel per BHP hour lbs	1.385	.595	.655	.740	.567
Thermal effy. per cent. on BHP.	8.45	22.5	20.45	18.1	23.6
Cooling loss per cent	12.88	33.8	.....	....	37.5
Barometer—ins. Hg	29.98	30.01	30.05	30.05	29.79
Relative humidity—%	65	56	66	59	68

TABLE 11—ENGINE C

Tests M 4 to M 11—Schebler Carburettor

Exhaust Gas Analyses

Series	Air/Fuel	Cyl. No.	CO_2	O_2	CO	Series	Air/Fuel	Cyl. No.	CO_2	O_2	CO
M 4	8.9	1	6.25	.75		M 8	13.55	1	12.6	0.4	2.2
		2 & 3	8.8	.60				2 & 3	14.0	0.8	0.3
		4 & 5	6.2	.60				4 & 5	13.3	0.7	0.6
		6 & 7	8.2	.60				6 & 7	13.6	0.6	0.4
		8	4.6	.20				8	11.1	0.6	4.0
M 5	9.46	1	7.2	0.4		M 9	12.65	1	12.1	0.2	3.0
		2 & 3	9.2	0.5				2 & 3	13.1	0.5	1.3
		4 & 5	7.4	1.0				4 & 5	12.5	0.6	2.0
		6 & 7	7.6	.60				6 & 7	11.4	0.8	3.0
		8	4.5	.70				8	7.9	0.5	7.0
M 6	10.9	1	9.0	0.4	7.2	M 10	14.33	1	13.1	0.5	1.0
		2 & 3	11.0	0.8	3.8			2 & 3	14.1	0.9	0
		4 & 5	9.8	0.6	6.0			4 & 5	14.0	0.8	0
		6 & 7	10.8	0.2	5.1			6 & 7	13.6	0.8	0.1
		8	6.1	0.4	10.6			8	11.6	0.4	3.1
M 7	10.42	1	9.0	0.4	6.6	M 11	13.65	1	13.1	0.5	0.9
		2 & 3	10.5	0.5	4.0			2 & 3	13.8	0.6	...
		4 & 5	9.0	0.3	6.2			4 & 5	13.3	0.7	0.6
		6 & 7	7.7	0.3	7.8			6 & 7	11.9	0.4	2.6
		8	4.6	1.0	10.0			8	9.9	0.4	5.3

TABLE 11—ENGINE C—*continued*

Series	Air/Fuel	Cyl. No.	CO_2	O_2	CO	Series	Air/Fuel	Cyl. No.	CO_2	O_2	CO
M12	15.5	1	13.3	1.2	0.0	M 17	....	1	13.1	0	2.1
		2 & 3	....	...	...			2 & 3	11.5	0.3	3.3
		4 & 5	13.2	1.6	0.0			4 & 5	11.8	0	3.2
		6 & 7	13.5	1.0	0.0			6 & 7	13.8	0	1.0
		8	13.6	0.4	0.7			8	7.7	0	8.7
M 13	14.45	1	14.0	0.5	0.0	M 18	....	1	12.3	0	2.4
		2 & 3	13.8	0.7	0.0			2 & 3	11.4	0	3.5
		4 & 5	13.5	1.0	0.0			4 & 5	10.8	0	4.6
		6 & 7	13.1	0.4	1.4			6 & 7	12.8	0	2.1
		8	10.9	0.3	4.0			8	5.8	0	10.7
					Tests M 14 to 20—Solex Carburettor						
M 14	.....	1	6.7	0.1	9.8	M 19	....	1	11.2	0	5.0
		2-3	4.3	2.0	10.2			2-3	12.4	0	3.1
		4-5	...	...	....			4 & 5	10.1	0	6.2
		6-7	4.8	2.0	9.4			6-7	12.0	0	3.4
		8	8.1	0	9.6			8	5.4	0	12.8
M 16	.....	1	13.2	0	1.9	M 20	....	1	11.8	0.1	2.7
		2-3	11.2	0.4	3.5			2-3	11.9	0	2.7
		4-5	11.7	0	3.5			4-5	8.3	2.0	4.6
		6-7	13.3	0	1.6			6-7	11.5	0	2.9
		8	7.6	0	8.5			8	9.1	0	6.5

TABLE 12—ENGINE D

Test No.	1	2	3	4	5
Date	Feb. 5	Feb. 13	Feb. 13	Feb. 13	Feb. 13
Air temp. ° F	91.7	96	96.0	95.4	92.0
Exhaust temp. ° F. from engine	1105	1195	1227	1157	1094
Fuel per hour—lbs	36.6	23.4	24.9	29.75	33.6
gals	4.96	3.17	3.37	4.05	4.56
Cooling water per hour lbs	1231	1175	1209	1209	1200
" " inlet temp. ° F	44.8	46.0	45.0	45.5	44.3
" " outlet temp. ° F.	169.6	186.6	192.5	183	177
" " temp. rise ° F	124.8	140.6	147.5	137.5	132.7
" " BTU per hour	153500	165000	178100	166200	159100
Brake load—net lbs	21.12	20.87	21.0	21.0	21.12
Brake MEP lbs/sq. in	67.2	66.6	66.9	66.9	67.3
" " torque lbs. ft	226.5	224.5	226.5	226.0	227.0
Speed—r.p.m	861.5	821.2	857.0	865.3	867
Brake horse power	37.2	35.15	36.9	37.2	37.4
Fuel per BHP .bs	.984	.667	.675	.800	.867
gals	.1332	.0903	.0914	.1082	.1215
Thermal effy. per cent. on BHP.	13.6	20.1	19.86	16.75	14.94
Cooling losses BTU per BHP hr.	4125	4690	4840	4470	4250
Cooling losses per cent	22.1	37.0	37.7	29.4	24.9
Exhaust and radiation %	64.3	42.9	42.4	53.8	60.1
Air	10.1	15.4	14.7	11.3	10.0
Gas					
Barometer—ins. hg	....	29.46	29.45	29.44	29.43
Relative humidity—%	40	.....	.....	.....	.....

TABLE 13—ENGINE D

EXHAUST GAS ANALYSES

Test No.	Cylinder No.	CO_2	O_2	CO	N_2
1	1	7.5	0.8	5.7	86.0
	2	7.6	0.4	6.5	85.5
	3	7.6	0.7	5.7	86.0
	4	7.3	1.0	5.7	86.0
2	1	13.0	2.9	0	84.1
	2	14.0	1.1	0	84.9
	3	13.4	1.8	0	84.8
	4	12.9	2.2	0	84.9
3	All	13.7	0.9	0	85.4
4	1	10.0	0.9	5.5	83.6
	2	9.7	1.5	5.5	83.3
	3	9.5	2.2	4.7	83.6
	4	9.8	1.0	5.2	84.0
5	All	7.6	1.9	7.7	82.8

A STUDY OF CERTAIN PROPERTIES AND REACTIONS OF PHENYLHYDRAZINE

By E. G. R. Ardagh[1], B. Kellam, F. C. Rutherford and H. T. Walstaff

Part I

Determination of Phenylhydrazine. — The need for a simple and dependable method for the quantitative determination of phenylhydrazine arose as a result of the use of this reagent by Ardagh and Williams[2] in the quantitative determination of the carbonyl group in organic compounds. The present paper should be read in conjunction with the two just referred to, since it is impossible, due to lack of space, to repeat much of the detail given there.

In the quantitative determination of acetophenone and benzophenone we employed ethyl alcohol + water as solvent for these ketones since they are too sparingly soluble in water alone for practical purposes.

The method recommended by Ardagh and Williams for the determination of phenylhydrazine[2] by titration with iodine in slightly acid solution, the course of which reaction is represented by the equation given at the end of this paragraph, gives low results when more than 10% by volume of alcohol is present. This is particularly the case when the phenylhydrazine is very dilute, say, 0.01 *M*.

$$C_6H_5HN{\cdot}NH_2 + 2I_2 \longrightarrow C_6H_5I + 3HI + N_2$$

To accelerate the rate of the reaction between the ketone and the phenylhydrazine, heating the mixture to 60° under an atmosphere of nitrogen is very effective. At this temperature some phenylhydrazine is always destroyed when alcohol is present even in solutions as acid as *P*H 2, but if the *P*H does not exceed 4 the destruction of phenylhydrazine is reduced practically to a constant (about 2% of the amount present after heating to 60° for three hours, and 5% for nine hours) even though the alcohol present may make up half the total volume, as shown in the accompanying graph (Fig. 1).

The effects of considerable quantities of alcohol on the figures obtained for phenylhydrazine may be summed up as follows.

(1) On heating the solution containing the phenylhydrazine to 60° for three hours, the decomposition of phenylhydrazine in faintly acid

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[2]Ardagh and Williams, *J. Am. Chem. Soc.* **47**, 2976, 2983 (1925).

solution (*P*H about 4) is independent of the proportion of alcohol present and is roughly a constant at 2% of the amount of phenylhydrazine present. It is most important that the alcohol be free from aldehydes and traces of other impurities which will react with iodine. Sometimes absolute alcohol is prepared by the use of calcium carbide. In this case traces of acetylene will be present.

(2) When alcohol and buffer are both present (we used NaH_2PO_4), numerous factors appear to affect the iodine-phenylhydrazine reaction, *e.g.*, rate of addition of iodine, excess of reagents, time allowed for reaction, etc.

(3) When alcohol without buffer is present, the titration gives more trouble than when buffer without alcohol is present, but either is less uncertain than when both alcohol and buffer are present.

(4) The phenylhydrazine content of the solution to be titrated should not fall below approximately 0.05 *M*, especially if alcohol is present.

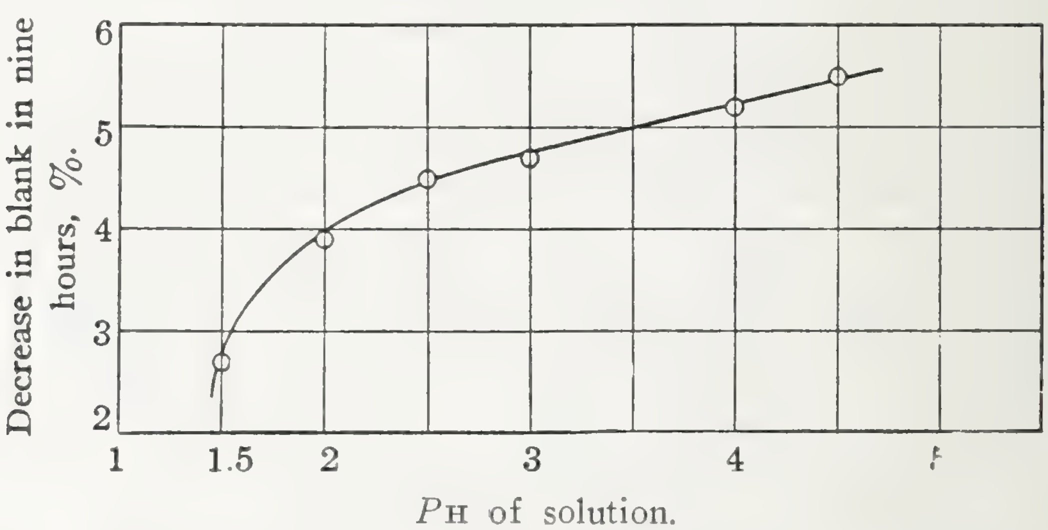

Fig. 1.—Change in phenylhydrazine value with *P*H of solution of $C_6H_5NHNH_2.HCl$ heated for nine hours at 60° (50% alcohol).

(5) The phenylhydrazine solution should be added to an excess of the standard iodine and the mixture allowed to stand for five minutes before titrating the excess iodine.

(6) When alcohol is present the end-point is improved by adding 2 to 3 cc. of thiosulfate in excess and shaking gently to remove the iodine dissolved in the droplets of iodobenzene. If these droplets are very minute a more certain end-point is secured by omitting the ether formerly recommended.[2]

Owing to the inconvenience as well as the instability of the phenylhydrazine base, we prepared the hydrochloride, which we used in all our work. To obtain the salt free from impurity, the base, after redistilling at 16 mm. under nitrogen followed by diluting with alcohol, must be added slowly with constant stirring to an excess of the acid, also diluted

with alcohol. If the reverse method is followed a product containing aniline hydrochloride and even ammonium chloride is likely to be obtained as a result of the reaction shown here, especially if the mixture is allowed to become warm.[3]

$$2C_6H_5HN{\cdot}NH_2 + C_6H_5HNNH_2{\cdot}HCl \longrightarrow 3C_6H_5NH_2 + NH_4Cl + N_2$$

Proceeding in the manner described we had no difficulty in preparing beautiful, lustrous, white scales of the hydrochloride which, after washing, first with alcohol, then with diethyl ether, and drying to constant weight over porous calcium chloride in a desiccator filled with nitrogen, required for titration the precise proportion of 0.1 *N* iodine calculated for the pure salt.

Experiment proved that 25 cc. of a 0.25 *M* solution in water of the hydrochloride diluted to 100 cc. when exposed to air turned brown and turbid in a few days, as a result of oxidation.

In our attempts to find an inhibitor against oxidation we tried adding 0.1 g. of each of the following to 100 cc. batches of the above: pyridine, quinoline, *o*-toluidine, *m*-phenylenediamine and β-naphthylamine. None of these appeared to exercise any inhibitory effect. The addition of 0.5 cc. of 6 *N* hydrochloric acid to a 100-cc. batch appeared, on the contrary to be most effective. Doubtless this was due to the resulting increase in hydrogen-ion concentration.

In our work we preferred to prepare our phenylhydrazine solutions from day to day from the pure, dry hydrochloride. This practice we considered to be more free from possible sources of error than any other method.

PART II

$$C_6H_5CH_3C{=}O + C_6H_5HNNH_2{\cdot}HCl \rightleftharpoons C_6H_5CH_3C{=}NNHC_6H_5 + HCl + H_2O$$

Optimum Conditions for the Preparation of Acetophenone Phenylhydrazone.—In the earlier work referred to, the authors' object was the development of an analytical method for the quantitative determination of aldehydes and ketones; in Part II and Part III of the present article the purpose is to discover the conditions that will result in a high yield of hydrazone in a short time.

Ardagh and Williams found that the conditions necessary to give a high percentage yield of hydrazone in a reasonable time from acetophenone and from benzophenone must be maintained within narrower limits than are required in the case of the aldehydes and ketones of the aliphatic series, and furthermore that the rate of hydrazone formation is

[3]M. Busch, *J. Prakt. Chem.*, **116**, 39, (1927).

too slow at room temperature to be satisfactory. This is particularly the case with benzophenone.[4]

At 60° the reaction proceeds much more rapidly. The hydrogen-ion concentration of the solution, as one would expect, also has an important influence upon the rate.

The accompanying graph (Fig. 2) constructed from our results shows the effect of the hydrogen-ion concentration upon the rate of the formation of acetophenone phenylhydrazone at 60°. The optimum *P*H evidently lies between the limits 4 and 5.

The following method we found to be very successful.

To a 100-cc. ground-glass stoppered graduated flask were added 25 cc. of 0.25 *M* $C_6H_5HNNH_2 \cdot HCl$, 25 cc. of buffer solution (0.5 *M* NaH_2PO_4), to which enough phosphoric acid had been added to give the buffer a *P*H of 4.6, 10 cc. of acetophenone (1 cc. contained 0.0246 g. of acetophenone) dissolved in 40% alcohol by volume and 40 cc. of sodium chloride containing 370 g. per liter (saturated). The air was displaced by nitrogen and the tightly stoppered flask immersed in a water-bath at 60°. The yield was 100% of the theoretical in less than thirty minutes. The time required to obtain 100% yield at room temperature as shown in Fig. 5 was twelve hours in a solution having a *P*H of 6.1. No doubt

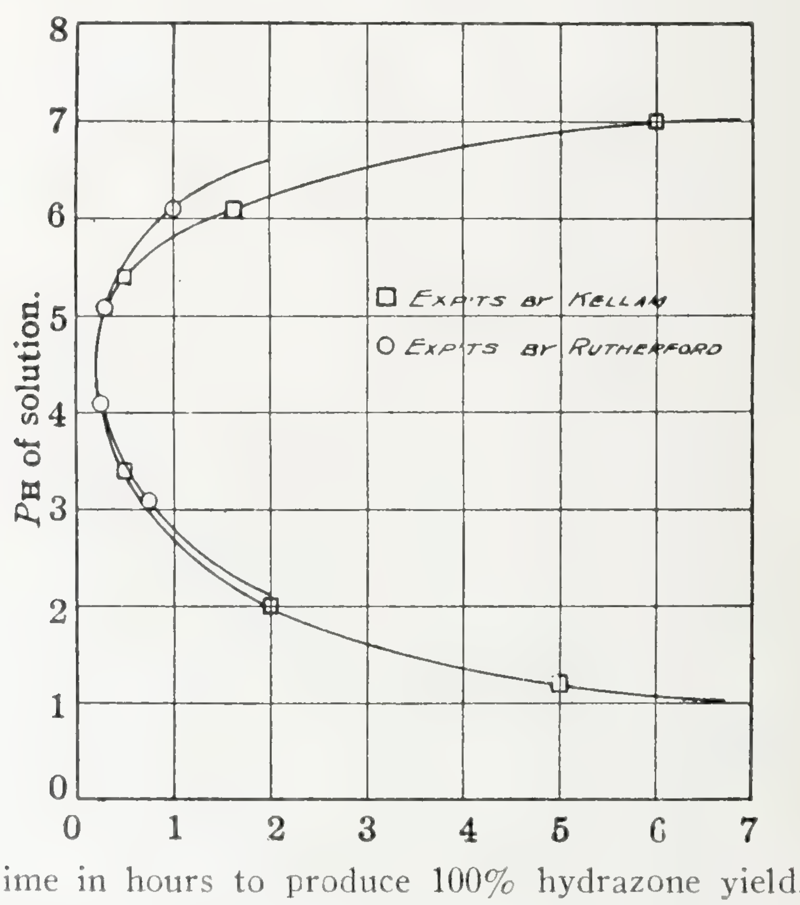

Fig. 2.—Relation between *P*H of solution and time required to produce 100% yield of acetophenone phenylhydrazone.

[4]Ref. 2, p. 2988.

the time required would have been materially reduced if the P_H had been adjusted to between 4 and 5.

Figures 3 and 4 show the rate of reaction at 60° at a P_H of 7.0 and of 7.8, respectively. These rates are quite obviously very much slower than for lower P_H's. In fact it is possible that at P_H 7.8 the reaction might never arrive at completion.

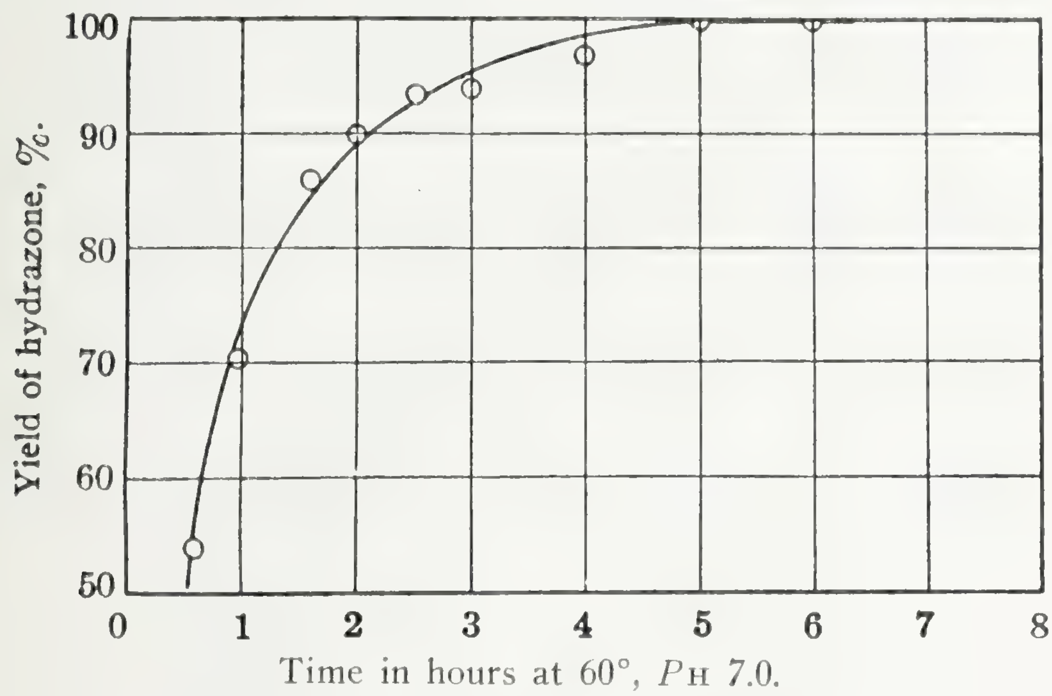

Fig. 3.—Acetophenone phenylhydrazone, time-yield, P_H 7.0.

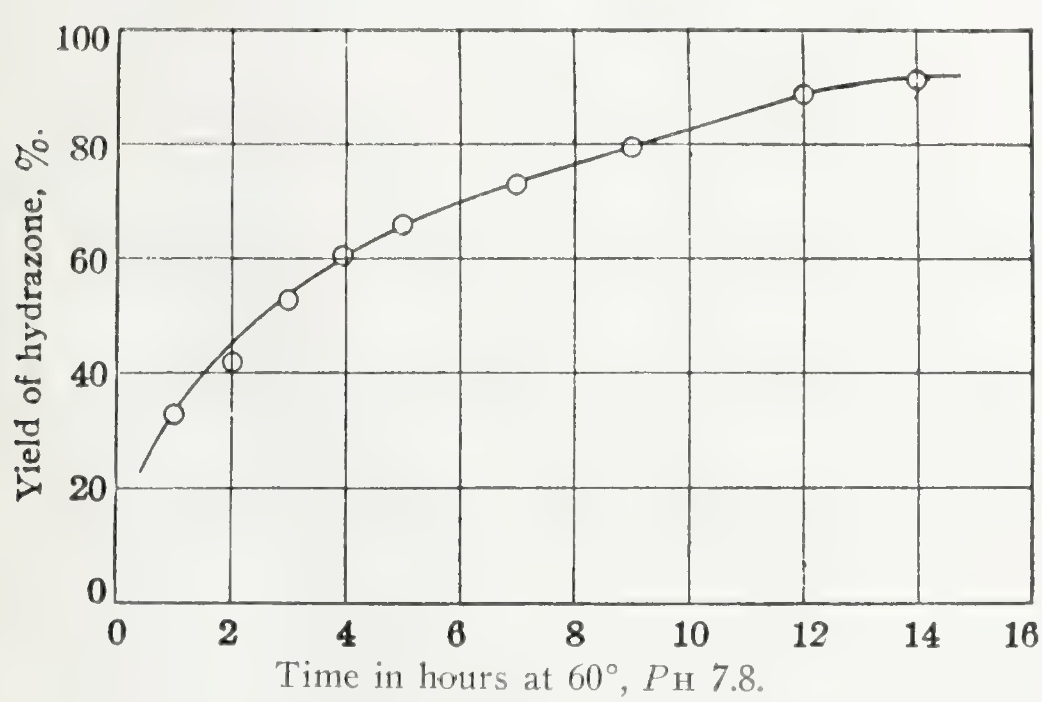

Fig. 4.—Acetophenone phenylhydrazone, time-yield, P_H 7.8.

All the determinations of phenylhydrazine were made by the method described in the earlier paper referred to,[5] with such modifications as have herein been shown to be necessary in the presence of alcohol. In each determination we added 10 cc. of the phenylhydrazine solution to an excess of the 0.1 N iodine, usually 30 cc.

[5]Ref. 2, p. 2986.

PART III

$$(C_6H_5)_2C=O + C_6H_5HNNH_2{\cdot}HCl \rightleftharpoons (C_6H_5)_2C=NNHC_6H_5 + HCl + H_2O$$

Optimum Conditions for the Preparation of Benzophenone Phenylhydrazone.—To secure 100% yield of this hydrazone from the ketone in a reasonable time necessitates very careful control of conditions. To keep the ketone in solution, the alcoholic content of the mixture must be not much below 50% by volume. Even at 60° and with 50% alcohol the reaction proceeds much more slowly than for acetophenone.

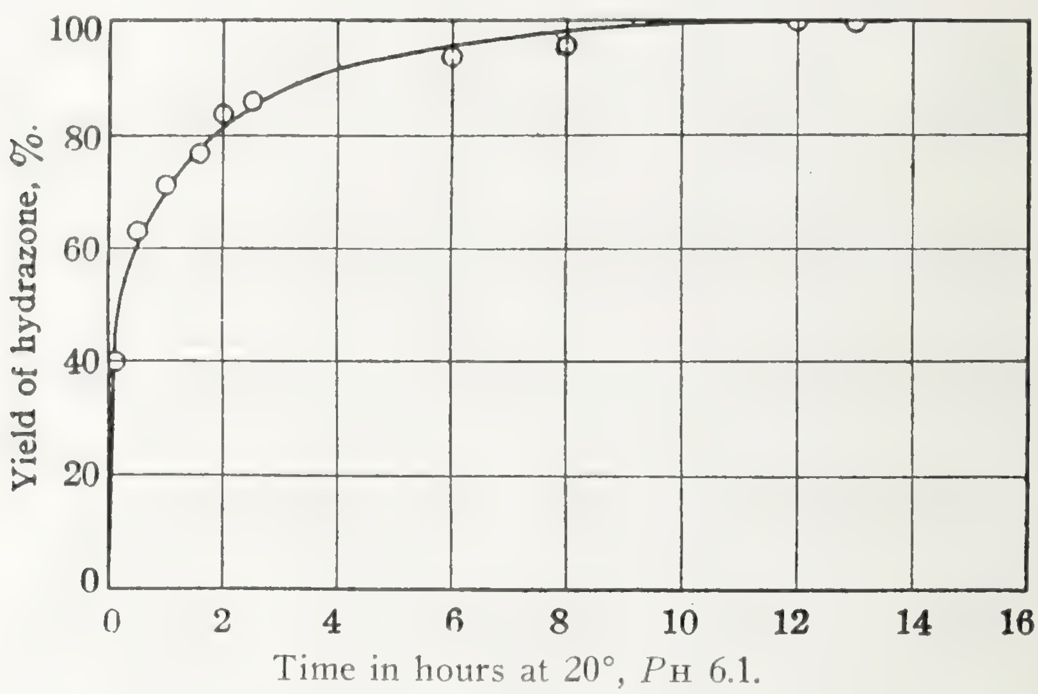

Fig. 5.—Acetophenone phenylhydrazone, time-yield, *P*H 6.1.

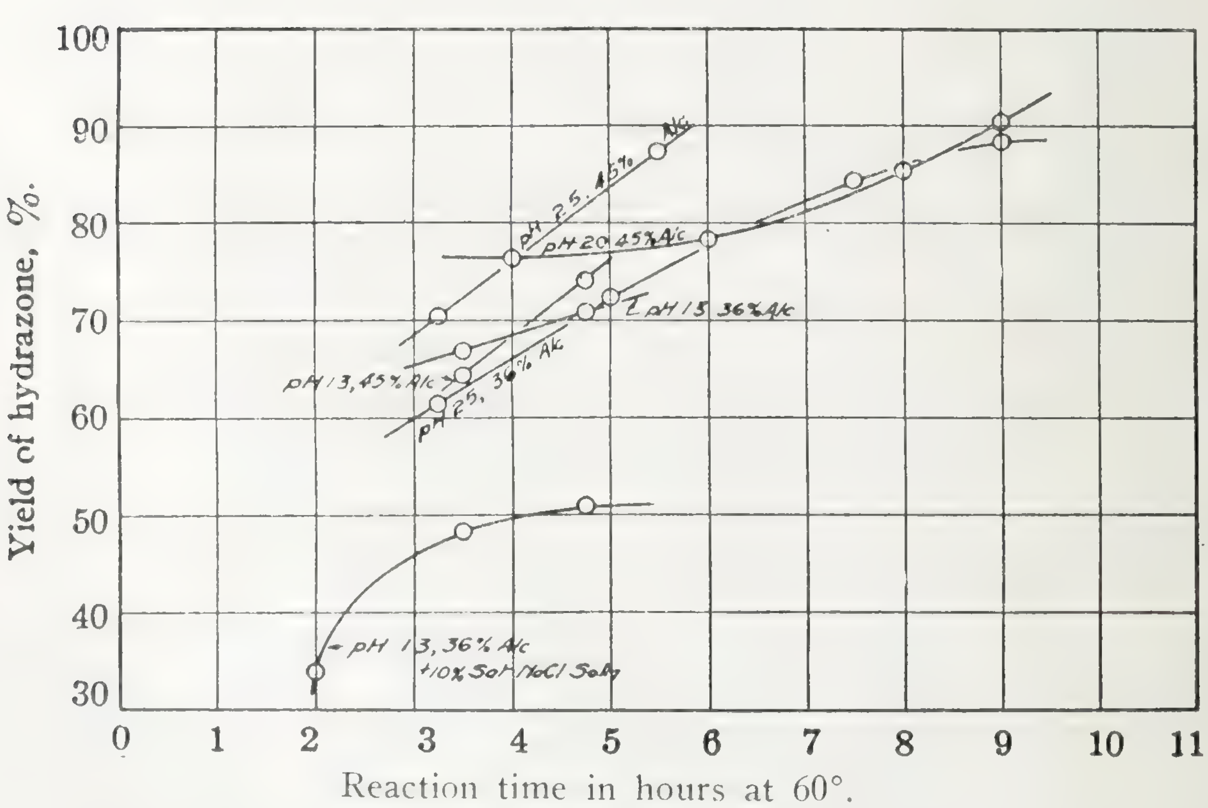

Fig. 6.—Yields of benzophenone phenylhydrazone in solutions of different composition, %.

In the formation of the hydrazone, water is split off. One would expect, therefore, that by using absolute alcohol the rate of reaction would rise. The solubility of the hydrazone would, however, increase with increase in alcohol concentration. As a result the yield of hydrazone would tend to diminish above a certain alcoholic concentration in spite of the increase in the rate of its formation. We had not time to determine experimentally, from the viewpoint of both time and yield, the most effective alcoholic

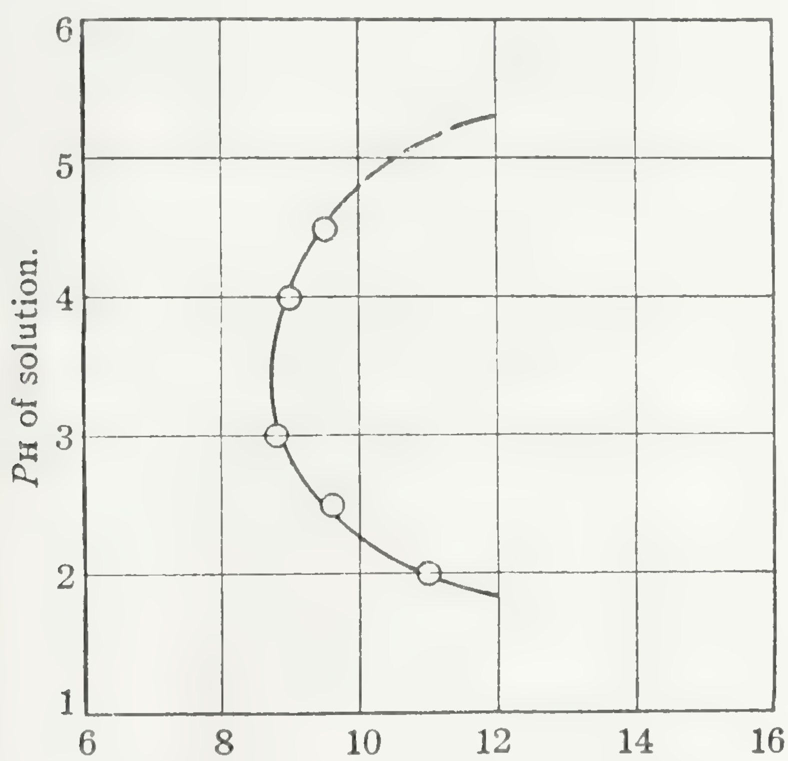

Fig. 7.—Relation between *P*H of solution (50% alcohol) and time required to produce 100% yield of benzophenone phenylhydrazone.

concentration. In any case the opinion of the experimenter would also have an important bearing upon the interpretation of the word *effective*. An increase in alcohol content would, unfortunately, add to our analytical troubles, and since the study was originally undertaken for the purpose of developing an analytical method, we have endeavored for this reason to keep the alcohol concentration down to a convenient figure. Figure 6 gives some indication of the influence at 60° of the hydrogen-ion concentration and of the alcoholic content upon the rate of the reaction.

Figure 7 shows the effect of the hydrogen-ion concentration on the rate of the reaction at 60° when the alcoholic content of the solution is 50% by volume. The optimum hydrogen-ion concentration evidently lies between *P*H 3 and *P*H 4, which is very close to the hydrogen-ion concentration of 0.25 *M* phenylhydrazine hydrochloride itself (*P*H 3.5).

It seems strange that the addition of saturated sodium chloride

retards the rate of formation of this hydrazone, since in all the other hydrazones we prepared the reverse is the case.

One peculiar phenomenon observed in dealing with both the acetophenone and benzophenone phenylhydrazones was that, while both hydrazones are capable of supersaturation to a high degree during formation, nevertheless when they were forced out of solution as soon as they were formed (by constant agitation) the rate of formation was not increased. In the case of the benzophenone phenylhydrazone it even seemed that the longer the elapsed time before the crystals of hydrazone appeared (depending on the menstruum employed), the higher was the final yield.

SUMMARY

1. The conditions requisite for obtaining satisfactory results in the iodimetric determination of phenylhydrazine, more particularly in the presence of ethyl alcohol, have been worked out.

2. The effect of hydrogen-ion concentration upon the minimum time required to secure a high yield of the hydrazones of acetophenone and benzophenone has been determined.

3. Details of a method by which very pure phenylhydrazine hydrochloride can be prepared have been worked out.

RIVETED TENSION MEMBERS

By T. R. Loudon[1]

In presenting the following paper, the writer wishes to acknowledge the very helpful work done by Messrs. K. B. Jackson; G. W. Smart; W. Turner and H. St. P. Butler as research assistants.

Considerable attention has been given from time to time to the problem of arriving at a rational deduction formula for use with riveted tension members. Some of the attempts that have been made to derive a formula from a strictly analytical basis had an apparent amount of success; but the writer showed in the results of a research published in the Canadian Engineer of August 26, 1924,[2] that certain assumptions that had been made in these investigations were not correct. These results were obtained by using the photo elastic method; and while they served to disprove the assumptions in question, there was no clear indication from them as to an easy method of determining the actual stress distribution in riveted tension members.

The research was continued, using the photo elastic method, with a view to determining whether or not there were certain simple factors governing the stress distribution from which a reasonable deduction formula could be put together.

There are two avenues of approach to the problem. The first, and apparently the more popular of the two, is to use tension plates with rivet holes in them and to observe their behaviour when tested to destruction. The second method is to observe what happens when tension plates which are actually riveted together are treated in a similar manner. Both kinds of models were investigated; as it was felt at first that there might be some fundamental difference in stress distribution between these two types which would affect the general workability of a formula drawn up from observations on only one type of plate.

Some idea of the general difference in stress distribution in the two types of plates may be had from Figs. 1 and 2. Fig. 1 shows one set of principal stress lines for a plate with merely a rivet hole in it; Fig. 2 shows the corresponding set of principal stress lines for a plate into which the load is transferred by means of a rivet. The type of variation of stress intensity on a right section through these holes in both Figs. 1

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[1]Professor of Applied Mechanics, University of Toronto.

[2]Reprinted at end of this article.

and 2 is illustrated in Fig. 3. There is a region of high stress concentration close to the rivet or rivet hole depending upon which plate is used; and the highest unit stress in either case may be three or four times the average unit stress on the uncut right section of the plate. This means, of course, that the ordinary elastic limit may be exceeded in these areas around the holes even when the average stress on the gross sections is well below this limit.

It was intended originally to explore the stress distribution for both types of models; but it was seen, as will be shown, that a certain factor common to both types was the governing effect, so that merely plates with holes in them were used finally.

The photo elastic method gives on a screen properly placed, a colour variation scheme from which the values of the algebraic differences

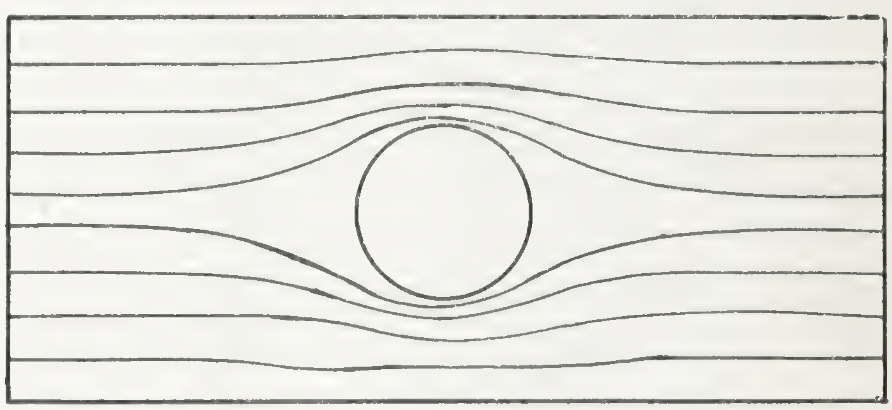


Fig. 1.

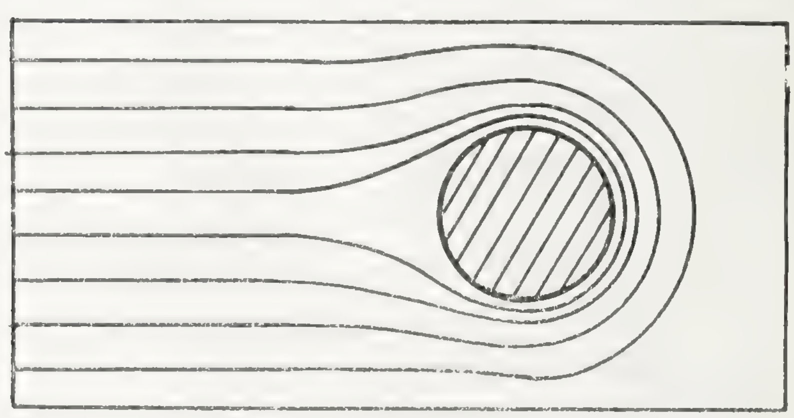


Fig. 2.

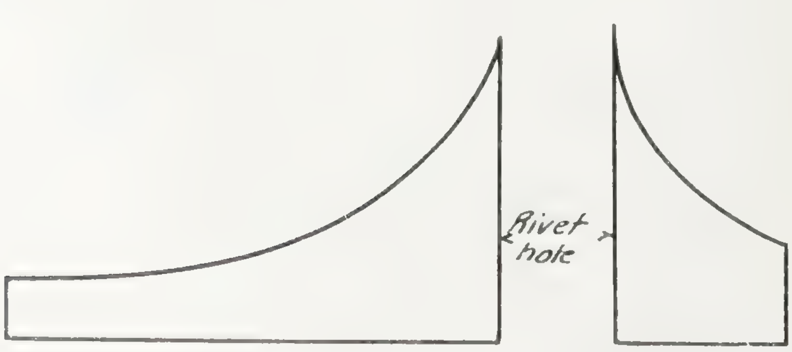


Fig. 3.

between the two principal stresses in the main plane of the specimen may be obtained. As the load is placed on a specimen, there appear in succession the colours yellow, red and blue followed by the repeated sequence of the same colours as the stress becomes greater and greater. The outlines shown on Figs. 4 and 5 may be called isochromes as they give roughly the central lines of the various colours for a given load. These colour bands may be taken as representing the lines along which the principal stress differences have certain values. The principal stress differences shown on these isochromes are in pounds per square inch, and if one of the principal stresses is small, as is the case in this problem, it is not very far from the truth to say that the isochromes show the regions along which the other principal stress has a given value. The celluloid tension models used in making Figs. 4 and 5 were 1½ inches wide and approximately 3/16-inch thick with ¼-inch rivet holes. The edge distances were 3/8-inch and the gauge lines ¾-inch apart. The rivet stagger is ¾-inch in Fig. 4, and 2 inches for Fig. 5.

The average unit stress on the uncut right section in Figs. 4 and 5 is below the elastic limit of the material; but in the areas around the edges of the rivet holes the elastic limit is exceeded. This was done in order to give a more striking stress picture; although it is realized that the elastic limit having been exceeded, the specimens could not have been used for accurate stress determination. These diagrams are merely used here to illustrate a point that is not so convincingly seen from isochromes in which low stress values are used; although it must be clearly understood that almost the same configuration of results obtains for loads which keep the stresses below the elastic limit.

Many sections such as those illustrated in Figs. 4 and 5 were examined. Various stagger ratios were used and it was found that the salient features in all of these, conformed to the same type of stress distribution as shown in these figures.

It was while investigating the stress along various sections of these models that a certain persistent fact was noticed; namely: that there is a region of very high stress around the rivet holes which is so much higher than anywhere else that one would naturally expect first failure always to take place in or about that locality. *This is not new evidence; but it is doubtful whether its full significance has ever been taken into account as far as the present problem is concerned.*

When this was noticed, an attempt was made by the photo elastic method to deduce the effect of varying the stagger and gauge on these maximum stresses at the edges of the rivet holes; but it can be seen from Figs. 4 and 5 that there is not a very great decrease in stress value at the rivet holes for an increase of stagger from ¾-inch to 2 inches. Fig. 6 shows the stress variations on the edge distances of right cross

sections through rivets for three specimens of the same dimensions given for Figs. 4 and 5, the staggers being varied as shown and only one-half the load applied as in Figs. 4 and 5. It is evident that the failures of the specimens would most likely be in the order of No. 1 to No. 3; but the actual variation of maximum stress at the edge of the rivet hole is not very great as we go from No. 1 to No. 3. The reason why No. 1 would fail first follows, of course, from the fact that there is a greater average stress on the section. But the difference between any two of these cases is not such as to give rise to the conviction that it would require considerable variation of material on the right section to look after it. Indeed, one would almost form the conclusion that having allowed for the worst case, the remaining cases require very little variation of material. *The condition at the edge of the rivet holes is startling enough in all cases.*

The variation of stress on a right cross section on the other side of the rivet hole is of the same nature in all cases as shown in Fig. 3. The average stress in this case is very much lower than for the edge distance

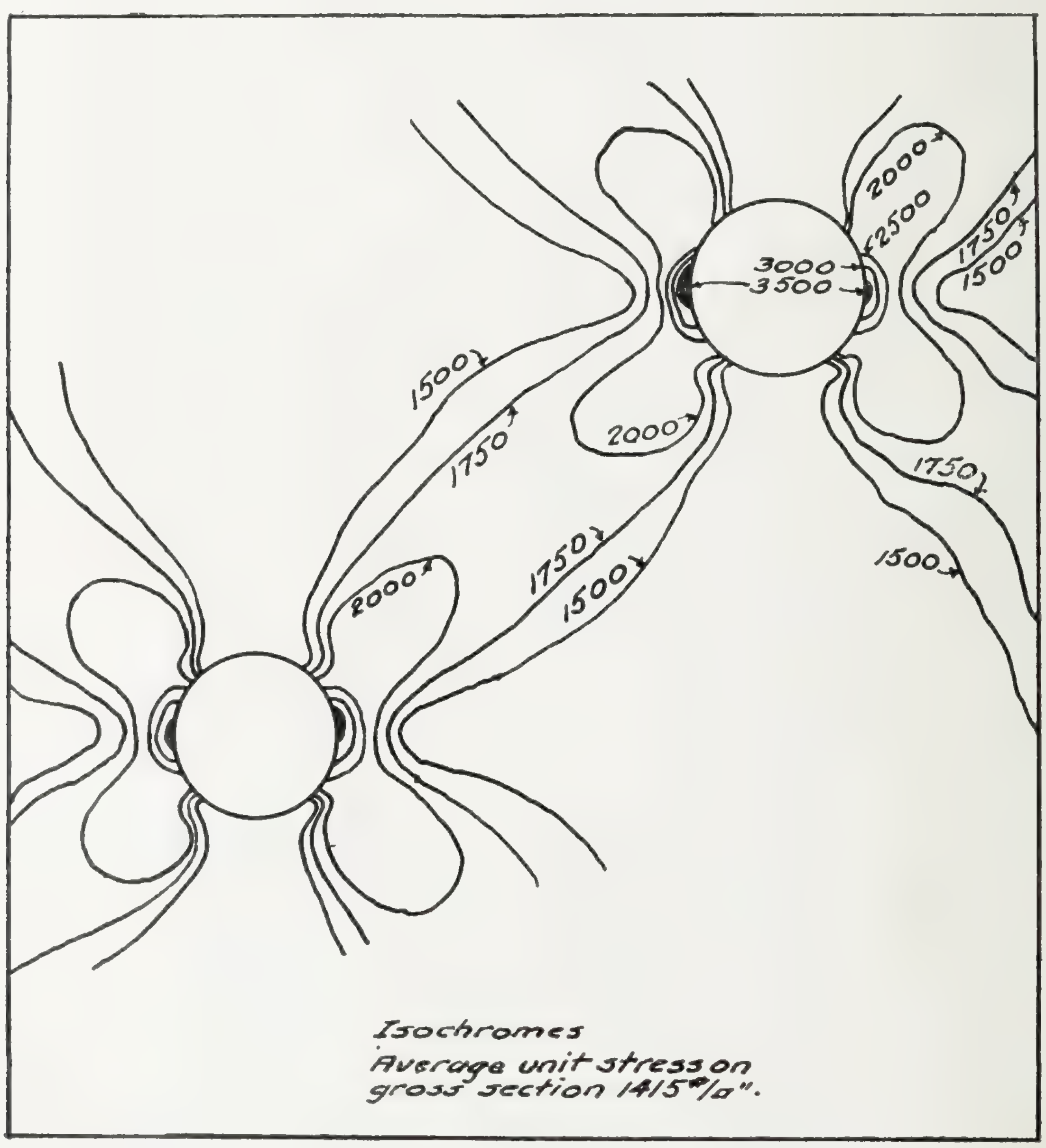


Fig. 4.

side; and, therefore, one would not expect first failure to begin on that side.

TABLE I

SPECIMENS 24″×4″×¼″; ½″ HOLES; GAUGE LINES 2″ APART

Specimen	Stagger	First Failure
A	0	47300
B	¼ in.	46800
C	½ in.	47850
D	¾ in.	47500
E	1 in.	48600

The supposition that the dangerous area was located at the edge of the rivet hole *on the edge distance side* was borne out in a few preliminary tests of steel plates. *In every case, failure commenced at that locality;* that is, with practical edge distances and rivet spacing values.

It was decided then to find out by means of a series of such tests on perforated steel plates whether or not any more pronounced law could be determined than was evident from the photo elastic method. The results of two of a series of ten tests is given in Table I. The plates used were 24 inches by 4 inches by ¼ inch with ½-inch holes in them on gauge lines 2 inches apart. These results are typical of the series. Certainly there is nothing convincing in these figures. They are the variations one might expect as being due either to the slight variations of the internal steel structure itself or to the errors in placing the specimens in the testing machine even with the greatest of care being taken.

Another series of tests is given in Table II. These are the results from using steel plates 15 inches by 3 inches by 1/8-inch. The edge distances in all cases were the same on each side for a given test, two holes 3/8-inch diameter being used for all specimens.

The variation of figures in both Table I and Table II may be taken as typical of the results obtained in all the tests. Altogether, a total of two hundred and ninety-two plates were tested, with more or less the same type of results.

There is one very curious fact that seems to be persistent although it has not any great bearing on the present problem. It will be noticed that the plates seem to be weakest not when the two rivet holes are on

TABLE II

SPECIMENS 15″×3″×1/8″; 3/8″ HOLES

Stagger	Gauge Distance								
	1 3/8	1 1/8	1 1/8	7/8	7/8	7/8	5/8	5/8	5/8
0	16540	16350	16160	16560	16340	16500	15070	16300	16440
1/8		16250	15870	15800	16290	16020		16220	16700
1/4	16550	16150	16070	16240	16250	16150	15550	16360	16590
3/8		16160	16520	16100	16700	15910		16470	16550
1/2	16340	16480	16510	16550	16300	15780	15740	16370	15920
5/8		16080	16350	16040	16340	15640		16380	
3/4	16360	16550	16460	16360	16720	16320	16380	16640	16330
1	16830	16630	16500	17130	16630	16450	16440	16860	
1 1/2	16410	16690	16570	16880	16650	16880	16240	16770	16450
2	16690	17330	16410	16500	16880	16200	16250	16650	16810

Loads in Table are First Failure loads

the same right section, but when the holes are slightly staggered. This same fact was suspected during the photo elastic investigations.

It is obvious that if the basis of considering first failure as the governing factor in the problem is admitted, that there is no necessity for a very complicated deduction formula. And what other basis of argument is required? It is this first failure which must be prevented. What happens after first failure is of no practical importance. It might be thought, of course, that the method of failure of the material after this first failure takes place would give some clue to the conditions before first failure. But the writer cannot hold with this idea. The stress distribution after first failure is so different that it is not possible to correlate the after effects with those before first failure. Fig. 7 shows the isochrome lines around one hole for a specimen of the same dimensions as that used in Fig. 5 with only one half of the load placed upon it and the edge distance cut through to the boundary of the plate. It is obvious that the failure on the edge distance side has increased the stress at the rivet hole beyond all proportion and the stress distribution in the remaining material bears no similarity to that in Fig. 5.

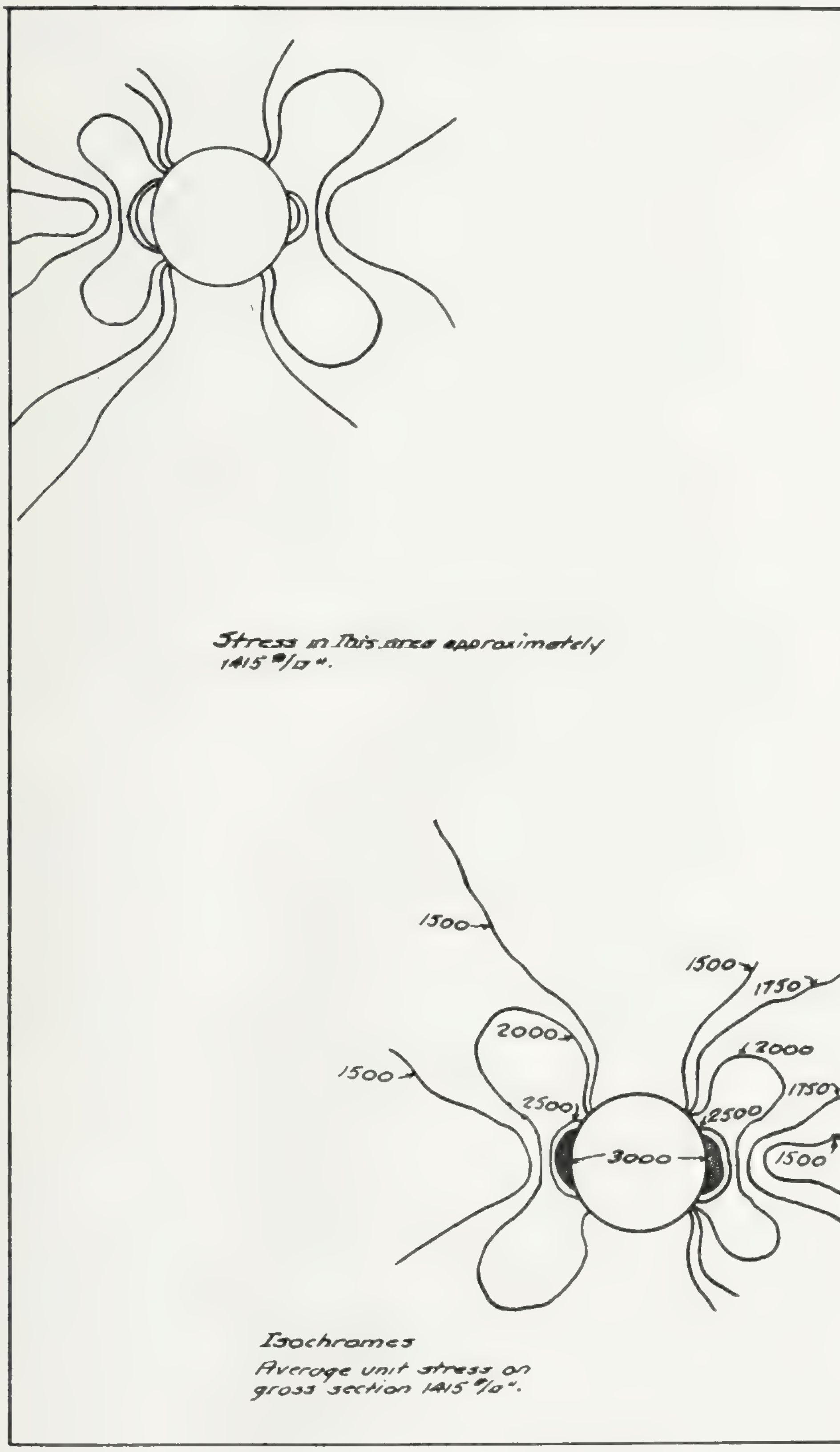


Fig. 5.

Before the tests in Table II were made, several plates of the same material with no holes in them were tested to determine their strengths. The average of these tests gave the ultimate strength as 57,000 pounds per square inch. If two 3/8-inch rivet holes be deducted from a given right cross section and the ultimate strength calculated in the customary manner assuming an average strength of 57,000 pounds per square inch, a figure of 16,100 pounds is obtained which is sufficiently close to the results of Table II for zero stagger to pass without comment. On the other hand, if merely one rivet is deducted, an ultimate strength of

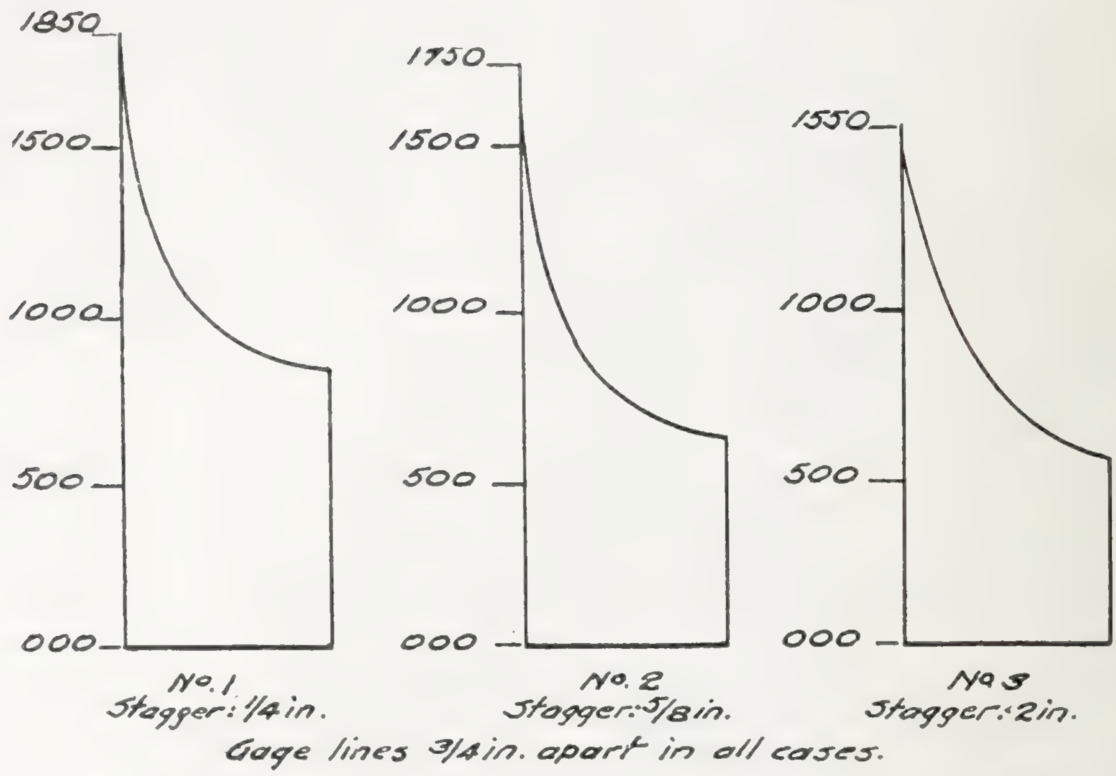


Fig. 6.

18,700 pounds is given which is considerably higher than any of the tests show for first failure. This can be explained by the high stress concentration at the rivet holes which is not very much affected by increased stagger as has been shown by the photo elastic results; and also by the fact that the ultimate failure of the riveted tension plates would be a higher value than first failure, in the neighbourhood of the holes.

One might almost feel safer after looking at these results in always deducting two rivets in order to allow for the effect of damage done to rivet holes in the shop. Certainly the practice of adding 1/8-inch to the rivet diameter in figuring deductions is a factor of safety in the right direction when the high stress in the neighbourhood of the rivet is remembered. Edge distances certainly should not be made too small.

Examining the results in Table II and other results of a similar character, the writer makes the following arbitrary observations:

(*a*) As the stagger ratio, $\frac{s}{g}$, increases in value from zero to about $\frac{6}{10}$ here is very little increase in strength of the tension member. The first failure values in Table II above the double lines lie between these limits of stagger ratio.

(*b*) There seems to be very little increase in strength of the tension member for increase of stagger ratio beyond a value of 1.6.

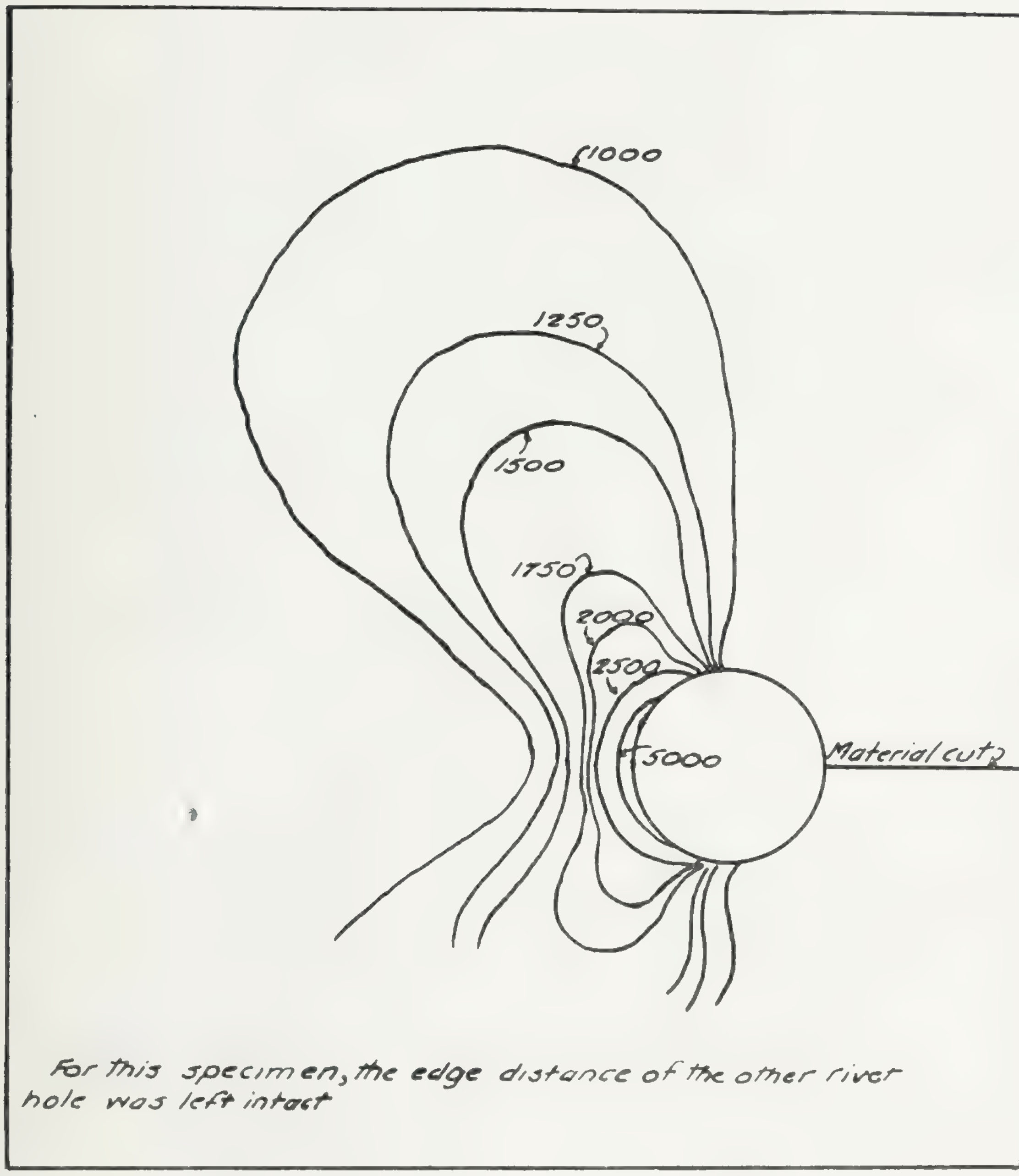


Fig. 7.

Based upon these opinions, the writer at first thought of proposing the following deduction formula:

$$x = 1.6 - \frac{s}{g}$$

where x is the deduction to be added to one rivet deduction, x in no case to exceed a value of unity.

s = the stagger in inches
g = the gauge distance in inches.

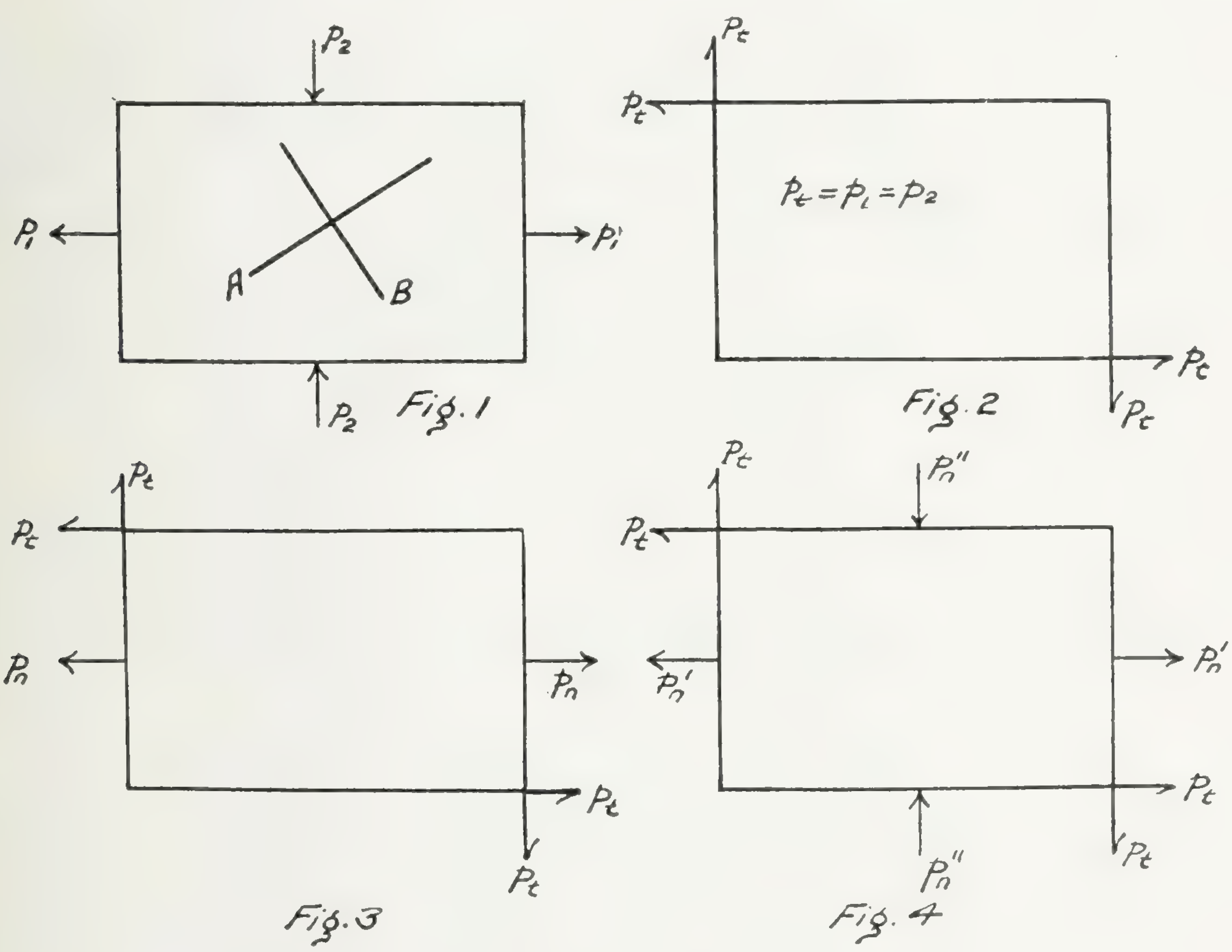
This formula, of course, gives a deduction for two rivet holes up to a considerable stagger ratio which will appear excessive to many structural designers. The fact that 1/8-inch is usually added to the rivet diameter in calculating net areas lends weight to the objection; and in view of this practice, the writer feels that a concession should be made and proposes the following formula:

$$x = 1.5 - \frac{s}{g}$$

NET SECTION OF RIVETED TENSION MEMBERS[1]

By T. R. Loudon[2]

It has been recognized for some time that many of the specifications for calculating net sections of riveted tension members are inadequate in certain respects. Considerable interest has been aroused in the subject by the discussions and results of investigations that have been published in technical journals during the last two years or so; but so far no agreement seems to have been reached.



Figs. 1 to 4

Some of the results above referred to are based upon a theoretical assumption as to stress distribution that does not seem to the writer to be quite warranted. It is true that from the theory very admirable results seem to have been arrived at in the way of deduction formulae, but this in itself is not a proof that the theory is correct.

The following discussion is submitted rather with the idea of reviewing the situation than with the intention of deducing a new theory; which, however, it is hoped to do when the results of the investigation

[1]Published in *The Canadian Engineer*, Vol. 47, No. 9.

[2]Professor of Applied Mechanics.

now being carried on in the Photo-Elastic Laboratory of the University of Toronto are completed.

In order to see more readily the points to be reviewed, the following facts relative to a certain type of compound stress should be kept in mind.

If, acting on an elementary rectangle of material, there are two principal stresses, one tension and the other compression as indicated in Fig. 1, by p_1 and p_2, then, on any other pair of planes A and B perpendicular to one another within the rectangle, there will exist certain stress conditions whose properties will depend upon the relative magnitudes of the principal stresses p_1 and p_2 and the inclinations of the planes A and B to these principal stresses.

If $p_1 = p_2$ and the planes A and B are inclined at 45° to the principal planes, then the stresses on A and B will be pure shear of magnitude equal to either p_1 or p_2, the planes A and B, Fig. 1, being taken as the sides of the elementary rectangle shown in Fig. 2.

If the inclination of the planes A and B bear a certain relation to p_1 and p_2, then on one plane there will be found a normal and a shearing stress; and on the remaining plane, merely a shearing stress as illustrated by Fig. 3. This case of stress occurs when $\frac{p_2}{p_1} = \tan^2\phi$, where the angle ϕ is the inclination to p_1 of the plane on which merely shear is found.[1]

Generally, however, the relations between p_1 and p_2 and the inclinations of the planes A and B, Fig. 1, are such that the stresses are equivalent to normal and shearing stresses on both planes A and B as indicated in Fig. 4.

Conversely, if the conditions of stress on the planes A and B, Fig. 1, are known, it is possible to determine the magnitudes of the principal stresses and the inclinations of the principal planes. But it must be definitely understood that the conditions on both planes A and B must be known before the principal stresses and planes can be determined. It is not sufficient to know merely the conditions on either plane A or B alone, for it is seen readily that the cases shown in Figs. 3 and 4, both have one plane on which there is a normal and a shearing stress, and in order to differentiate the cases, it is necessary to know the conditions on the other plane.

Keeping in mind the above facts, which illustrate the type of

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[1]If p_1 and p_2 are two principal stresses, p_1 being tension and p_2 compression, then the unit normal stress on any plane inclined at ϕ to p_1 is given by the relation:

$$f_n = p_1 \sin^2\phi - p_2 \cos^2\phi$$

If there is to be merely shear on this plane, this normal stress must be zero. Putting the above value equal to zero, the relation of p_1 to p_2 in order to have merely shear on a plane is $\frac{p_2}{p_1} = \tan^2\phi$

principal stresses likely to be found in a tension member of the section that will be examined, the theoretical analysis of riveted tension members upon which a great amount of discussion has been based may now be examined. The method is as follows:

Let Fig. 5 represent a simple flat tension member which for simplicity of argument has a thickness of unity. The diameter of the rivet holes for the purpose of deduction is taken as h (rivet diameter $+\frac{1}{8}$ of an inch).

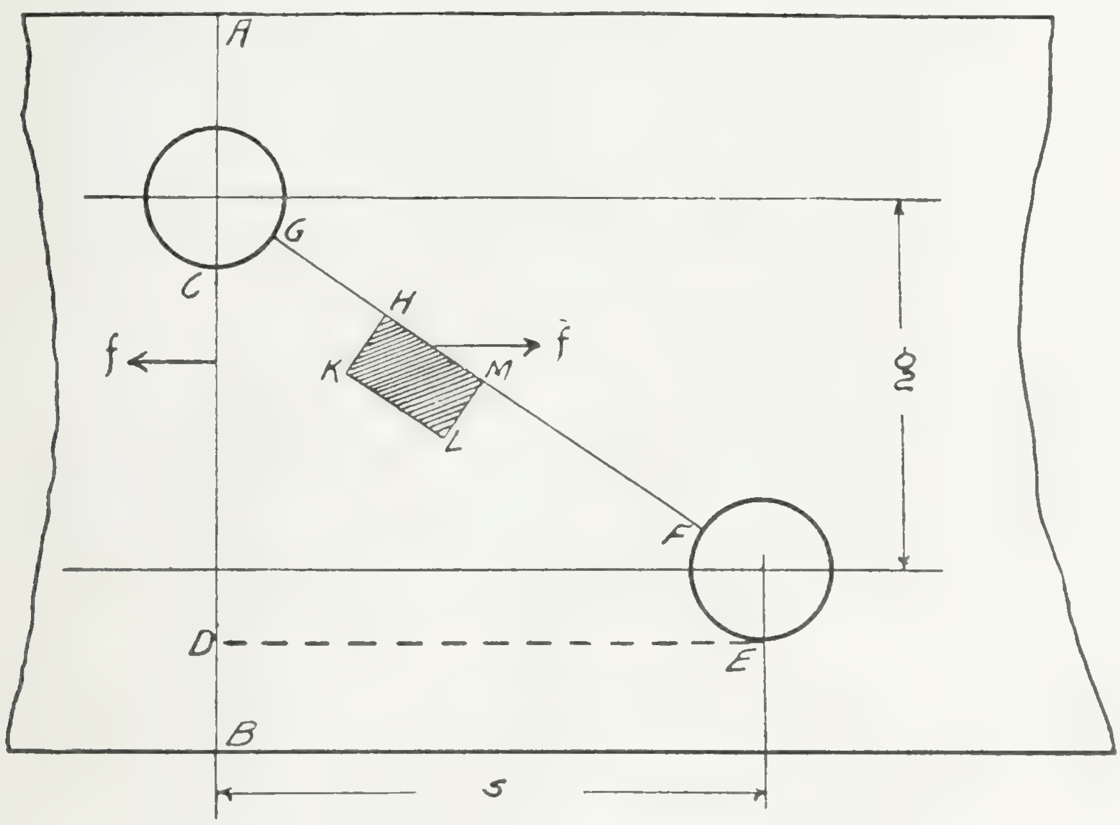

FIG. 5

Consider the equilibrium of the portion $CDEFG$, Fig. 5. If the average stress across the right section AB is f lbs. per sq. in., then the total stress on CD will be fg, where g is the gauge of the rivets. The assumption is made then that the total stress fg on CD is resisted entirely by an equal and opposite total stress acting obliquely on FG, the average of which will be

$$f^1 = \frac{fg}{\sqrt{g^2+s^2}-h}$$

This average stress f' which has been assumed as acting parallel to the longitudinal axis of the member, may be resolved perpendicularly and parallel to FG giving stresses

$$f_n = f^1 \frac{g}{\sqrt{g^2+s^2}} = \frac{fg^2}{g^2+s^2-h\sqrt{g^2+s^2}}$$

$$f_t = f_n \times \frac{s}{g} = \frac{fgs}{g^2+s^2-h\sqrt{g^2+s^2}}$$

The next step in the theory under discussion is one that does not seem clear to the writer. The procedure is to say that the following well known formula for determining maximum normal stress may be applied:

$$f_{max} = \tfrac{1}{2}[f_n + \sqrt{f_n^2 + 4f_t^2}] \qquad (1)$$

Now this formula is one that is deduced from the case illustrated in Fig. 3, where there exists a shear on one plane and a shear and a normal stress on the other plane, and is the formula by means of which the principal stresses are determined from the information of Fig. 3. But does Fig. 3 represent the stress condition that exists at section *GF*, Fig. 5?

If a small elementary rectangle be taken on *GF*, Fig. 5, as indicated on the enlarged scale by *HKLM*, it is quite clear that with the assumed resultant stress f' there is a shear and a normal stress on *HM*; but from the facts of the argument presented so far, it is not at all clear that there is merely a shear on *HK* and *ML*.

Fig. 6 represents diagrammatically the condition of stress that may exist at a given region on the diagonal *GH* between rivets. Let p_1 and p_2 be the two principal stresses at the region being discussed, which for convenience sake is enlarged to the portion *DB*. *ABCD* is an elementary rectangle formed by principal planes on which p_1 and p_2 act, and which includes the portion *DB* of the diagonal between rivets on which resultant stress f' is desired.

The inclination α of the resultant stress f' to the principal stress p_1 may be shown to be given by the relation

$$\text{Tan } \alpha = \frac{p_2}{p_1} \cot \theta \qquad (2)$$

where θ is the inclination of *DB* to p_1, or the angle *CBD* Fig. 6. Then, if the direction of f' is parallel to the longitudinal axis of the member as assumed, α is really the inclination of the principal stress p_1 to this axis of the member. This being the case, if the actual directions of the principal stresses can be found, it will be possible from (2) to determine the relation of p_1 to p_2 in order to give f' acting parallel to the longitudinal axis of the member.

Fig. 7 shows the directions of the p_1 principal stress lines for the case of a simple riveted tension bar as determined by the photo-elastic method. This diagram shows the stress lines between one pair of four rivets so spaced as to maintain the same net section diagonally between rivets as across a right section through one rivet.

Taking from Fig. 7 values of α and θ, it was found that when $\alpha = 22\tfrac{1}{2}$, $\theta = 61°$; and when $\alpha = 30°$, $\theta = 67°$, these being the two types of directions found on the diagonal section between rivets.

From this; by the use of equation (2), the following information

may be deduced relative to p_1 and p_2 in order that f' may act parallel to the longitudinal axis of the member:

$$(a) \qquad \frac{p_2}{p_1} = \frac{\tan 22\tfrac{1}{2}°}{\cot 61°} = \frac{.41}{.55} = .74$$

$$(b) \qquad \frac{p_2}{p_1} = \frac{\tan 30°}{\cot 67°} = \frac{.57}{.42} = 1.35$$

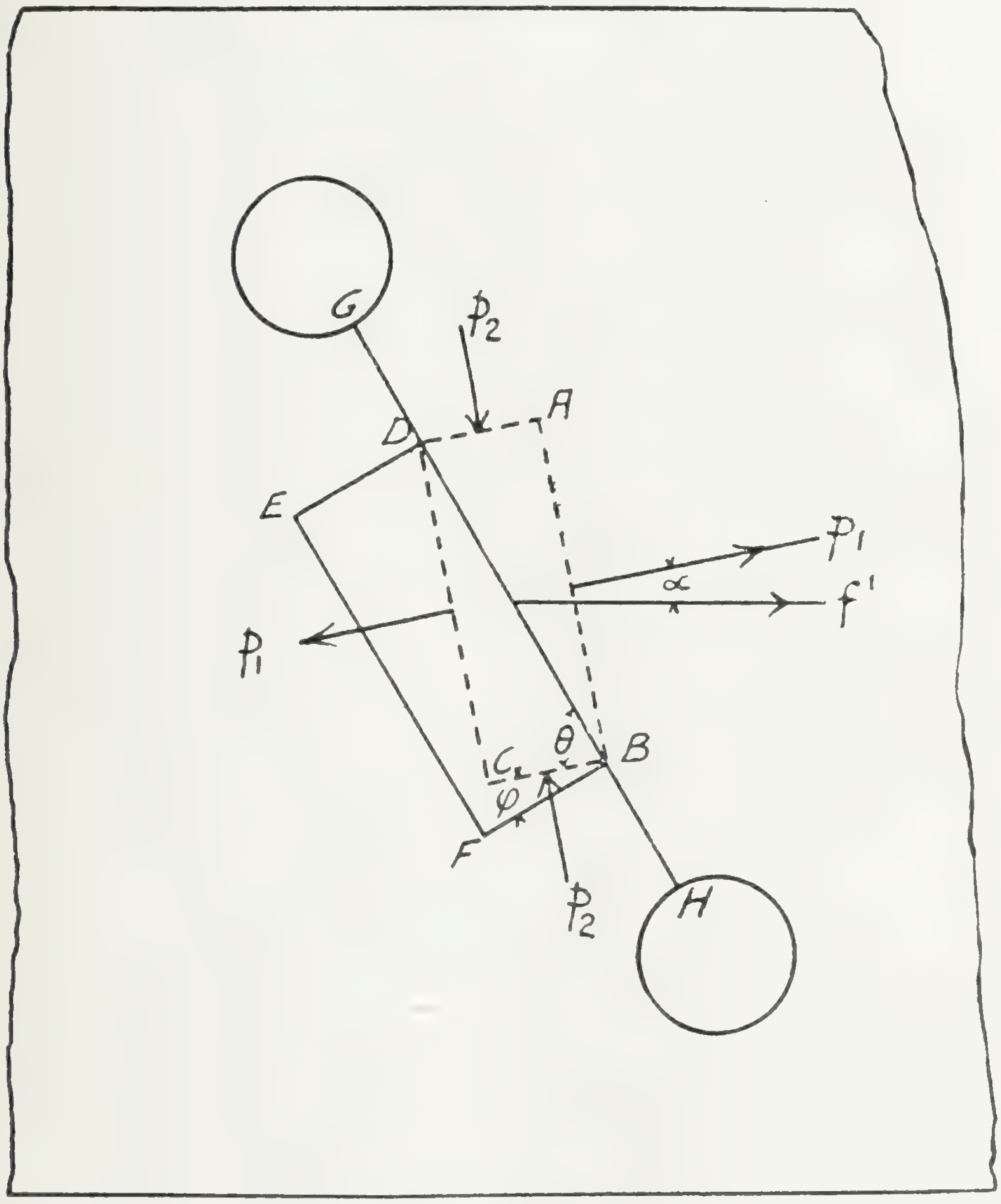

FIG. 6

Leaving this part of the discussion for the time being and turning to another phase of the problem; namely, as to whether or not the conditions of stress on the faces of an elementary rectangle *DBFE* Fig. 6 warrant the use of the formula (1) for maximum stress, another relation between p_1 and p_2 is obtained. The relation that should exist in order to justify the use of formula (1), as already stated, is:

$$\frac{p_2}{p_1} = \tan^2 \phi \qquad (3)$$

The angle ϕ, which in this case is equal to $(90° - \theta)$, is the inclination of the planes *FB* and *ED* Fig. 6 to the principal stress p_1, these being the planes on which merely shear should exist if formula (1) is to be applied. ϕ is shown in Fig. 6 as the angle *CBF*, since *CB* is parallel to p_1. Measurements of ϕ on Fig. 7, taken in the same regions as those in which α and θ were obtained for results (*a*) and (*b*), show ϕ to have values of 29° and 23° respectively, corresponding to $\alpha = 22\frac{1}{2}°$ and 30°. Putting these values of ϕ into (3), the following relations are obtained.

(*c*) $$\frac{p_2}{p_1} = \tan^2 29° = \frac{1}{3.3} = .30$$

(*d*) $$\frac{p_2}{p_1} = \tan^2 23° = \frac{1}{5.7} = .17$$

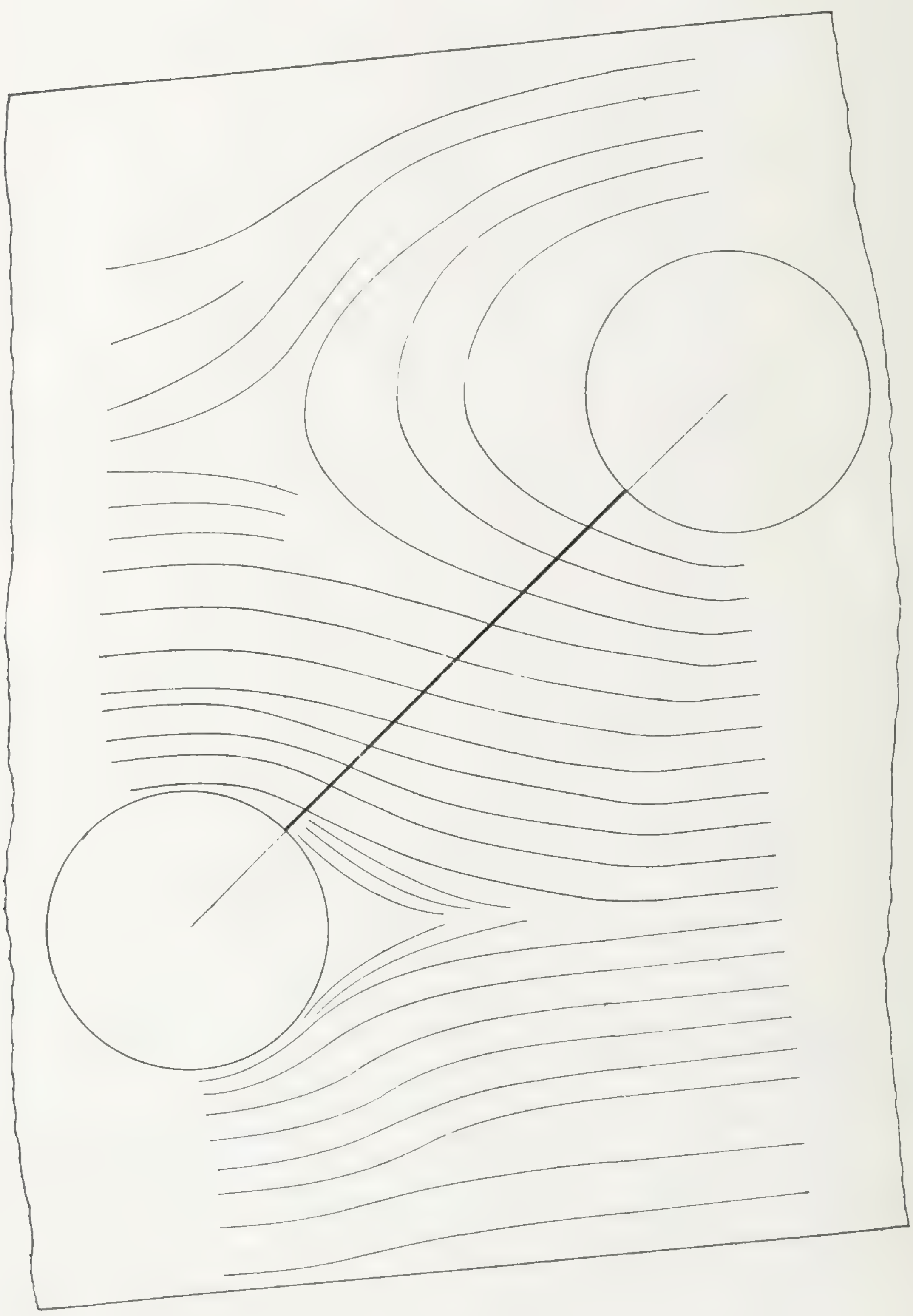

FIG. 7

Comparing results (*a*) with (*c*) and (*b*) with (*d*) since these are obtained from measurements in the same region, it is seen that there is no semblance of agreement in the relations between p_1 and p_2 for the two cases discussed. It is evident, then, that if the resultant stress f' is to act as assumed, there will not exist the condition of stress necessary to warrant the use of formula (1). On the other hand, if the conditions of principal stress are such as to allow the use of formula (1), then the direction of f' is not such as assumed. The theory, therefore, appears to be built up on contradictory assumptions.

It might be thought that Fig. 7 illustrates a case which is not typical; but an examination of other cases which are extremes of rivet pitch and guage seems to show the above conclusions to be true in all cases.

There is one other view that might be taken of the situation. It might be said that f' is the average of a total stress whose direction is as assumed, that is, parallel to the longitudinal axis of the member; and that this total stress is the resultant of a number of unit oblique stresses whose directions may or may not be parallel to the longitudinal axis of the member, but were such that the resultant of all of them had this assumed direction. It does not require much thought to see that to average such a resultant total stress over the section would be far from representing the true state of affairs. But a little investigation will show that if the relations between the principal stresses p_1 and p_2 at all points are such as called for to validate the use of formula (1), then the resultant total stress could not act parallel to the longitudinal axis of the member.

Turn the evidence whatever way one will there does not seem to be any basis on which the contradictory nature of the assumptions can be avoided.

It is doubtful, of course, if more than a rough approximation to the true result can be obtained by statical methods alone. The stress distribution in this case is highly complex and cannot be worked out by purely mathematical analysis; although with the aid of the photo-elastic method it is hoped to correlate the actual results of this stress distribution into some workable form in the near future.